April 15, 2017

How Earth Got its Moon
The article “How Earth got its moon” describes theories of how the Earth’s moon formed. Did it form as the result of a collision with a large protoplanet called Theia, or did multiple impacts create many mini moons that then merged over time? Computer modeling and isotopic analyses provide some data used to support or refute current theories. Scientists are still searching for other moon formation ideas and ways to test them.

Students can focus on details in the article, follow connections to earlier articles about the origins of Earth’s moon, explore cross-curricular connections to other major science topics and calculate relevant lunar properties for themselves. Science News for Students provides another version of this article written at a lower Lexile level (7.9 readability score): “How Earth got its moon.” Power Words are defined at the end of the Science News for Students article.

Want to introduce your students to an interesting STEM career related to this article? Check out Cool Jobs: Probing Pluto by Science News for Students.

Women’s contributions to scientific endeavors, including planetary science, have been the subject of award-winning movies this year including Hidden Figures, which focused on African-American mathematicians who were crucial to the first launch of an American into space orbit. Want to introduce your students to some other women pursuing STEM careers? Check out “A woman’s place is in science” in Science News for Students.
the articles. The section is divided roughly by science subdiscipline for educators who would like to focus on one topic area. The extension prompts for each subdiscipline include some that are topic-specific and others that are more conceptually advanced. **Earth and Space Sciences questions** address types of rocks and minerals found on Earth, how they formed and how they might provide clues to the moon’s formation. **Physical and Chemical Sciences questions** involve defining isotopes and physical applications of measuring isotope ratios. **Biological Sciences and Engineering questions** concern biomedical applications of certain isotopes and questions regarding isotopic measuring techniques and other applications.

**Activity:** Students can calculate the angular momentum and density of the moon and consider the implications of the results for various theories of the moon’s origin.

### Standards Alignment

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Directions: After reading the article “How Earth got its moon,” answer these questions:

1. What was the moon-formation idea proposed in the mid-1970s?

2. Why does the author describe Earth’s moon as an “oddball”?

3. A study in 2001 analyzed rocks collected during the Apollo mission to the moon. How did these lunar rocks support the hypothesis that the moon was formed by multiple impacts and contradict the giant-impact hypothesis?

4. What did planetary scientist Raluca Rufu and her colleagues learn recently that supports the multi-impact hypothesis? Explain how their findings support this hypothesis and why scientists were not able to figure this out previously.

5. According to planetary scientist Nicolas Dauphas, how does the isotopic combination of materials that make up Earth tell a story that supports the idea of a single impact? What does Dauphas say supplied the Earth’s mass?
6. Planetary scientist Sarah Stewart states that we need to test all the new ideas about the moon’s formation. Describe the recent test that used temperature to help explain how the moon formed, and explain which moon-formation idea is consistent with the results.

7. Explain the similarities and differences between the graphic titled “Making moons” and the computer simulation images shown below.

Making moons The multi-impact hypothesis says several small hits sent terrestrial materials into orbit that eventually formed our large moon.
1. What was the moon-formation idea proposed in the mid-1970s? Possible student response: Scientists proposed the “giant-impact hypothesis.” According to this idea, a Mars-sized body called Theia collided with the Earth 4.5 billion years ago with such force that debris from both Earth and Theia was thrown into orbit, eventually combining to form the Earth’s moon.

2. Why does the author describe Earth’s moon as an “oddball”? Possible student response: Most of the solar system’s moons orbit gas giant planets that are farther away from the sun than Earth is. Earth’s moon is large compared with the moons of Mars, the only other terrestrial planet with moons. Mars’ moons are small enough to be asteroids that were caught by its gravity, but Earth’s moon is probably too big to have been captured by gravity.

3. A study in 2001 analyzed rocks collected during the Apollo mission to the moon. How did these lunar rocks support the hypothesis that the moon was formed by multiple impacts and contradict the giant-impact hypothesis? Possible student response: The 2001 study of lunar rocks shows that Earth and its moon have identical mixes of oxygen isotopes. If the moon was a mixture of material from the two planets, the moon’s composition should be different than Earth’s.

4. What did planetary scientist Raluca Rufu and her colleagues learn recently that supports the multi-impact hypothesis? Explain how their findings support this hypothesis and why scientists were not able to figure this out previously. Possible student response: When the proposal of multiple impacts was first suggested in 1989, the computer power needed to run the necessary simulations was not available. Recently, Rufu and colleagues designed a computer simulation of multiple impacts to Earth. In the simulation, impactors that hit Earth directly transferred energy deep into Earth, sending terrestrial material into space. This material combined over centuries to form small moons, and over roughly 100 million years, about 20 small moons merged to form one moon.

