General Relativity's Catch 22

We need determinism to do physics, but when we get determinism, we find we can't do physics

by Dietrick E. Thomsen

A belief in determinism is basic to physical science as we know it. If we did not believe that we could predict the future of a physical system from a knowledge of its past, we would not have physical science.

We tend to take determinism for granted, on a lofty philosophical level, probably because everybody who grew up west of Baluchistan has been conditioned by theologies and philosophies that regard everything that happens as part of a divine plan, and on a more empirical level because it seems to work.

But our friendly neighborhood guru might tell us that that is the illusion of our senses. If physics had developed in India—if physics could have developed in India—it would bear a much different aspect. So it behooves us to ask whether our universe is indeed deterministic or whether we are fooling ourselves and the last laugh will be had by some 18-armed god of cosmic caprice.

In the scientific compartments we have, the question is properly a cosmological one. As such it becomes a question in general relativity, which is the theoretical fundament of modern cosmology. General relativity is in one of its aspects a description of the geometry of space-time, and it is in that context that Rainer Sachs of the University of California at Berkeley and those working with him on the subject pose the question of determinism: Is it possible to distinguish a class of nondeterministic space-times from a class of deterministic ones, and, if so, is the space-time of the actual universe one or the other? Sachs reported some of his recent findings at the symposium in honor of Peter G. Bergmann held at Syracuse University last month. He finds that we're damned if we do and damned if we don't.

The space-times in question, singularly or plurally, share a number of basic characteristics. They are four dimensional. Three spacelike dimensions represent the three dimensions of our ordinary spatial experience; the fourth dimension is timelike (not simply time, by the way; other mathematical factors go into its definition). It is impossible to draw a four-dimensional space on a two-dimensional piece of paper, so, in visualization, four dimensions are reduced to two (or three with perspective drawing), the timelike in the vertical and the spacelike in the horizontal.

A point in space-time (let us use the singular from now on in describing common characteristics) defines both a location and a time, and is called technically an event. The history of a physical system is a series of events occurring as time passes, which traces out a world-line in space-time. World-lines are generally vertical on the graph since everything is always moving in time; changes in position appear as sidewise wiggles.

A line connecting two events in space-time can represent a message sent between them. Thus, for example, if a man standing on Polk Street in San Francisco sends a carrier pigeon to a woman on Valencia Street, the pigeon's flight can be represented by a line between the two events (its departure from Polk Street and its arrival at Valencia Street). The slope of the line represents the velocity of the flight. (It is determined by dividing one dimension, space, by the other, time.)

Thus skewed lines in space-time represent velocities, and in playing this game, the constants of physics are finagled so that the line at 45 degrees, angled halfway between the spacelike and timelike directions, represents the speed of light. Velocities less than light make lines more vertical than 45 degrees; velocities greater than light make lines more horizontal.

If we take a given event point and draw all the 45-degree lines we can through it, we will find that we have drawn (in the three-dimensional representation) the surface of a cone, or rather of two cones with a common vertex at the event. One cone stretches into the past, the other into the future. This light cone, as it is called, is an extremely important boundary; it defines the past and future of the given event. Because nothing can go faster than light, the event can send and receive messages only along lines more vertical than the surface of the cone. Thus it can communicate only with other events that lie inside the cone. It cannot receive messages from past events outside the cone nor send messages to future events outside the cone.

For determinism the question is whether knowledge and influence coincide. As Sachs puts it: "It's a theory of who can communicate with whom." As a system proceeds from event to event along its world-line, its light cone moves with it and may change in volume. If I, on the basis of what I know of my past make a prediction about my state ten minutes from now, am I likely to be surprised when I get there? Will I find that my past light cone has shifted so that an event that I could not know about when I made the prediction is nevertheless capable of reaching and influencing my new state? If so, the space is nondeterministic and we are in danger of having to roll up the scrolls of physics.

But when we try to define a deterministic space to save ourselves, Sachs and his co-workers find, other dangers arise. The definition of such a space is deceptively simple. A space, call it, M, is deterministic if every event, x, can predict its own future from its own past. It seems reasonable, says Sachs, but it leads to extraordinarily stringent constraints on space-time, perhaps too stringent to do physics according to the laws we think we do physics by.

Determinism places some restrictions on the behavior of world-lines, indeed on all timelike curves (those that move mostly in the timelike direction) whether they be world-lines or not. If they enter our light cone in the future, they must also do so in the past. They may stray out of the light cone for a while, but while out they must not do certain things. They may not disappear into a singularity (that is, fall through a hole in space, for example). They may not, in the past, bend themselves so as to run parallel (or asymptotic to) the surface of our light cone, staying out of it indefinitely.

The latter circumstances could bring us a nasty surprise from the infinite past. Sachs' example is an enormously energetic photon that is always coming toward us and will blow up the room and disturb our predictions. We can't have things like that in a deterministic universe. Nor can we have things vanishing—pop goes the weasel—out of the universe at random, which is what the singularity business involves.

In the light of these criteria Sachs goes on to consider two of the most important space-times in contemporary physics, Minkowski space-time, which is closely related to special relativity and important for the dynamics of elementary particle physics, and Robertson-Walker space-time, which is associated with the big-bang type of cosmology that is widely believed in nowadays. He finds that they both suffer from surprise or singularity difficulties or both. "Why is it we can do physics in these space-times?" he asks. Manifestly, experimenters blithely go ahead and do physics in them, and so far nobody has been surprised by an infinitely old photon or at least lived to tell the tale. In theory, the bumblebee can't possibly fly, and yet it does.

"Perhaps we are not supposed to ask these questions of a nonquantum theory," Sachs suggests. General relativity lacks a quantized formulation that would describe its application to the subatomic world (SN: 4/12/75, p. 245). It is thus an incomplete theory, and when and if it is completed, the solution to this problem may appear. The additional information (from the surprises) may be very small, Sachs speculates, enough to make life interesting but not enough to upset predictions. Or physics might contain laws that are laws about initial conditions. (This last would be philosophically upsetting to many because laws about initial conditions would be arbitrary, and physical laws are not supposed to be arbitrary.)

Those questions pending, Sachs goes on to ask: "Are there any space-times where we don't run into this dilemma? He finds some, but when he defines their characteristics, they turn out to be equally appalling for traditional physics. First they change the definition of prediction. Not only can every observer predict his own future, he can predict everything about the whole space-time, a kind of universal omniscience. Most surprising according to Sachs is that a preferred frame of reference appears. "You are at rest with respect to the universe as a whole if you are moving toward the future in such a way that the volume of your past increases as slowly as possible." A privileged frame of reference is a big no-no in most contemporary physicists' view. Furthermore, the deterministic space-times violate certain cherished physical laws, such as the energy conditions of fluid dynamics.

"The nasty problem is how can we do any physics at all?" Sachs asks. How indeed? Does the geometry really perhaps not apply? Physics is after all done. Or is the physics an illusion? Will the quantization of general relativity give us a way to go between the horns of the dilemma *y matar el toro?* Or shall we all go to a pagoda and meditate? The problem is a serious one. Physics, like justice, must not only be done, it must be seen to be done according to first principles. We can hope the future will supply the solution. □