Four-color conjecture verified

One of the most fascinating unsolved problems of mathematics has finally been resolved by an intricate proof based on a unique symbiosis of mathematician and machine. The problem, known as the four-color conjecture, has tantalized and rebuffed mathematicians throughout the last hundred years. The solution, completed just last week, required a four-year analysis with over 10 billion logical steps.

The four-color conjecture says, roughly, that no more than four colors are required to color a map in such a way that four contiguous regions are assigned different colors. While the solution to this problem is of no practical use to cartographers, the century-long endeavor to solve it generated a whole new branch of mathematics called graph-theory that has had significant impact on fields such as computer science and operations research.

The proof of the four-color conjecture was carried out at the University of Illinois by Kenneth Appel and Wolfgang Haken. Their strategy involved a sophisticated variation of a method used by A. B. Kempe in 1879 in the first of many false "proofs" of the conjecture. The coloring of certain complex configurations is deduced from the coloring of other configurations involving fewer regions, thereby "reducing" certain cases to certain simpler ones. Whereas Kempe thought that only a few reducible cases were required, Appel and Haken (and their computer) catalogued over 1,900 of them.

It took three and a half years for Appel and Haken to devise an algorithm that would, so to speak, prepare a graph for color. They developed this algorithm in close cooperation with a computer: Whenever the computer got stuck on a particular configuration, the mathematicians used their theory and intuition to reprogram it with sufficient new insight to overcome the impediment. After the preparation algorithm was completed last February, it took six months of relatively routine computer-assisted computation to complete the proof by developing and verifying the catalog of reducible cases.

Most mathematicians who worked on this problem in the past believed that the conjecture was true, although many felt that it might never be proved due to the extraordinary complexity of large maps. Fifty years of intensive work in the first half of the 20th century produced proof of the conjecture only for maps containing fewer than 40 regions. Thorough search of coloring possibilities for maps larger than this is computationally impossible since there may be as many as 10^{40} different cases. □

Light and reproduction

Female reproduction is influenced by light, and several studies using light in varying applications have provided further evidence for this. For instance, changes in daylength are primarily responsible for synchronizing breeding with time of year in female mammals and birds. Female animals can be brought into heat by exposing them to long periods of light during the winter, and their estrus can be prematurely terminated if they are exposed to short periods of light during the summer. Light has even been used to correct women's irregular menstrual periods and to help them become pregnant.

The precise way that light does affect female reproduction is becoming increasingly clear. Three areas of the brain and at least one neurotransmitter chemical are involved in mediating the effects.

The two areas of the brain that have been long known to control the release of sex hormones in the body are the hypothalamus and pituitary gland—both part of the more primitive (limbic) area of the brain. The hypothalamus makes peptide releasing factors that in turn influence the release of the sex hormones from the pituitary. One might easily assume the light influences reproduction by acting directly on these brain structures. However, this does not seem to be the case. A 1956 experiment, for example, showed that lesions of the hypothalamus do not interfere with the effects of light on estrus. Researchers began looking for some other area of the brain that might mediate the effects of light and then transmit the effects to the hypothalamus and the pituitary. The area they hit upon was the pineal—a pine-cone-shaped gland located in the limbic area near the hypothalamus and the pituitary—the gland that the French philosopher Descartes thought was "the seat of the soul." Removal of the pineal gland prevents the usual effect of artificial light in accelerating the onset of estrus in ferrets, J. Herbert of the University of Cambridge reported in 1970.

Meanwhile, other researchers looked for chemicals in the brain that might mediate the effects of light on female reproduction. They found one that looked promising—a neurotransmitter called 5-hydroxytryptamine (5-HT). Their results: The pineal contains large concentrations of 5-HT; 5-HT concentrations in the pineal vary with light exposure; and 5-HT controls daily biological rhythms such as eating, drinking and running, suggesting that it might also control seasonal and annual rhythms such as female reproduction. Concentrations of 5-HT vary with light exposure not only in the pineal but also in the hypothalamus, Herbert and Carol Yates of Cambridge now report in the July 15 NATURE.

The implications of this most recent discovery about light and reproduction, in view of past findings, is that 5-HT in the pineal first detects the amount of light in the environment, then transmits the message to 5-HT in the hypothalamus. Hypothalamic factors might then pass the message on to the pituitary, which would release or withhold sex hormones. □

Earthquakes: Radon link

Geologists believe they may have a new tool for predicting earthquakes—a sharp rise in the amount of dissolved radon in well-water. This radioactive, inert gas is formed when the trace amounts of uranium found in most rocks decay. As rocks expand in response to mounting strain just before an earthquake, more radon is released and dissolved in groundwater.

The first observation of this phenomenon was made in Russia during the mid-1960s, but it recently came to prominence when the Chinese apparently used radon measurements (as well as other factors) to successfully predict a quake within a few hours of its occurrence. Four projects to study the matter are currently underway in the United States.

One of these efforts, conducted by a team of scientists at the Lawrence Berkeley Laboratory, was described at the Health Physics Society's annual meeting last month in San Francisco by team leader Alan R. Smith. Field data collected from the site of the 1975 Oroville, Calif., quake suggest "a correlation between changes in the radon content of several wells and the ongoing seismic activity," says Smith. Similar measurements are underway at the "Palmdale Bulge" in southern California, where surface elevation is thought to herald a major quake within the next couple of years. □

Deep sea Leg 48

Leg 48 of the internationally sponsored Deep Sea Drilling Project has shown that an ancient mountain range, which has now sunk to a depth of 4,000 feet below the sea surface, once existed between Greenland and Europe. Project scientists also discovered that the edge of the European continent, near Spain, was once bounded by a hitherto unknown swamp and shallow sea, abounding with coral reefs. Both are now 10,000 feet below the Bay of Biscay.

The drilling was done by the research vessel Glomar Challenger, which ended Leg 48 of its ongoing mission July 13. Recent cruises have concentrated on finding out why the edges of the continents are as we see them today, and on understanding how the forces that caused continental drift apparently raised and lowered mountain ranges like the one just discovered. □