5. According to planetary scientist Nicolas Dauphas, how does the isotopic combination of materials that make up Earth tell a story that supports the idea of a single impact? What does Dauphas say supplied the Earth’s mass? Possible student response: Dauphas reported that most of the material making up Earth comes from the same material as a single type of meteorite called an enstatite chondrite. The placement of different elements helped Dauphas develop a timeline showing when different types of space rocks added to the Earth’s mass. For example, iron-loving elements, such as ruthenium, sink deeper into the Earth’s crust. Therefore, any ruthenium near the Earth’s crust likely arrived later in the Earth’s development. The timeline shows that around three-fourths of Earth’s
mass came from enstatite chondrite and its precursors. If Theia formed at the same approximate distance from the sun as Earth, its composition would be similar.

6. Planetary scientist Sarah Stewart states that we need to test all the new ideas about the moon's formation. Describe the recent test that used temperature to help explain how the moon formed, and explain which moon-formation idea is consistent with the results. Possible student response: The moon's isotopic abundances were compared with that of glasses formed by a nuclear blast. Due to the extremely high temperatures created by the blast, the glasses lacked light isotopes of zinc, which had been leached out. Lunar rocks did not have zinc isotopes either, suggesting they had been exposed to extreme temperatures. Both moon-formation ideas are consistent with this hypothesis. Scientists say that either one large impact or many smaller impacts could have caused high enough temperatures to leach the light zinc isotopes from the rock.

7. Explain the similarities and differences between the graphic titled “Making moons” and the computer simulation images shown below. Possible student response: The simulation images show the result of a collision between early Earth and a Theia-like protoplanet to form a beltlike array of debris from both planets. The “Making moons” graphic also shows how a beltlike array of debris forms from an impactor. The impactor in this diagram is presumably smaller, as the diagram is outlining the multi-impact formation idea. This diagram also shows the results of subsequent impacts and moonlet merger.

Making moons The multi-impact hypothesis says several small hits sent terrestrial materials into orbit that eventually formed our large moon.
How Earth Got its Moon

Quest Through the Archives

Directions: After reading the article “How Earth got its moon,” use the archives at www.sciencenews.org to answer these questions:

1. Isotopic analysis is one tool for comparing the origin of different materials. Isotopes of elements are also used in radioactive dating to determine the age of materials and artifacts. Search for another article, in addition to “How Earth got its moon,” that discusses how researchers use isotopes to help determine when Earth’s moon formed. What isotopes did the article mention were used for dating purposes?

2. The Apollo missions provided samples of moon material that helped researchers form hypotheses about the moon’s origin. Search for an article about the return of the Apollo 11 mission. What information about the moon was determined from collected materials?

3. Search for another article that discusses the moon’s composition and explain it.
Responses to Quest Through the Archives

1. Isotopic analysis is one tool for comparing the origin of different materials. Isotopes of elements are also used in radioactive dating to determine the age of materials and artifacts. Search for another article, in addition to “How Earth got its moon,” that discusses how researchers use isotopes to help determine when Earth's moon formed. What isotopes did the article mention were used for dating purposes? Possible student response: “The moon is still old,” published 1/11/2017, says that researchers examined the isotopic composition of hafnium as well amounts of lead, uranium and lutetium contained in the mineral zircon found in samples that Apollo 14 astronauts brought back from the moon.

2. The Apollo missions provided samples of moon material that helped researchers form hypotheses about the moon's origin. Search for an article about the return of the Apollo 11 mission. What information about the moon was determined from collected materials? Possible student response: “Apollo returns: the work begins,” published 8/2/1969, discusses scientists’ agreement that the rocks were igneous and probably formed from the heat of meteor impacts or possibly by volcanic activity. Dissimilar to the concentration of titanium found in Earth, large amounts of titanium were observed from moon rock. Glasslike beads were abundant and suggested that meteorite impacts disturbed silicon-rich deposits on the moon's surface.

3. Search for another article that discusses the moon's composition and explain it. Possible student response: “Iron-loving elements tell stories of Earth's history,” published 7/27/2016, discusses the idea that rare elements such as gold, platinum and iridium are attracted to Earth's iron core. Studying these iron-loving, or siderophile, elements may help scientists determine how the Earth's core formed.
Cross-Curricular Discussion

After students have had a chance to review the article “How Earth got its moon,” lead a classroom discussion based on the questions that follow. You can copy and paste only the questions that apply to your classroom into a different document for your students.

Here is other relevant Science News journalism that your students can explore:

Science News articles:
- Two-stage process formed moon, simulations suggest
- Fast-spinning young Earth pulled the moon into a yo-yo orbit
- Moon’s origins revealed in rocks’ chemistry
- Moon material on Earth

Science News for Students articles:
- Student programs computer to predict path of space trash (6.4 readability score)
- Comet probe may shed light on Earth’s past (7.5 readability score)

EARTH AND SPACE SCIENCES

Discussion Questions:

1. List as many possible hypotheses as you can think of for how the moon formed and how it ended up in its current orbit around Earth. Discuss the evidence that would support or refute each hypothesis.

