CLASSICS OF SCIENCE: Carnot Cycle

Physics

An Account of Carnot's Theory of the Motive Power of Heat; with Numerical Results deduced from Regnault's Experiments on Steam, by William Thompson (Lord Kelvin), Professor of Natural History in the University of Glasgow. Proceedings of the Royal Society, Edinburgh, 1848-49.

CARNOT'S *Theory of the Steam-Engine*

Let C D F_2 E_2 be a cylinder, of which the curved surface is perfectly impermeable to heat, with a piston also impermeable to heat, fitted in it; while the fixed bottom C D, itself with no capacity for heat, is possessed of perfect conducting power. Let K be an impermeable stand, such that when the cylinder is placed upon it the contents below the piston can neither gain nor lose heat. Let A and B be two bodies permanently retained at constant temperatures, S^0 and T^0, respectively, of which the former is higher than the latter. Let the cylinder, placed on the impermeable stand, K, be partially filled with water, at the temperature S, of the body A, and (there being no air below it) let the piston be placed in a position E F, near the surface of the water. The pressure of the vapour above the water will tend to push up the piston, and must be resisted by a force applied to the piston.* till the commencement of the operations, which are conducted in the following manner:

(1.) The cylinder being placed on the body A, so that the water and vapour may be retained at the temperature S, *let the piston rise any convenient height* E E_1, *to a position* E_1 F_1, *performing work by the pressure of the vapour below it during its ascent.*

[During this operation a certain quantity, H, of heat, the amount of latent heat in the fresh vapour which is formed, is abstracted from the body A.]

(2.) The cylinder being removed, and placed on the impermeable stand K, *let the piston rise gradually, till, when it reaches a position* E_2 F_2, *the*

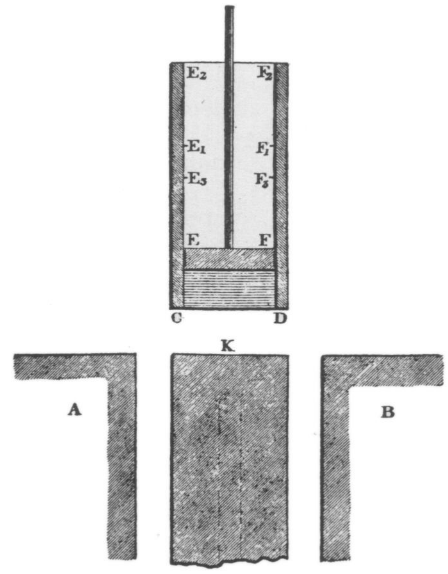

Diagram of the apparatus postulated in Carnot's Theory

temperature of the water and vapour is T, *the same as that of the body* B.

[During this operation the fresh vapour continually formed requires heat to become latent; and, therefore, as the contents of the cylinder are protected from any accession of heat, their temperature sinks.]

(3.) The cylinder being removed from K, and placed on B, *let the piston be pushed down, till, when it reaches the position* E_3 F_3, *the quantity of heat evolved and abstracted by* B *amounts to that which, during* the first operation, was taken from A.

[During this operation the temperature of the contents of the cylinder is retained constantly at T^0, and all the latent heat of the vapour which is condensed into water at the same temperature, is given out to B.]

(4.) The cylinder being removed from B, and placed on the impermeable stand, *let the piston be pushed down from* E_3 F_3 *to its original position* E F.

[During this operation, the impermeable stand preventing any loss of heat, the temperature of the water and air must rise continually, till (since the quantity of heat evolved during the third operation was precisely equal to that which was previously absorbed), at the conclusion it reaches its primitive value, S, in virtue of Carnot's fundamental axiom.]

At the conclusion of this cycle of operations† the total thermal agency has been the *letting down* of H units of heat from the body A, at the temperature S, to B, at the lower temperature T; and the aggregate of the mechanical effect has been a certain amount of *work produced,* since during the ascent of the piston in the first and second operations, the temperature of the water and vapour, and therefore the pressure of the vapour on the piston, was on the whole higher than during the descent, in the third and fourth operations. It remains for us actually to evaluate this aggregate amount of work performed; and for this purpose the following graphical method of representing the mechanical effect developed in the several operations, taken from Mons. CLAPEYRON'S paper, is extremely convenient.

Let O X and O Y be two lines at right angles to one another. Along O X measure off distances O N_1, N N_2, N_2 N_3, N_3 O, respectively proportioned to the spaces described by the piston during the four successive operations described above; and, with reference to these four operations respectively, let the following constructions be made:—

(1.) Along O Y measure a length O A, to represent the pressure of the saturated vapour at the temperature S; and draw A A_1 parallel to O X, and let it meet an ordinate through N_1, in A_2.

(2.) Draw a curve A_1 P A such that, if O N represent, at any instant during the second operation, the distance of the piston from its primitive position, N P shall represent the pressure of the vapor at the same instant.

(3.) Through A_2 draw A_2 A_3 parallel to O X, and let it meet an ordinate through N_3 in A_3.

(4.) Draw the curve A_3 A such that the abscissa and ordinate of any point in it may represent respectively the distances of the piston from its primitive position, and the pressure of the vapor, at some instant during the fourth operation. The last point of this curve must, according to Carnot's fundamental principle, coincide with A, since the piston is, at the end of the cycle of (*Turn to next page*)

*In all that follows, the pressure of the atmosphere on the upper side of the piston will be included in the applied forces, which, in the successive operations described, are sometimes overcome by the upward motion, and sometimes yielded to in the motion downwards. It will be unnecessary, in reckoning at the end of a cycle of operations, to take into account the work thus spent upon the atmosphere, and the restitution which has been made, since these precisely compensate for one another.

