MATHEMATICS

Classics of Science: Cartesian Co-ordinates

Suggested by Dr. Mark H. Liddell

In the extract here quoted, Descartes laid the foundation of analytic geometry. It will be observed that he chose for reference a vertical instead of a horizontal line, that he calls the "origin" A instead of O, and that his ordinates and abscissas seem reversed to those used to the modern designations, but aside from these minor points the derivation of his equation of the curve is essentially the same process that the student might use today, should he wish to show his prowess on such a complicated case.

THE GEOMETRY OF RENE DESCARTES, translated from the French and Latin by David Eugene Smith and Marcia L. Latham with a facsimile of the first edition, 1637. Chicago, 1925.

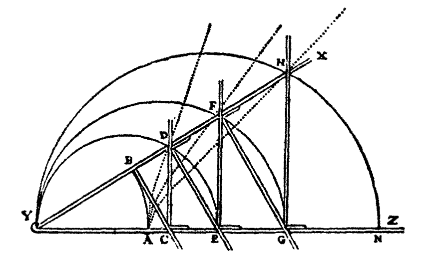
Complex Curves

Consider the lines AB, AD, AF and so forth, which we may suppose to be described by means of the instrument YZ. This instrument consists of several rulers hinged together in such a way that YZ being placed along the line AN the angle XYZ can be increased or decreased in size, and when its sides are together the points B, C, D, E, F, G, H, all coincide with A; but as the size of the angle is increased, the ruler BC, fastened at right angles to XY at the point B, pushes toward Z the ruler CD which slides along YZ always at right angles. In like manner, CD pushes DE which slides along YX always parallel to BC; DE pushes EF; EF pushes FG; FG pushes GH, and so on. Thus we may imagine an infinity of rulers, each pushing another, half of them making equal angles with YX and the rest with YZ.

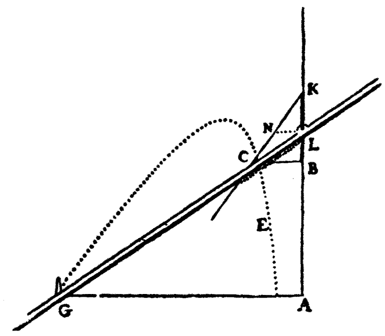
Now as the angle XYZ is increased the point B describes the curve AB, which is a circle; while the intersections of the other rulers, namely, the points D, F, H describe other curves, AD, AF, AH, of which the latter are more complex than the first and this more complex than the circle. Nevertheless I see no reason why the description of the first cannot be conceived as clearly and distinctly as that of the circle, or at least as that of the conic sections; or why that of the second, third, or any other that can be thus described, cannot be as clearly conceived of as the first: and therefore I see no reason why they should not be used in the same way in the solution of geometric problems.

Relation to a Straight Line

I could give here several other ways of tracing and conceiving a series of curved lines, each curve more complex than any preceding



Descartes' curves drawn by his hinged instrument



Descartes' drawing of the hyperbola referred to his co-ordinates

one, but I think the best way to group together all such curves and then classify them in order, is by recognizing the fact that all points of those curves which we may call "geometric," that is, those which admit of precise and exact measurement, must bear a definite relation to all points of a straight line, and that this relation must be expressed by means of a single equation. If this equation contains no term of higher degree than the rectangle of two unknown quantities, or the square of one, the curve belongs to the first and simplest class, which contains only the circle, the parabola, the hyperbola, and the ellipse; but when the equation contains one or more terms of the third or fourth degree in one or both of the two unknown quantities (for it requires two unknown quantities to express the relation between two points) the curve belongs to the second class; and if the equation contains a term of the fifth or sixth degree in either or both of the unknown quantities the curve belongs to the third class, and so on indefinitely.

Line of Reference

Suppose the curve EC to be described by the intersection of the ruler GL and the rectilinear plane figure CNKL, whose side KN is produced indefinitely in the direction of C, and

(Just turn the page)

INVENTIONS

Thermometer Runs Clock

A self-winding clock, run by what is virtually a glycerine thermometer, has been invented by a Swiss engineer, Karl Heinrich Meier. It utilizes the energy captured by the daily fluctuations in temperature to raise the weights that drive its mechanism, and it is stated that one of the clocks has been kept going for a year on a daily range of not more than eight degrees Fahrenheit. The essential mechanism consists of a long coiled tube filled with glycerine, connected with a cylinder, into which a piston it fitted. when the glycerine is warmed and expands, it forces out the piston which, in turn, lifts the clock weight. It is expected that this device will be especially useful in operating outdoor clocks in public places. The types now in common use are usually electrically driven and are therefore expensive to install, besides requiring frequent attention.