[Students may have other ideas, or variations or combinations of the ideas listed below.]

a. The moon formed far away from Earth and was later captured by Earth’s gravity. The moon’s composition is similar to that of Earth, so the moon would have coalesced from material extremely similar to that from which Earth coalesced. On the other hand, some scientists think that Theia and/or asteroids that hit Earth could have mostly been formed from the same type of material. It is also very challenging to imagine that the incoming moon would not pass by Earth without being captured, and not collide with Earth, or that it would end up where it is today.

b. The moon and Earth coalesced from the same localized clump of debris orbiting the recently formed sun, with the moon and Earth orbiting together. This would account for the nearly identical composition,
but how would a debris clump have so much angular momentum that it would divide into two objects and not form one?

c. While a clump of debris orbiting the sun was still combining to form Earth, it was disrupted by something else (like a passing planet), causing the debris to combine into two bodies (Earth and the moon) instead of just one. This idea might account for the very similar compositions, the coalescence into two bodies and not one, and the lack of evidence for a collision. On the other hand, it does presume that a planet happened along at exactly the right time and on exactly the right trajectory to perturb the still-coalescing proto-Earth.

d. Earth formed and was spinning so fast that it flung off part of itself to form the moon. As will be calculated in the activity later, Earth would have to have been spinning far faster than it is now, and in fact so fast that it would have torn itself apart or never would have coalesced in the first place.

e. Earth formed and then was hit by a large object (the protoplanet Theia); the collision produced the current Earth and moon. As noted in the article, to make Earth and the moon so similar to each other in composition, Theia would have to have been extremely similar to Earth. Alternatively, the Theia-Earth collision would have to have mixed the contents of those two initial bodies so well that the resulting moon and Earth ended up with nearly identical ratios of the same materials, instead of having one that was more like Theia and one that was more like proto-Earth.

f. Earth formed and then was hit by a succession of impactors that gradually skimmed off enough material that combined to form the moon. As noted in the article, this could account for the extreme similarity of Earth and the moon. On the other hand, if Earth got hit that many times, why didn’t Mercury, Venus and Mars? And why did all of Earth’s mini moons combine into just one moon?

Extension Prompts:
2. How did Earth form? [Scientists believe that Earth formed at the beginning of the Hadean Eon, beginning about 4.5 billion years ago. Collisions occurred between cosmic objects — such as asteroids and larger planetesimals — within a spinning cloud of dust and gases called an accretion disk. At the center of this spinning disk was the previously formed sun. Other pieces in the mix continued to collide, forming early molten planets including Earth. As the molten material cooled, the iron in the mix sank to the center of the mass and formed a core, while lighter elements floated to the surface. The oldest known zircon crystals on Earth date back about 4.4 billion years.]

3. What are the three main types of rocks on Earth and how are they formed? [Rocks are formed during what’s known as the rock cycle. Specifically, there are three types of rocks that are formed by different processes. Igneous rocks form from cooled lava or magma. Depending on whether the molten material cools inside the Earth or above the surface, the rock will have different characteristics. Sedimentary rocks are formed from particles of shell, sand and other sediments that get compressed in layers over time. Fossils are commonly found in sedimentary rock. Metamorphic rocks form inside the Earth when igneous, sedimentary or other metamorphic rocks undergo physical and chemical changes due to pressure and exposure to]
extreme heat. Metamorphic rocks frequently have ribbon-like layers and may have crystals growing as part of their composition. Through processes such as weathering and erosion, the rock cycle explains how rock masses are moved around, and eventually melt back into magma when edges of the Earth's plates move and sink into the mantle.]

4. How do plate tectonics on Earth differ from geologic activity on the moon? [Pieces of the Earth's crust and upper mantle, together called the lithosphere, move over time. The movement of these plates, known as tectonic plates, slowly changes the surface of the planet. As convection currents occur in the molten material of the mantle, causing hot rocks to rise and cooler rocks to sink, the tectonic plates move. Larger terrestrial planets, including Earth, Venus and Mars, have active inner molten layers, so their surfaces continue to deform (though not all have plate tectonics). Mercury and our moon, however, are smaller and have cooled enough to be considered tectonically inactive. It is believed that the moon has been tectonically inactive for the last 3 billion years.]

