

The tenth principle gives you the opportunity of applying the budget system to gambling, but the results are discouraging. Thus if you stake your hundred francs, fifty francs at a time, you have the chance, if you are lucky both times, of coming out with 300 francs, but you have also the chance of breaking even or of going broke. And the latter events are more apt to happen.

## PART TWO

A PHILISOPHICAL ESSAY ON PROBABILITIES. By Pierre Simon, Marquis de Laplace. Translated from the sixth French edition by Frederick Wilson Truscott and Frederick Lincoln Emory. New York: 1902.

## Concerning Hope

The probability of events serves to determine the hope or the fear of persons interested in their existence. The word *hope* has various acceptations; it expresses generally the advantage of that one who expects a certain benefit in suppositions which are only probable. This advantage in the theory of chance is a product of the sum hoped for by the probability of obtaining it; it is the partial sum which ought to result when we do not wish to run the risks of the event in supposing that the division is made proportional to the probabilities. This division is the only equitable one when all strange circumstances are eliminated; because an equal degree of probability gives an equal right to the sum hoped for. We will call this advantage *mathematical hope*.

*Eighth Principle.*—When the advantage depends on several events it is obtained by taking the sum of the products of the probability of each event by the benefit attached to its occurrence.

Let us apply this principle to some examples. Let us suppose that at the play of heads and tails Paul receives two francs if he throws heads at the first throw and five francs if he throws it only at the second. Multiplying two francs by the probability  $\frac{1}{2}$  of the first case, and five francs by the probability  $\frac{1}{4}$  of the second case, the sum of the products, or two and a quarter francs, will be Paul's advantage. It is the sum which he ought to give in advance to that one who has given him this advantage; for, in order to maintain the equality of the play, the throw ought to be equal to the advantage which it procures.

If Paul receives two francs by throwing heads at the first and five francs for throwing it at the second



A GAMBLER WHO DID NOT RECKON ON PROBABILITIES, from Hogarth's "Rake's Progress." Hogarth was 52 years older than Laplace, and died when the mathematician was 15 years old

throw, whether he has thrown it or not at the first, the probability of throwing heads at the second throw being  $\frac{1}{2}$ , multiplying two francs and five francs by  $\frac{1}{2}$  the sum of these products will give three and one half francs for Paul's advantage and consequently for his stake at the game.

*Ninth Principle.*—In a series of probable events of which the ones produce a benefit and the others a loss, we shall have the advantage which results from it by making a sum of the products of the probability of each favorable event by the benefit which it procures, and subtracting from this sum that of the products of the probability of each unfavorable event by the loss which is attached to it. If the second sum is greater than the first, the benefit becomes a loss and hope is changed to fear.

Consequently we ought always in the conduct of life to make the product of the benefit hoped for, by its probability, at least equal to the similar product relative to the loss. But it is necessary, in order to attain this, to appreciate exactly the advantages, the losses, and their respective probabilities. For this a great accuracy of mind, a delicate judgment, and a

great experience in affairs is necessary; it is necessary to know how to guard one's self against prejudices, illusions of fear or hope, and erroneous ideas, ideas of fortune and happiness, with which the majority of people feed their self-love.

The application of the preceding principles to the following question has greatly exercised the geometers. Paul plays at heads and tails with the condition of receiving two francs if he throws heads at the first throw, four francs if he throws it only at the second throw, eight francs if he throws it only at the third, and so on. His stake at the play ought to be, according to the eighth principle, equal to the number of throws, so that if the game continues to infinity the stake ought to be infinite. However, no reasonable man would wish to risk at this game even a small sum, for example five francs. Whence comes this difference between the result of calculation and the indication of common sense? We soon recognize that it amounts to this: that the moral advantage which a benefit procures for us is not proportional to this benefit and that it depends upon a thousand (*Turn to next page*)

## Probabilities—Continued

circumstances, often very difficult to define, but of which the most general and most important is that of fortune.

Indeed it is apparent that one franc has much greater value for him who possesses only a hundred than for a millionaire. We ought then to distinguish in the hoped-for benefit its absolute from its relative value. But the latter is regulated by the motives which make it desirable, whereas the first is independent of them. The general principle for appreciating this relative value cannot be given, but here is one proposed by Daniel Bernoulli which will serve in many cases.

*Tenth Principle.*—The relative value of an infinitely small sum is equal to its absolute value divided by the total benefit of the person interested. This supposes that every one has a certain benefit whose value can never be estimated as zero. Indeed even that one who possesses nothing always gives to the product of his labor and to his hopes a value at least equal to that which is absolutely necessary to sustain him.

If we apply analysis to the principle just propounded, we obtain the following rule: Let us designate by unity the part of the fortune of an individual, independent of his expectations. If we determine the different values that this fortune may have by virtue of these expectations and their probabilities, the product of these values raised respectively to the powers indicated by their probabilities will be the physical fortune which would procure for the individual the same moral advantage which he receives from the part of his fortune taken as unity and from his expectations; by subtracting unity from the product, the difference will be the increase of the physical fortune due to expectations: we will call this increase *moral hope*. It is easy to see that it coincides with mathematical hope when the fortune taken as unity becomes infinite in reference to the variations which it receives from the expectations. But when these variations are an appreciable part of this unity the two hopes may differ very materially among themselves.

This rule conduces to results conformable to the indications of common sense which can by this means be appreciated with some exactitude. Thus in the preceding question it is found that if the fortune of Paul is two hundred francs, he ought not reasonably to stake more than nine

francs. The same rule leads us again to distribute the danger over several parts of a benefit expected rather than to expose the entire benefit to this danger. It results similarly that at the fairest game the loss is always greater than the gain. Let us suppose, for example, that a player having a fortune of one hundred francs risks fifty at the play of heads and tails; his fortune after his stake at the play will be reduced to eighty-seven francs, that is to say, this last sum would procure for the player the same moral advantage as the state of his fortune after the stake. The play is then disadvantageous even in the case where the stake is equal to the product of the sum hoped for, by its probability. We can judge by this of the immorality of games in which the sum hoped for is below this product. They subsist only by false reasonings and by the cupidity which they excite and which, leading the people to sacrifice their necessities to chimerical hopes whose improbability they are not in condition to appreciate, are the source of an infinity of evils.

The disadvantage of games of chance, the advantage of not exposing to the same danger the whole benefit that is expected, and all the similar results indicated by common sense, subsist, whatever may be the function of the physical fortune which for each individual expresses his moral fortune. It is enough that the proportion of the increase of this function to the increase of the physical fortune diminishes in the measure that the latter increases.

**Pierre Simon Marquis de Laplace** (1749-1827) at the age of 18 gained the attention of d'Alembert by his mathematical genius. Most of his work was on problems in celestial mechanics. The results of his studies were written first in his *Exposition du Système du Monde*. At the age of 50 he published the first two volumes of his great *Mécanique Céleste*, following with the third and fourth in the next six years. The fifth volume did not appear until twenty years later. Advancement of the theory of probabilities was a by-product of his studies of planetary motions, and in other chapters of the interesting book from which this extract is taken, he goes into more detail on this and other practical phases of the subject.

*Science News-Letter, November 16, 1929*

A statistical study shows that about eighteen per cent. of the population are first-born members of their families.

Earthquakes have shaken Asia Minor approximately 10,000 times since the beginning of our era.

During the past five years more than 3,000,000 Europeans have sought homes in other countries.

Two hundred different American dialects have been recorded by phonograph on the campus of Columbia University.

Rome, Naples and Milan are striving for the honor of being the first Italian city to have 1,000,000 inhabitants.

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