General Relativity: Geometry Meets Physics

Is physics really a branch of geometry? We’ll see.

by Dietrick E. Thomsen

Those who practice general relativity—if that is the proper word—are intimately concerned with geometry. In fact one could say that the program of general relativity is the geometrization of physics. Workers in the field are concerned not only with the geometry of the real space-time in which actual physical events take place, but abstract multidimensional spaces in which they cut, twist and slice in order to come up with geometric descriptions of many things, including the properties of elementary particles and not-so-elementary particles (spinning black holes). Recent work on topics of this kind was described at a symposium held recently at Syracuse University in honor of the 60th birthday of one of the foremost practitioners of the art, Peter G. Bergmann.

General relativity is both a theory of gravitation and a description of the geometry of space-time. With it Einstein showed how a physical reality, the force of gravity, could be explained as an effect of geometry, a result of the curvature of space. That led him and many others to wonder how much of the rest of physics could be explained geometrically. A lot, it seems.

One long-standing project of general relativists is to quantize the theory, to derive a form appropriate to the subatomic domain. Einstein’s formulation dealt with the continuous quantities and processes of the macroscopic world. Gravity also operates in the microworld, and for completeness a quantum general relativity is needed. But progress has been slow and workers few in part because of the extreme intellectual difficulty of the endeavor, in part because of a certain lack of urgency. Gravity’s effects on the doings of elementary particles are so tiny and so overwhelmed by those of the other forces acting (electromagnetism and the two kinds of force peculiar to the microworld, the strong and the weak) that its effects can not only be safely neglected, it is virtually impossible to find them. The particle physicists can continue to play billiards without worrying about how the relativists come in and twist up the green baize. In the end, however, the table may be more important to the game than the balls, or rather the distinction between table and balls may vanish. As the geometrizations of general relativity invade the particle-physics domain, it began to appear that they may provide ways toward a unified theory of the various forces involved.

In quantizing any theory it is necessary to translate from the smooth world of the macrocosm to the jumpy world of quanta in such a way that the two halves of the theory are compatible and slide into one another in the so-called classical limit, the range of sizes in between the macroworld and microworld where the two meet. Physicists traditionally begin in a very physical way by minding their p’s and q’s. P is the traditional symbol for an object’s momentum; q for its position. The formulation of the theory is derived from the relationship between them, which is one thing in classical physics and another in quantum physics. The trick is in going from one to another.

The traditional way, in the words of Arthur Komar of Yeshiva University, has about it something of a “magical operation,” a hocus-pocus mental twist that often troubles students when they first encounter it and sometimes continues to trouble senior professors like Komar years after they first experienced it. Komar decided to search for a way to make the transition that was easier to accept and less flawed by “magic” mental leaps, a system in which the translation to a quantum theory becomes a different mathematical formulation of the same theory.

The search leads him into some very abstruse philosophico-mathematical territory in which, he says, “I began to realize I didn’t know what an integer was.” He works with different models (in the mathematical sense) of arithmetic (SN: 2/15/75, p. 109). “All of physics seems based on arithmetic when it comes down to it,” he says, and he investigates “a possibility of taking physics as stated as a formal language and realizing it on different models of arithmetic.” The different realizations might give him a more rigorous way of going from a classical to a quantum theory.

He reports some success in thinking along these lines, and it is incidentally with a system that starts not from the relationship of the p’s and q’s but from the relationship among the constraints imposed on their actions by the geometry of the situation. It leads to qualitatively new ways to look at the quantum world including gravitational arguments to get around the famous problem of the Einstein box, a thought experiment in which Einstein used a box emitting a burst of energy and gravitational arguments in an attempt to knock down the uncertainty principle, one of the bases of quantum physics.

In a somewhat less metamathematical vein Roger Penrose of Oxford University asks: “What do you want of quantum theory? What one wants is a quantum, some concept of a graviton.” Every force field that gets quantized has a field quantum, a particle that embodies the field so to speak and carries its forces from place to place. Electromagnetism has the photon; the strong (nuclear) interaction has various mesons. Gravity has the graviton. Eye hath not seen nor ear heard a graviton, nor is it ever likely to. What is more important is that the detectors of particle physics do not catch them, either. There is no physical evidence for their existence; as of now they are purely hypothetical.