5. How do geologic layers help create a timeline? [A basic law of geology called the Law of Superposition, or geochronology, says that in undisturbed layers of rock, the oldest layer or strata will be laid down first. Since the rock that forms on Earth's surface is mostly sedimentary, the rocks are in layers that were laid down at different times. These layers can give clues about geologic events, like volcanic eruptions, faults and folds; show the results of climatic changes, such as floods and freezes; and give us fossils that reveal clues to biological events. Since the placement of the layer relatively dates its formation (with older rocks buried deeper), these layers can serve as a timeline of the geologic history of the planet.]

Earth and Space Sciences Question Bank

List as many possible hypotheses as you can think of for how the moon formed and how it ended up in its current orbit around Earth. Discuss the evidence that would support or refute each hypothesis.

How did Earth form?

What are the three main types of rocks on Earth and how are they formed?

How do plate tectonics on Earth differ from geologic activity on the moon?

How do geologic layers help create a timeline?

PHYSICAL AND CHEMICAL SCIENCES

Discussion Questions:

1. What are isotopes? What is meant by the term isotopic abundance? What does it mean when isotopes are called stable or unstable? [Isotopes are atoms of the same element that have the same number of protons but differing numbers of neutrons. Therefore, isotopes are atoms of the same element with a different atomic mass. Most elements in nature contain mixtures of different isotopes in relatively set percentages. The set percentage of an isotope with a specific atomic mass is called the isotopic abundance. Unstable isotopes undergo nuclear decay over time and emit radiation, while stable isotopes do not.]
2. What are the stable isotopes of oxygen? Name them and give their approximate abundances in Earth's atmosphere as a percent. [By definition, oxygen atoms contain 8 protons (and 8 electrons if they are not ionized or combined with other atoms). Different isotopes have different numbers of neutrons. The stable isotopes of oxygen (those that do not radioactively decay) are: Oxygen-16 (^{16}\text{O}, 8\text{ protons} + 8\text{ neutrons} = 16\text{ amu}), which makes up approximately 99.76% of oxygen in Earth's atmosphere. Oxygen-17 (^{17}\text{O}, 8\text{ protons} + 9\text{ neutrons} = 17\text{ amu}) makes up approximately 0.04% of oxygen in Earth's atmosphere. Oxygen-18 (^{18}\text{O}, 8\text{ protons} + 10\text{ neutrons} = 18\text{ amu}) makes up approximately 0.20% of oxygen in Earth's atmosphere.]

3. Why aren't radioactive oxygen isotopes relevant for comparing rock samples from different celestial bodies? [Oxygen isotopes other than ^{16}\text{O}, ^{17}\text{O} and ^{18}\text{O} are unstable with half-lives of minutes, seconds or even less, so they would not be found in ancient rock samples as they would have already undergone nuclear decay to become another element.]

Extension Prompts:

4. Why would rocks that have been through different processes, or formed on different celestial bodies, have different oxygen ratios? [Because of the extra neutrons, ^{17}\text{O} and ^{18}\text{O} are more massive than ^{16}\text{O}. That makes them, or molecules containing them, move at a slower relative average velocity and less likely to diffuse, evaporate or boil away when they are heated. Oxygen isotope ratios in a sample vary depending on what physical processes the sample has been through. Note that the article also mentions using the same general principle with isotopes of other elements such as zinc. Though isotopes of an element tend to have similar chemical properties (the same number of electrons, for instance), they have different physical properties.]

PHYSICAL AND CHEMICAL SCIENCES QUESTION BANK
What are isotopes? What is meant by the term isotopic abundance? What does it mean when isotopes are called stable or unstable?

What are the stable isotopes of oxygen? Name them and give their approximate abundances in Earth's atmosphere as a percent.

Why aren't radioactive oxygen isotopes relevant for comparing rock samples from different celestial bodies?

Why would rocks that have been through different processes, or formed on different celestial bodies, have different oxygen ratios?

BIOLOGICAL SCIENCES AND ENGINEERING
Discussion Questions:

1. What would be some biomedical applications of unstable oxygen isotopes? [Unstable isotopes would react chemically just like regular oxygen, but would be radioactive and emit detectable radiation. The longest-lived radioactive oxygen isotope is oxygen-15 (^{15}\text{O}, 8\text{ protons} + 7\text{ neutrons} = 15\text{ amu}), which has a
half-life of 122 seconds and emits positrons (antimatter electrons). The positrons make $^{15}$O very useful for positron emission tomography (PET), which uses positrons to produce three-dimensional images showing the activity of tissues and organs. Oxygen-15 could also be used to label and track various oxygen-containing biomolecules and their reactions, but due to its very short half-life, you would have to do an experiment quickly!

2. **How can computer simulations reveal surprises in science?** [Computer simulations reproduce the behavior of a system using programmed algorithms. Scientists can use computer simulations to test new drugs that are not ready for animal testing, for example, and gain insight on their effectiveness. Modeling systems with a computer allows scientists to watch how a process plays out based on different starting conditions, which can provide new and surprising information that wasn’t in the original algorithms.]

