Beating the Game

Game theory compares blackjack systems and proposes to teach a computer backgammon

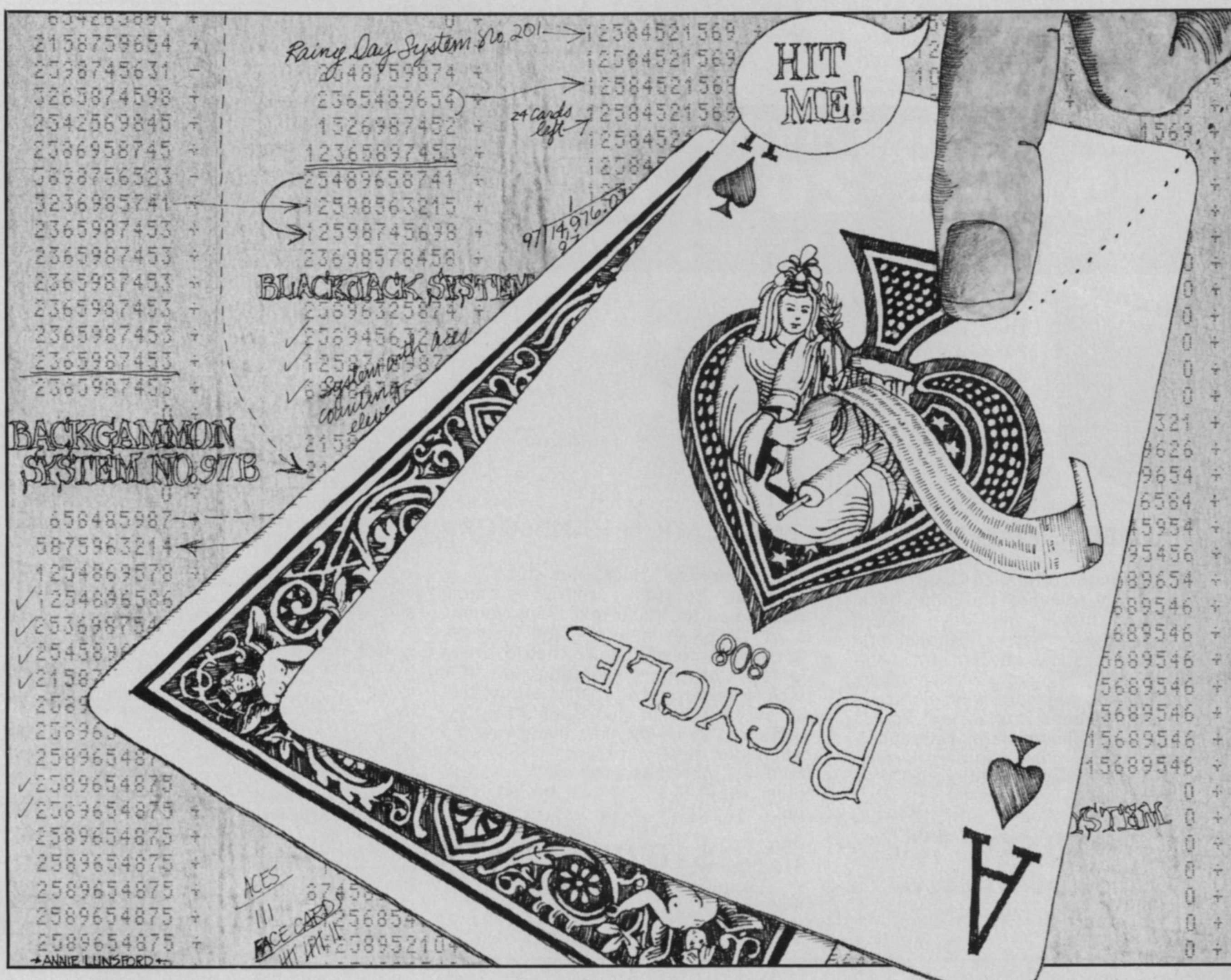
by Dietrick E. Thomsen

The man who broke the bank at Monto Carlo is a musical fantasy that grew out of the avid interest many people have in the things that happen on green baize tables. The man who, according to folklore, was told not to return to Las Vegas because he had won too much money there is real. He is Edward Thorp, a professor of mathematics at the University of California at Irvine.

