New mathematics in field theory

Work with nonpolynomial Lagrangians gives field theorists hope of solving problems

by Dietrick E. Thomsen

$$\mathcal{L}_{\text{int}} = e\bar{\psi}\gamma_\mu\psi A_\mu$$

$$L_{\text{int}} = [\det(\eta^{\mu a} + \kappa h^{\mu a})]^{-1}\,(e\bar{\psi}\gamma_a A^a\psi)$$

$$L_{\text{int}} = [1 + \kappa h(x)]^{-1}\,(e\bar{\psi}\gamma_\mu\psi A_\mu)$$

Salam and Strathdee

P. A. M. Dirac's Lagrangian for electrodynamics and two examples of modifications introduced by considering gravitational effects.

Particle physicists distinguish two basic classes among the elementary particles: hadrons and leptons. Hadrons are the particles (including the proton, the neutron and many others) that are susceptible to the so-called strong interaction, the force that binds atomic nuclei together. The leptons are the particles that do not respond to the strong force. The known leptons are the electron, the mu meson or muon, the electron neutrino and the muon neutrino.

In addition to the strong force, nature provides at least three other forces to animate the world of particles: the weak interaction, the electromagnetic interaction and the gravitational interaction. The last two are familiar from macroscopic physics; they can affect large bodies. The weak interaction, which comes into play between leptons and also in certain radioactive decays of hadrons, is, like the strong interaction, confined to the microscopic dimensions of particle physics.

Field theory is the branch of physics that attempts to derive mathematical descriptions of these forces and how they cause the particles to move, to change, to be created or to be annihilated.

The subject is bedeviled by mathematical difficulties: It cannot derive straightforward solutions to many of the problems it poses, and the field theories that do get derived often fail to correspond to the actual physics in important ways.

Field theorists like to start work by writing down a mathematical summary statement called a Lagrangian, which expresses the energies of a general group of particles under the influence of whatever force is being considered. From the Lagrangian theorists hope to derive a detailed description of the force with the ultimate aim of predicting what will happen in any particular case.

Unfortunately the Lagrangians that can be written have proved very intractable to the usual methods of mathematical derivation. The electromagnetic interaction is mathematically the most tractable one, and it is in this theory, called quantum electrodynamics, that the most progress has been made. But the theory that has come out still doesn't fit the physics at all points.

Over the last 16 years a method for dealing with the less tractable Lagrangians, the so-called nonpolynomial Lagrangians, has been developing. It started with some work of Dr. Susumu Okubo of the University of Rochester and was taken up by a number of people in the United States, the Soviet Union and Europe. It has progressed to the point where Dr. Abdus Salam, director of the International Center for Theoretical Physics at Trieste, could tell the Coral Gables Conference on Fundamental Interactions at High Energy, held at the University of Miami in January, that in the past, "We had a lot of bad math mixed with good physics, but at long last we have got to good math."

There are critics who do not believe the good math is applicable to the physics in question, notably Dr. Benjamin W. Lee of Yale University, but if it is applicable it has interesting consequences. The ultimate hope is that it can work out field theories for all the natural forces. One of the first uses to which Dr. Salam and a number of other theorists have put it is to remove some of the difficulties of quantum electrodynamics by mixing in gravity. The result not only cures the electrodynamic problems they say, but also makes some far reaching alterations in the theory of gravity.

An example of the fundamental deficiencies of quantum electrodynamics that Dr. Salam and his collaborators say they can cure is the infinite self-mass of the electron. The electron possesses negative electric charge. If it is imagined to occupy a particular volume, then all parts of the electron are charged, and each repels all the others since they all have negative charge. Something is necessary to counteract this self-repulsion. Whatever it is that holds the electron together is called the self-energy or the self-mass.

The smaller the electron is, the greater must the self-mass be to hold it together. Unfortunately the theory requires that the electron be a geometric point, that is, have zero dimensions. With zero dimensions the self-mass becomes infinite. Experimentally this is absurd. The observed mass of the electron, which includes the self-mass and other contributions, is not only finite, but extremely small.

In order to make the physics of quantum electrodynamics come out better, Dr. Salam and several others decided to see whether bringing the effects of gravity on the electron into the theory would remove the infinity. To consider both forces together required writing a nonpolynomial Lagrangian, and thus the problem had to be solved by the new method. Pursuing the solution not only removed the electrodynamic infinity, it also led to modifications in the theory of gravitation, described for the Coral Gables conference by Dr. C. J. Isham, also of the Trieste center.

In his general relativity Albert Einstein presented a theory of gravitation in which gravity operates on all objects in the same way and has a single mathematical description. The name for the class of mathematical entities into which Einstein's description of gravity falls is a tensor. Einstein's particular tensor is referred to as G, and on the particle level, theorists suppose that Einstein's force is mediated by a particle called a graviton or g-particle.

(Theorists of particle physics, including field theorists, regard forces between bodies at some distance from each other as being carried by intermediary particles that the bodies exchange. Each kind of force has one or more different intermediaries.)

To Einstein's formulation, Dr. Isham told the conference, the new formulation adds another tensor, which they call F. This produces its own intermediary particle. f. In a recent paper in THE PHYSICAL REVIEW, Drs. Isham, Salam and J. A. Strathdee suggest that this f particle may correspond to certain particles that happen to be known to experimenters by the same letter, or it may be an entirely new particle with a mass between 1,400 million and 1,700 million electron-volts and a spin of 2 units.

Gravity in the new formulation does not affect all particles in the same way, and the difference lies between hadrons and leptons. G interacts with leptons, says Dr. Isham; F with hadrons. Overall effects of gravitation are described by using a mixture of the two proper to the situation.