Extension Prompts:

3. **Generally, what technology is used to measure isotopes and their relative abundances in a sample?** [Mass spectrometers are used to measure isotopic composition. These machines separate different isotopes based on their mass-to-charge ratio, typically by accelerating ions through an electric or magnetic field.]

4. **Other than answering questions of planetary formation, what are other applications of measuring ratios of certain isotopes?** [Answers will vary here due to the vast number of applications. Answers may include applications in the following fields: archaeology, forensic science, other geologic applications, climatology, hydrology and ecology.]

**Biological Sciences and Engineering Question Bank**

What would be some biomedical applications of unstable oxygen isotopes?

How can computer simulations reveal surprises in science?

Generally, what technology is used to measure isotopes and their relative abundances in a sample?

Other than answering questions of planetary formation, what are other applications of measuring ratios of certain isotopes?
Teacher Guide: Goodnight Moon

Class time: 30-50 minutes.

Purpose: To calculate the conditions that would have been required for the moon to “spin off” from Earth and end up with its current orbital angular momentum, and to consider the implications of those calculations for hypotheses about the moon's origin.

Background: The article “How Earth got its moon” discusses several hypotheses of how the moon formed. One idea in the 1800s was that perhaps Earth was spinning faster after it formed, and that material flew off Earth “like children tossed from an out-of-control merry-go-round” and then merged to form the moon. But how fast would Earth need to spin for this to be true? And would the moon end up rotating with the same angular momentum that it has today in this scenario? Angular momentum measures the tendency of a body that is rotating to keep rotating.

After students have reviewed the article “How Earth got its moon,” pass out the accompanying student guide, Blackline Master 3. Students may benefit from access to calculators with scientific notation capability. Also, please discuss with your students that approximations are being made in the calculations for simplicity.

Depending on the level and background of your students, you may want to review the concepts below beforehand. The equations required to answer each question are included in the student guide. Feel free to select only the questions that relate to your class; however, once all of the given calculations are complete, students will be able to relate their answers to moon formation hypotheses. Another option is to give your students some of the equations and require them to remember or derive the rest.

This exercise uses the following skills:

- Scientific notation
- Converting between kilometers and meters
- Converting between days, hours, minutes and seconds
- Taking the square root of a number
- Calculating the square of a number
- Calculating the circumference of a circle
- Calculating the volume of a sphere
- Calculating density
- Calculating gravitational acceleration
- Calculating centrifugal (or centripetal) acceleration
- Calculating the velocity of a circular orbit
- Calculating angular momentum
Instructions: In this exercise, students will test the “spin off” hypothesis by calculating basic properties of Earth and the moon and then using them to calculate the orbital velocity of the moon and its angular momentum. They will use this information to calculate how much faster Earth would have to be spinning for material to be flung off it (the “breakaway velocity”). Finally, students will examine how that spin rate would affect Earth and decide whether the hypothesis is supported.

Here are some assumptions that are made in this exercise: The moon is in an approximately circular orbit around Earth. The Earth is much more massive than the moon, so a reasonable approximation is to treat the moon as orbiting around the center of Earth. (In fact, the orbit is slightly elliptical, and the Earth wobbles a bit since it is not infinitely more massive than the moon, but we are approximating here for simplicity.)

Here are some important values:

- Mass of Earth = $m_{\text{Earth}} \approx 5.97 \times 10^{24}$ kg
- Mass of the moon = $m_{\text{moon}} \approx 7.34 \times 10^{22}$ kg
- Radius of the moon = $r_{\text{moon}} \approx 1740$ km
- Radius of Earth = $r_{\text{Earth}} \approx 6370$ km
- Radius of the moon’s orbit = $r_{\text{orbit}} \approx 384,000$ km
- Newton’s gravitational constant = $G \approx 6.67 \times 10^{-11}$ kg

Questions:

1. In terms of radius, how small is the moon compared with Earth?

   $$\frac{r_{\text{moon}}}{r_{\text{Earth}}} = 0.273 = 27.3\%, \text{ The moon’s radius is 27.3\% that of Earth.}$$

2. In terms of mass, how small is the moon compared with Earth?

   $$\frac{m_{\text{moon}}}{m_{\text{Earth}}} = 0.0123 = 1.23\%, \text{ The moon’s mass is 1.23\% that of Earth.}$$

3. What is the volume of Earth?

   $$V_{\text{Earth}} = \frac{4}{3} \pi r_{\text{Earth}}^3 = 1.08 \times 10^{21} m^3$$