Even as a hypothesis they are not well understood. Penrose is trying to achieve a description of a single graviton. He conceives, as several people objected from the floor, that that is not enough. For a real situation you need many gravitons, perhaps infinitely many. A description of one does not necessarily extrapolate to many by multiplying by x. Nevertheless, it is a beginning. Penrose, like many others, proceeds in analogy with electromagnetics, the most successfully quantized of theories, and tries to make the description of the graviton proceed in parallel with that of the photon. As he considers the invariance laws the graviton must obey, he makes use of an abstract space, a space that is derived from the properties of the “real” space-time in which events take place and is related to it. In this abstract space he makes slices and twists and thereby begins to derive the properties of the graviton. It is a very geometric way to start to describe what is supposed to be a piece of matter. As Penrose remarks: “If a graviton is to mean anything from a physical point of view, each must carry its own curvature with it, some measure of space-time curvature.” It is this crucial characteristic that Penrose is looking for as he seeks solutions of

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Einstein's equations that connect to the slices and twists of his abstract space. If the graviton is such a geometrized piece of matter, how about the other bits of matter called elementary particles? Some people have gone so far as to suggest that particles are miniature black holes, extremely sharp little twists of space-time. The application of the ideas of general relativity to the particle world in the hope of perhaps finding a theory that unifies all the particles and force fields is a lively activity nowadays. One of its foremost practitioners is Abdus Salam of the International Centre for Theoretical Physics at Trieste. In an extension of Salam's work, Richard Arnowitt of Northeastern University studies gauge transformations in an abstract space he calls supersymmetry space. Gauge is the term for a quantity related to how you measure distance in a given space. It is possible to imagine spaces with different gauges; it is also possible to imagine a single space in which the gauge varies from page to page. Transformations of gauge are operations basic to a number of the theories of the different force fields of particle physics.

In supersymmetry space Arnowitt finds one basic gauge field that does everything. It includes the ordinary gauge transformations of general relativity and a large number of those used in high-energy theory, especially those of electromagnetism and the Yang-Mills theory that applies to the weak interaction. "Maybe we can start developing something that unifies all these things," Arnowitt remarks.

Everybody is seeking solutions, especially solutions for Einstein's equations. Penrose wants them to describe gravitons. Back in the macrocosm they are needed too. Einstein gave the world a new general field equation. To describe physics, one must solve them for specific situations. What exactly happens when a gravitating body meets another in the context of Einstein's thought?

The situation is a very sticky one. For 50 years only one solution to Einstein's equations was known. It was derived by Karl Schwarzschild shortly after Einstein published, and it concerns the simplest case, the gravitational monopole, a single body sitting in space. Ten years ago came the Kerr solution, for a rotating monopole. The Schwarzschild and Kerr solutions are important to astrophysics because they predict black holes and rotating black holes respectively. Beyond those two, none.

The situation in the great analogy theory, electromagnetism is quite different. Many solutions to Maxwell's equations, the electromagnetic counter-
part of Einstein's, are known, and a rather complete description of electromagnetic phenomena with all its practical consequences is possible. This fact and a curious thing that happened to the Schwarzschild and Kerr solutions led Ezra T. Newman of the University of Pittsburgh and co-workers to a "heaven" where they hope to find other solutions to Einstein's equations and can do other curious things.

They discovered some years ago that if they took the Schwarzschild solution and performed a coordinate transformation, simply changing the definition of the dimensions of the space in which it was expressed, they got the Kerr solution out. They didn't quite know what this meant until they started to do it with electromagnetics and found that they could generate whole chains of solutions in this way. The best coordinate change they found was to complexify the coordinates of ordinary space-time, replacing real numbers with complex numbers (sums of real and imaginary numbers). This gave them a space they call heaven. (Yes, that really is the term they use.)

Going backward and forward between heaven and the cosmos, you can do a number of interesting things. Maybe someday you can find solutions of Einstein's equations in addition to the Schwarzschild and Kerr solutions. You can also get geometrical definitions for such physical quantities as mass, angular momentum and charge.

So the whole world is being geometrized. The experimental physicist will object (perhaps quicker than the ordinary pedestrian): What has all this got to do with the world I kick around in my apparatus? Answer: Not much right now. The relativists recognize the limitation. Jeffrey Winicur of the University of Pittsburgh, introducing Penrose, remarked, "Roger Penrose is internationally famous for his calligraphy." Penrose himself conceded: "If none of this makes sense physically, it constitutes a nice piece of pure mathematics."

Even if it makes sense physically, it has to be confirmed. Right now the best hope for confirmation of any part of general relativity lies in astrophysics where black holes are a hot topic. But Engelbert Schucking of New York University, a relativist who has worked with astronomers for a long time, is lugubrious about the prospects. He points out that astronomers cannot reduce systematic errors enough to be really convincing and that the time scale of observations is extremely short compared to the time it takes things to happen, a human lifetime against 100 million years. "It is extremely difficult to get something that amounts to proof," he says. "To expect certainty is wishful thinking."