The F-G formulation of gravity with its two intermediary particles and their division of labor provides an analogy with current developments in electrodynamics. Physicists had supposed that the photon, or light particle, was the sole intermediary of electromagnetic forces. But experimental evidence has shown that photons approaching hadrons tend to turn themselves into one of the mesons called phi, rho or omega. Thus, it appears that electromagnetic forces are mediated to leptons directly by photons and to hadrons by phi, rho or omega.

Deriving analogies and cross-connections of this sort feeds the hope that a comprehensive theory may be found that includes all or several of the forces. One of the aims of the current work, says Dr. Salam, is "a unification of nuclear physics and gravitation by writing the same equation for nuclear physics as Einstein did for gravitation but with a different coupling constant." (Coupling constants measure the strength of one kind of force relative to the other kinds.)

Dr. Stanley Deser of Brandeis University hastened to assure the conference that macroscopic gravitational effects are not in danger of alteration. To preserve the stability of the solar system, a mixture for long-range interactions can be chosen so that G dominates and the Einsteinian description is virtually unchanged. "The boat is not rocked as far as Kepler's laws are concerned," he says. (Kepler's laws govern the shapes of planetary orbits.)

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Salam: Good mathematics at last.

At short ranges, the picture may be radically altered. "If you are a hadron or a lepton you will move in different paths," says Dr. Deser. In fact the F tensor may provide a negative or repulsive gravity at short distances. If this happens it could have important cosmological consequences.

Neutron stars and black holes are two supposed kinds of astronomical objects formed by the collapse of ordinary stars (SN: 2/27/71, p. 151). A black hole, particularly, is supposed to be an object undergoing endless, unstoppable collapse under the influence of the mutual gravitational attraction of its parts.

In the interior of both these kinds of objects, supposedly, are collections of hadrons compressed until the distances between them are extremely small. If repulsive F gravity dominates this situation it could help explain the stability of neuron stars and provide a natural stopping place for gravitational collapse, thus resolving some of the paradoxes about black holes that arise from contemplating the results of unending collapse.

Einstein's theory describes the strong gravitational fields of very massive bodies by saying that space-time is strongly curved in their neighborhood and that the curvature alters the motion of other bodies that happen to come near. Dr. Isham points out that under F gravity strong curvature of space-time can occur in the neighborhood of hadrons. He suggests that hadrons may prove to be black holes in the F field, analogous to the supposed astronomical black holes, which are black holes in the G field. □

films OF THE WEEK

NETSILIK ESKIMO SERIES. Nine films in 21 parts, 16mm, color, sound, between 27 min. and 36 min. each. Series focuses on the traditional Eskimo life among the Netsiliks, the People of the Seal, before European acculturation. Films reveal the live reality of traditional Eskimo life dependent entirely upon their own ingenuity. Titles include: At the Caribou Crossing Place (2 parts), At the Autumn River Camp (3 parts), At the Winter Sea Ice Camp (4 parts), Group Hunting on the Spring Ice (3 parts), Jigging for Lake Trout (1 part), At the Spring Sea Ice Camp (3 parts), Stalking Seal on the Spring Ice (2 parts), Building a Kayak (2 parts), and Fishing at the Stone Weir (2 parts). Audience: upper elementary. Purchase information from Modern Learning Aids, Dept. SN, P.O. Box 302, Rochester, N.Y. 14603.

MICROELECTRODES IN MUSCLE. 16mm, color, sound, 19 min. Shows an intracellular microelectrode technique being used to measure transmembrane potential changes. Preparation of microelectrodes from hard glass tubing of about two millimeters external diameter is illustrated. A frog is dissected and the Sartorius muscle in the thigh isolated. The muscle, connecting nerve and piece of the pelvic bone are transferred to the recording amplifier and placed in contact with a solution surrounding the muscle fiber. Measurements of the electrical potential across the membrane muscle fiber begin as the microelectrode tip enters the cell. Audience: college. Purchase $225 or rental $30 from John Wiley and Sons, Dept. SN, 605 3rd Ave., New York, N.Y. 10016.

ELSE AND HER CUBS. 16mm, color, sound, 31 min. This is the personal documentary film photographed in Kenya by Joy and George Adamson themselves throughout their extraordinary friendship with the lioness, Else, from the time she was a cub until she in turn had cubs. Of that enduring singular friendship between human being and wild animal, Joy Adamson wrote "Born Free," followed by "Living Free" and "Forever Free." Audience: elementary, secondary, adult. Purchase $390 or rental $40 from Benchmark Films, Dept. SN, 145 Scarborough Rd., Briarcliff Manor, N.Y. 10510.

MECHANICS OF LIFE: MUSCLES AND MOVEMENT. 16mm, color, sound, 9½ min. Muscles move by contracting. No work is being done as the muscle relaxes, so every muscle must have an opposing muscle. Muscles may be voluntary; the heart is an example of involuntary muscle. Good posture and proper exercise help our muscles work efficiently. Audience: elementary. Purchase $140 or rental $8 from BFA Educational Media, Dept. SN, 2211 Michigan Ave., Santa Monica, Calif. 90404.

NEW MAN IN THE FOREST. 16mm, color, sound, 26 min. Presents suggestions and techniques for deriving maximum benfits from small or large plots of forest land. A new owner of a small forest and his young son explore their land and talk with professional advisers to learn what they should do. They enjoy recreation among the many trees, but they also plan to harvest those trees for profit. The final portion of the film shows the way to plan for and accomplish this use of trees without destroying the forest. Audience: general science, botany, agriculture, as well as owners of forest land. Purchase $335 or rental $20 from International Film Bureau, 332 S. Michigan Ave., Chicago, Ill. 60604.

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