4. What is the average density of Earth? How does that compare with the density of water (1,000 kg/m$^3$)?

   $$\frac{m_{\text{Earth}}}{V_{\text{Earth}}} = 5.51 \times 10^3 \text{ kg/m}^3 = 5.51 \text{ times the density of water}$$

5. What is the volume of the moon?

   $$V_{\text{moon}} = \frac{4}{3} \pi r_{\text{moon}}^3 = 2.21 \times 10^{19} m^3$$

6. What is the average density of the moon? How does that compare with the density of water (1,000 kg/m$^3$)?

   $$\frac{m_{\text{moon}}}{V_{\text{moon}}} = 3.33 \times 10^3 \text{ kg/m}^3 = 3.34 \text{ times the density of water}$$
7. How does the average density of the moon compare with that of Earth?

\[
\frac{\text{moon density}}{\text{Earth density}} \approx 0.604 \approx 60.4\%
\]

8. Why do you think there is a density difference?

*Earth and the moon are composed of different elements or have different percentages of elementary components. For example, Earth has a much larger iron core than the moon does.*

9. What is the radius of the moon's orbit using meters and scientific notation?

\[r_{\text{orbit}} \approx 3.84 \times 10^5 \text{ km} = 3.84 \times 10^8 \text{ m}\]

10. What is the circumference of the moon's orbit around Earth?

\[2 \pi r_{\text{orbit}} \approx 2.41 \times 10^9 \text{ m}\]

11. For a stable circular orbit of the moon around Earth, the inward gravitational acceleration of the moon toward Earth \((G m_{\text{Earth}}/ r_{\text{orbit}}^2)\) must balance the outward centrifugal acceleration of the moon \((v_{\text{moon}}^2/ r_{\text{orbit}})\). Assuming this is true, solve the equation for the orbital velocity of the moon.

\[
v_{\text{moon}}^2/ r_{\text{orbit}} = G m_{\text{Earth}}/ r_{\text{orbit}}^2, \quad \text{or}
\]

\[v_{\text{moon}} = \sqrt{G m_{\text{Earth}}/ r_{\text{orbit}}}
\]

12. From the equation you solved above, what is the orbital velocity of the moon around Earth?

\[v_{\text{moon}} \approx 1.02 \times 10^3 \text{ m/sec} = 1.02 \text{ km/sec}\]

13. One orbit of the moon around Earth takes \(t_{\text{orbit}} \approx 27.3\) days. How many seconds is that?

\[t_{\text{orbit}} \approx 2.36 \times 10^6 \text{ sec}\]

14. Using the circumference of the moon's orbit and the time required for one orbit, what is the orbital velocity of the moon around Earth?

\[v_{\text{moon}} = \frac{2 \pi r_{\text{orbit}}}{t_{\text{orbit}}} \approx 1.02 \times 10^3 \text{ m/sec} = 1.02 \text{ km/sec}\]

15. Do your answers for questions 12 and 14 agree?

*They should match at least to about three significant figures.*

16. What is the angular momentum of the moon, \(L_{\text{moon}}\), due to its orbit around Earth?

\[L_{\text{moon}} = m_{\text{moon}} v_{\text{orbit}} r_{\text{orbit}}
\]

\[= 2.87 \times 10^{34} \text{ kg m}^2/\text{sec}\]

17. Angular momentum is conserved or stays constant unless an opposing force is present. If the moon had the same angular momentum, \(L_{\text{moon}}\), that you just calculated but the entire mass of the moon were concentrated at Earth's surface \(r_{\text{Earth}}\) instead of the usual distance \(r_{\text{orbit}}\) from Earth, what would the moon's velocity, \(v\), be? This velocity can represent the breakaway velocity of the moon.

\[v_{\text{breakaway}} = \frac{L_{\text{moon}}}{(m_{\text{moon}} r_{\text{Earth}})} \approx 6.15 \times 10^4 \text{ m/sec} = 61.5 \text{ km/sec}\]
18. What is the circumference of Earth?

\[ 2 \pi r_{\text{Earth}} = 4.00 \times 10^7 \text{ m} = 40,000 \text{ km} \]

19. Earth makes one rotation in \( t_{\text{Earth}} = 24 \text{ hours} \). How many seconds is that?

\[ t_{\text{Earth}} = 8.64 \times 10^4 \text{ sec} \]

20. What is the velocity of the surface of Earth at the equator, due to Earth’s rotation?

\[ v_{\text{Earth}} = \frac{(2 \pi r_{\text{Earth}})}{t_{\text{Earth}}} = 463 \text{ m/sec} \]

21. If the moon were joined to Earth but just on the verge of breaking away from Earth’s surface, how much faster would Earth have to be moving at the equator for the moon to have the correct angular momentum it ended up with?