Science News-Letter, February 4, 1928

ENGINEERING

Engine May Save Millions

American automobile owners can save $400,000,000 annually by using engines that operate at a constant instead of a variable pressure, Prof. H. M. Jacklin of Purdue University has reported to the Society of Automotive Engineers.

Present auto engines operate so that the volume of the gaseous fuel exploded is constant. Prof. Jacklin's experiments were made upon an experimental engine constructed with a movable cylinder-head that was used to reduce the volume of the cylinder as the speed of the engine decreases. This maintained the same pressure within the cylinder at all speeds. No adjustment of spark was necessary.

Gains of up to 50 per cent. in miles per gallon might be expected if the new type engine were substituted for the ordinary engine now in use. Fuel bills would be cut a third, according to Prof. Jacklin's computations. Assuming a complete substitution of the constant compression engine in the 20,000,000 cars now running 6,000 miles a year on 20 cents per gallon gasoline, Prof. Jacklin sees the possibility of conserving our natural resources and the national pocketbook to the extent of some four hundred millions annually.

Science News-Letter, February 4, 1928

The Arctic tern migrates from the north polar regions to the Antarctic.

News-Letter Features

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Cartesian Co-ordinates

(*Continued from page* 71)

which, being moved in the same plane in such a way that its side KL always coincides with some part of the line BA (produced in both directions), imparts to the ruler GL a rotary motion about G (the ruler being hinged to the figure CNKL at L). If I wish to find out to what class this curve belongs, I choose a straight line, as AB, to which to refer all its points, and in AB I choose a point A at which to begin the investigation. I say "choose this and that," because we are free to choose what we will, for, while it is necessary to use care in the choice in order to make the equation as short and simple as possible, yet no matter what line I should take instead of AB the curve would always prove to be of the same class, a fact easily demonstrated.

Finding the Equation

Then I take on the curve an arbitrary point, as C, at which we will suppose the instrument applied to describe the curve. Then I draw through C the line CB parallel to GA. Since CB and BA are unknown and indeterminate quantities, I shall call one of them y and the other x. To the relation between these quantities I must consider also the known quantities which determine the description of the curve, as GA, which I shall call a; KL, which I shall call b; and NL parallel to GA, which I shall call c. Then I say that as NL is to LK, or as c is to b, so CB, or y, is to BK, which is therefore equal to $\frac{b}{c}y$. Then BL is equal to $\frac{b}{c}y - b$, and Al is equal to $x + \frac{b}{c}y - b$. Moreover, as CB is to LB, that is, as y is to $\frac{b}{c}y - b$, so AG or a is to LA or $x + \frac{b}{c}y - b$. Multiplying the second by the third, we get $\frac{ab}{c}y - ab$ equal to $xy + \frac{b}{c}y^2 - by$, which is obtained by multiplying the first by the last. Therefore, the required equation is

$$y^2 = cy - \frac{cx}{b}y + ay - ac.$$

Rene Decartes was born in the province of Touraine, France, March 31, 1596, and died in Stockholm, February 11, 1650. He attended school at La Fleche from the age of eight to the age of sixteen, and then went to Paris to live as his own master. There he enjoyed the friendship of Claude Mydorge, famous mathematician, and Marin Mersenne, a member of the order of Minim friars, which lasted until the death of those friends about 1648. After alternately tasting the pleasures of Paris social life and studying mathematics in strict seclusion, Descartes emigrated to the Netherlands, then a refuge for intellectuals who had run afoul the political and religious intrigues of their native countries. He enlisted in the army of the Prince of Orange, and spent the years 1617 to 1621 in one army or another in various parts of Europe. It was in 1619 that he worked out his mathematical ideas later published as the Discourse of Method, from which the above translation was made. In 1625 Descartes settled in Paris, but 1629 found him back in Holland, where he continued to live, though in thirteen different towns, until a year before his death. During this later residence in Holland, he abandoned mathematics for the study of physics and physiology.

Science News-Letter, February 4, 1928

Polar seas are bright green in color.

New York City spends over $3,000,000 a year removing snow from its streets.