†In Carnot's work some perplexity is introduced with reference to the temperature of the water, which, in the operations he describes, is not brought back exactly to what it was at the commencement; but the difficulty which arises is explained by the author. No such difficulty occurs with reference to the cycle of operations in the text, for which I am indebted to Mons. Clapeyron.

Carnot Cycle—*Continued*

operations, again in its primitive position, and the pressure of the vapor is the same as it was at the beginning.

18. Let us now suppose that the lengths, O N_1, N_1 N_2, N_2 N_3 and N_3 O, *represent numerically* the volumes of the spaces moved through by the piston during the successive operations. It follows that the mechanical effect obtained during the first operation will be *numerically represented* by the area A A_1 N_1 O; that is, the number of superficial units in this area will be equal to the number of "foot-pounds" of work performed by the ascending piston during the first operation. The work performed by the piston during the second operation will be similarly represented by the area A_1 A_2 N_2 N_1. Again, during the third operation a certain amount of work is spent on the piston, which will be represented by the area A_2 A_3 N_3 N_2; and lastly, during the fourth operation, work is spent in pushing the piston to an amount represented by the area A_3 A O N_3.

19. Hence the mechanical effect (represented by the area O A A_1 A_2 N_2) which was obtained during the first and second operations, exceeds the work (represented by N_2 A_2 A_3 A O) spent during the third and fourth, by an amount represented by the area of the quadrilateral figure AA_1 A_2 A_3; and, consequently, it only remains for us to evaluate this area, that may determine the total mechanical effect gained in a complete cycle of operations. Now, from experimental data, at present nearly complete, as will be explained below, we may determine the length of the line A A_1 for the given temperature S, and a given absorption H, of heat, during the first operation; and the length of A_2 A_3 for the given lower temperature T, and the evolution of the same quantity of heat during the fourth operation; and the curves A_1 PA_2, A_3 P' A may be drawn as graphical representations of actual observations. The figure being thus constructed, its area may be measured, and we are, therefore, in possession of a graphical method of determining the amount of mechanical effect to be obtained from any given thermal agency. As, however, it is merely the area of the figure which it is required to determine, it will not be necessary to be able to describe each of the curves A_1 P A_2 A_3 P' A, but it will be sufficient to know the difference of the abscissas corresponding to any equal ordinates in the two; and the following analytical method of completing the problem is the most convenient for leading to the actual numerical results.

Draw any line P P' parallel to O X, meeting the curvilineal sides of the quadrilateral in P and P'. Let § denote the length of this line, and p its distance from O X. The area of the figure, according to the integral calculus, will be denoted by the expression

$$\int_{p_3}^{p_1} \S d\,p,$$

where p_1, and p_3 (the limits of integration indicated according to FOURIER'S notation) denote the lines O A, and N_3 A_3, which represent respectively the pressures during the first and third operations. Now, by referring to the construction described above, we see that § is the difference of the volumes below the piston at corresponding instants of the second and fourth operations, or instants at which the saturated steam and the water in the cylinder have the same pressure p, and, consequently, the same temperature which we may denote by t. Again, throughout the second operation the entire contents of the cylinder possess a greater amount of heat by H units than during the fourth; and, therefore, at any instant of the second operation there is as much more steam as contains H units of latent heat, than at the corresponding instant of the fourth operation. Hence, if k denote the latent heat in a unit of saturated steam at the temperature t, the volume of the steam at the two corresponding instants must differ by $\frac{H}{k}$. Now, if 6 denote the ratio of the density of the steam to that of the water, the volume $\frac{H}{k}$ of steam will be formed from the volume $6\ \frac{H}{k}$ of water; and, consequently, we have sulphurous acid, or carbonic acid under high pressure, which approaches the physical condition of a vapor at saturation; and therefore, in general, and especially in practical applications to real air-engines, it will be unnecessary to make any modification in the expressions. In cases where it may be necessary, there is no difficulty in making the modifications, when the requisite data are supplied by experiment.

Either the steam-engine or the air-engine, according to the arrangements described above, gives all the mechanical effect that can possibly be obtained from the thermal agency employed. For it is clear, that, in either case, the operations may be performed in the reverse order, with every thermal and mechanical effect reversed. Thus, in the steam-engine, we may commence by placing the cylinder on the impermeable stand, allow the piston to rise, performing work, to the position E_3 F_3; we may then place it on the body B, and allow it to rise, performing work, till it reaches E_2 F_2; after that the cylinder may be placed again on the impermeable stand, and the piston may be pushed down to E_1 F_1; and, lastly, the cylinder being removed to the body A, the piston may be pushed down to its primitive position. In this inverse cycle of operations, a certain amount of work has been spent, precisely equal, as we readily see, to the amount of mechanical effect gained in the direct cycle described above; and heat has been abstracted from B, and deposited in the body A, at a higher temperature, to an amount precisely equal to that which, in the direct cycle, was *let down* from A to B. Hence it is impossible to have an engine which will derive more mechanical effect from the same thermal agency, than is obtained by the arrangement described above; since, if there could be such an engine, it might be employed to perform, as a part of its whole work, the inverse cycle of operations, upon an engine of the kind we have considered, and thus to continually restore the heat from B to A, which has descended from A to B for working itself; so that we should have a complex engine, giving a residual amount of mechanical effect without any thermal agency, or alteration of materials, which is an impossibility in nature. The same reasoning is applicable to the air-engine; and we conclude, generally, that any two engines, constructed on the principles laid down above, whether steam-engines with different liquids, an air-engine and a steam-engine, or two air-engines with different gases, must derive the same amount of mechanical effect from the same thermal agency.

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