\[ \frac{v_{\text{breakaway}}}{v_{\text{Earth}}} \approx 133 \text{ times faster} \]

22. If the surface of Earth were moving fast enough at the equator for the breakaway moon to have that much angular momentum, how long would it take for Earth to make one complete rotation?

\[ 24 \text{ hours} \left( \frac{v_{\text{Earth}}}{v_{\text{breakaway}}} \right) = 0.180 \text{ hours} \approx 10.8 \text{ min} \]

23. What is the inward gravitational acceleration at Earth’s surface?

\[ 1 \text{ gee} = G \frac{m_{\text{Earth}}}{r_{\text{Earth}}^2} = 9.81 \text{ m/sec}^2 \]

24. At Earth’s normal speed of rotation, what is the outward centrifugal acceleration at Earth’s equator?

\[ \frac{v_{\text{Earth}}^2}{r_{\text{Earth}}} \approx 0.0337 \text{ m/sec}^2 \]

25. What is the net acceleration a person feels when standing at Earth’s equator?

\[ 9.81 \text{ m/sec}^2 \text{ inward} - 0.0337 \text{ m/sec}^2 \text{ outward} = 9.78 \text{ m/sec}^2 \text{ inward} = 0.997 \text{ g} \]

26. If the Earth were rotating fast enough for the moon to spin off it with its correct angular momentum, what would be the outward centrifugal acceleration at Earth’s equator?

\[ \frac{v_{\text{breakaway}}^2}{r_{\text{Earth}}} \approx 594 \text{ m/sec}^2 \]

27. If Earth were rotating that fast, what would be the net acceleration a person would feel when standing at Earth’s equator? What would happen to Earth?

\[ 594 \text{ m/sec}^2 \text{ outward} - 9.81 \text{ m/sec}^2 \text{ inward} = 584 \text{ m/sec}^2 \text{ outward} = 59.5 \text{ g outward} \]

Everything would be flung off Earth, including the material making up Earth.

28. What do your calculations tell you about the validity of various hypotheses for the origin of the moon?

Possible student responses: The moon could not have been formed by a fast-spinning young Earth because the planet would have had to be spinning so fast to shed the material for the moon that much more material would have been flung from it.

Left on its own, the newly formed Earth would presumably not be rotating fast enough to spin off the moon.
One or more objects must have arrived carrying lots of angular momentum that ended up in the moon's orbit.

The entire moon could have come from elsewhere and entered into orbit around Earth with enough angular momentum, except it would likely be very difficult for Earth to capture the moon with its gravity.

If one or more objects hit Earth right in the center, they would not impart any angular momentum — it would give Earth a shove but not a spin.

If one or more objects came flying in, hit Earth off-center and then became the moon (possibly with some Earth material too), the moon could have the necessary angular momentum. Thus from our simplified calculations, this sort of scenario could explain the moon's origin and angular momentum.

Based on the average densities of Earth and the moon, Earth retained much of the dense iron core material from the proto-Earth, Theia or whatever else collided with it.
The article "How Earth got its moon" discusses several hypotheses of how the moon formed. One idea in the 1800s was that perhaps Earth was spinning faster after it formed, and that material flew off Earth "like children tossed from an out-of-control merry-go-round" and then merged to form the moon. But how fast would Earth need to spin for this to be true? And would the moon end up rotating with the same angular momentum that it has today in this scenario? Angular momentum measures the tendency of a body that is rotating to keep rotating.

Here are some assumptions that are made in this exercise: The moon is in an approximately circular orbit around Earth. The Earth is much more massive than the moon, so a reasonable approximation is to treat the moon as orbiting around the center of Earth. (In fact, the orbit is slightly elliptical, and the Earth wobbles a bit since it is not infinitely more massive than the moon, but we are approximating here for simplicity.)

Here are some important values:

- Mass of Earth = \( m_{\text{Earth}} \approx 5.97 \times 10^{24} \text{ kg} \)
- Mass of the moon = \( m_{\text{moon}} \approx 7.34 \times 10^{22} \text{ kg} \)
- Radius of the moon = \( r_{\text{moon}} \approx 1740 \text{ km} \)
- Radius of Earth = \( r_{\text{Earth}} \approx 6370 \text{ km} \)
- Radius of the moon's orbit = \( r_{\text{orbit}} \approx 384,000 \text{ km} \)
- Newton's gravitational constant = \( G \approx 6.67 \times 10^{-11} \text{ kg} \)

Questions:

1. In terms of radius, how small is the moon compared with Earth?
   \[
   r_{\text{moon}} / r_{\text{Earth}} \approx
   \]

2. In terms of mass, how small is the moon compared with Earth?
   \[
   m_{\text{moon}} / m_{\text{Earth}} \approx
   \]