Lately Thorp has been looking for the best way, in a theoretical and practical sense, to beat the blackjack table. He has devised a way of comparing the several blackjack systems against each other and a theoretically best possible system. At the same time his interest has turned to that ancient, but recently trendy, game, backgammon. He shared some of his latest insights on these topics with fellow mathematicians at the recent National Mathematics Meeting at Washington.

The blackjack systems depend on counting the cards as they fall. As play proceeds, the deck is depleted, and with the fall of each card the player's expectation of success changes. Removal of different denominations from the deck changes the expectations in different amounts. Removal of two nines does not have the same effect as removal of three fives.

From the way the expectations change, a particular numerical value can be assigned to each denomination of card. As the cards fall from the deck a running total of these numbers is kept. Different systems assign different numbers to different denominations. They also differ in how they use the



Ann Lunsford

running value total and the number of cards remaining in the deck. There are various combinations of addition and division. There is also a difference in whether the system keeps a separate count of aces. Aces have two values in blackjack, 1 and 11, and therefore some system makers like to tally them separately. The result of all this arithmetic is used to advise the player how to bet.

The question Thorp set himself was whether there is some method of comparing the different strategies without doing a massive computer simulation of a million hands. He finds one and he finds a criterion to compare them with each other and see how close they come to a theoretically possible optimum system.

First he needs a definition of "more advantageous." It may seem obvious that it means a greater chance of winning, but the case is complicated because a given system may give a greater expectation of winning when the play is in a particular stage, but it may be surpassed by another in a different situation. The final working definition of advantage is a system that gives at least as good a chance of winning over as wide a range of situations as an alternative with at least no more risk to the player.

Thorp finds that he can compare the quality of systems by defining an expectation function for each one that expresses its relative betterness. The expectation changes as play proceeds. It depends on the fraction of cards remaining in the deck, and it varies as they fall.

Graphically the expectations define a surface called a simplex, and the falling of cards causes motion from point to point on this surface as the expectation changes. Working with the geometry of the simplex Thorp can compare system to system, and he finds that he can define a single number, which he designates with the symbol lambda, that expresses a system's betterness relative to others and its closeness to a theoretically possible optimum system. Thus he has an analytic method for ranking blackjack systems and no longer has to simulate a million hands on a computer to compare them. But he does not tell us which is the best possible system.

Backgammon is among the most ancient games. A set dating to 2600 B.C. has been found. From the game theoretician's point of view, Thorp says, the hope of solving the problems that it presents is quite good.

The game consists of a ladder of 26 cells. Of these, 24 appear on an actual backgammon board. For the analysis, Thorp adds, the two "off the board" spaces at each end into which counters that have successfully completed their journeys are put. Each player's counters start at one end of the board, and the object is to get them all across the board and off it, passing the other player's counters coming in the opposite direction, before he gets his across. Moves are determined by throwing dice.

There is an important complication. If counters of both players arrive in the same cell, there are situations where one can be sent back to the beginning of its trip. This possibility of repeated restarts makes the game in a theoretical sense potentially infinite. In principle a backgammon match could last forever. It is "a fact which will impede analysis slightly," Thorp concedes.

The way to analyze the game is to set up partial models that are simplified, removing complexities of the real game, especially the one that makes it infinite, and then gradually to add back the complications. In Model I each player has one counter, and the bounce-back rule is suspended so that the counters can freely pass each other. When this is properly set up it produces a game of 167 steps. The first to reach step 167 wins. Computer simulation shows that when goals are equal, the first player to roll has a slight advantage, but this declines toward even chance as play proceeds. This is a very crude approach to a real backgammon game, but it leads to interesting insights, Thorp says.

In Model II one sets up an end game. Again there are two counters, but they have already passed each other so there is no further chance that they could be sent back to their starting points. This too is a finite game and is amenable to solution.

One complication of the real game is the doubling cube. As the game proceeds, if one player gains a certain advantage, he can use the doubling cube to double the stakes. This changes the consequences for the loser and alters the expectations and strategy of play. Recursion schemes can be devised to solve both Model I with the doubling cube and the end game with the doubling cube. (A recursion scheme is a system for calculating a series of related values. Knowing the first number in the series and the recursion scheme you can calculate the second. Putting the second number into the recursion scheme gets you the third. And so on.)

In actual backgammon play it is possible that the game might come down eventually to Model I or Model II, but these highly restricted situations are still far from the complexity of a full game, in which each player has several counters on the board at once and the bounce-back rule can operate. Still enough has been learned so far for Thorp to conclude that backgammon "can be played better by computer than by any person." But suppose the computer refuses to go to Reykjavik? □