3. What is the volume of Earth?
   \[
   V_{\text{Earth}} = (4/3) \pi r_{\text{Earth}}^3 \approx
   \]
4. What is the average density of Earth? How does that compare with the density of water (1,000 kg/m³)?
   \[ \frac{m_{\text{Earth}}}{V_{\text{Earth}}} = \]

5. What is the volume of the moon?
   \[ V_{\text{moon}} = \frac{4}{3} \pi r_{\text{moon}}^3 \approx \]

6. What is the average density of the moon? How does that compare with the density of water (1,000 kg/m³)?
   \[ \frac{m_{\text{moon}}}{V_{\text{moon}}} = \]

7. How does the average density of the moon compare with that of Earth?
   \[ \frac{\text{moon density}}{\text{Earth density}} \approx \]

8. Why do you think there is a density difference?

9. What is the radius of the moon’s orbit using meters and scientific notation?
   \[ r_{\text{orbit}} \approx \]

10. What is the circumference of the moon’s orbit around Earth?
    \[ 2 \pi r_{\text{orbit}} \approx \]

11. For a stable circular orbit of the moon around Earth, the inward gravitational acceleration of the moon toward Earth (G m_{\text{Earth}}/r_{\text{orbit}}^2) must balance the outward centrifugal acceleration of the moon (v_{\text{moon}}^2/r_{\text{orbit}}). Assuming this is true, solve the equation for the orbital velocity of the moon.

12. From the equation you solved above, what is the orbital velocity of the moon around Earth?
    \[ v_{\text{moon}} \approx \]

13. One orbit of the moon around Earth takes \( t_{\text{orbit}} \approx 27.3 \text{ days} \). How many seconds is that?
    \[ t_{\text{orbit}} \approx \]

14. Using the circumference of the moon’s orbit and the time required for one orbit, what is the orbital velocity of the moon around Earth?
    \[ v_{\text{moon}} = \frac{2 \pi r_{\text{orbit}}}{t_{\text{orbit}}} \approx \]

15. Do your answers for questions 12 and 14 agree?
16. What is the angular momentum of the moon, \( L_{\text{moon}} \), due to its orbit around Earth?

\[ L_{\text{moon}} = m_{\text{moon}} v_{\text{orbit}} r_{\text{orbit}} \]

17. Angular momentum is conserved or stays constant unless an opposing force is present. If the moon had the same angular momentum, \( L_{\text{moon}} \), that you just calculated but the entire mass of the moon were concentrated at Earth’s surface \( r_{\text{Earth}} \) instead of the usual distance \( r_{\text{orbit}} \) from Earth, what would the moon’s velocity, \( v \), be? This velocity can represent the breakaway velocity of the moon.

\[ v_{\text{breakaway}} = \frac{L_{\text{moon}}}{(m_{\text{moon}} r_{\text{Earth}})} \]

18. What is the circumference of Earth?

\[ 2 \pi r_{\text{Earth}} \]

19. The Earth makes one rotation in \( t_{\text{Earth}} = 24 \) hours. How many seconds is that?

\[ t_{\text{Earth}} = \]

20. What is the velocity of the surface of Earth at the equator, due to Earth’s rotation?

\[ v_{\text{Earth}} = \frac{(2 \pi r_{\text{Earth}})}{t_{\text{Earth}}} \]

21. If the moon were joined to Earth but just on the verge of breaking away from Earth’s surface, how much faster would Earth have to be moving at the equator for the moon to have the correct angular momentum it ended up with?

\[ \frac{v_{\text{breakaway}}}{v_{\text{Earth}}} = \]

22. If the surface of Earth were moving fast enough at the equator for the breakaway moon to have that much angular momentum, how long would it take for Earth to make one complete rotation?

\[ 24 \text{ hours} \left( \frac{v_{\text{Earth}}}{v_{\text{breakaway}}} \right) = \]

23. What is the inward gravitational acceleration at Earth’s surface?

\[ 1 \text{ gee} = \frac{G m_{\text{Earth}}}{r_{\text{Earth}}^2} = \]

24. At Earth’s normal speed of rotation, what is the outward centrifugal acceleration at Earth’s equator?

\[ v_{\text{Earth}}^2/r_{\text{Earth}} = \]

25. What is the net acceleration a person feels when standing at Earth’s equator?

26. If the Earth were rotating fast enough for the moon to spin off it with its correct angular momentum, what would be the outward centrifugal acceleration at Earth’s equator?

\[ \frac{v_{\text{breakaway}}^2}{r_{\text{Earth}}} = \]
27. If Earth were rotating that fast, what would be the net acceleration a person would feel when standing at Earth's equator? What would happen to Earth?

28. What do your calculations tell you about the validity of various hypotheses for the origin of the moon? Explain.