Physics Without Limits

Reworking physics without calculus may not be physics without tears, but it does provide some new insights and philosophical questions

BY DIETRICK E. THOMSEN

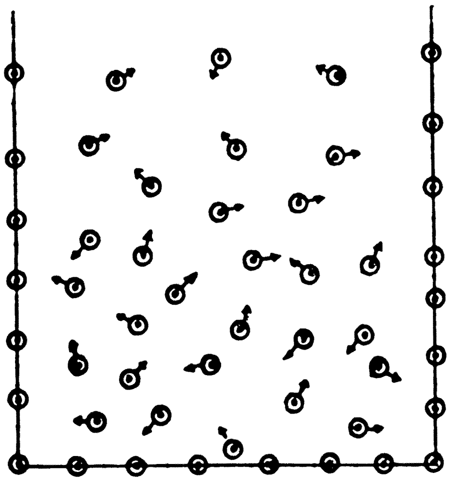
Isaac Newton invented the calculus. He did it in order to lay the foundations of classical mechanics. (Gottfried Wilhelm Leibniz also invented the calculus at the same time and for a similar purpose.) Newton and his contemporaries were caught up in the contemplation of continuous processes. They inhabited a space that looked like a constant unbroken extension from here to infinity. Time was an unbroken flow from eternity to eternity.

Newton was bemused by smooth, continuous unbroken motions like the orbiting of moons and planets. In Leibniz's case it was more the flow of heat, but the principle of continuity is the same. One might say that their motto was the ancient Greek saying "panta rhei," all things flow (which is in fact the motto of the modern Society of Rheology), and the mathematics they developed was designed to deal with continuous processes. The calculus was such a success that it has been the basis of classical and a lot of postclassical physics.

But nature doesn't always imitate art. Nature can be very bumpy in places. What looks continuous, like an iron bar or a flowing brook, may often, on microscopic examination, turn out to be a series of lumps. This is not necessarily so bad; often if a series of lumps come close enough together, they can be pushed into a semblance of continuity. However, if the lumps have kinks and sudden jumps, very often the continuous mathematics cannot deal with them. In a number of practical problems in applied physics, elements of the actual situation are disregarded in the calculations because the mathematics can't handle them. This can lead to mathematical models with a rather distant congruence to reality.

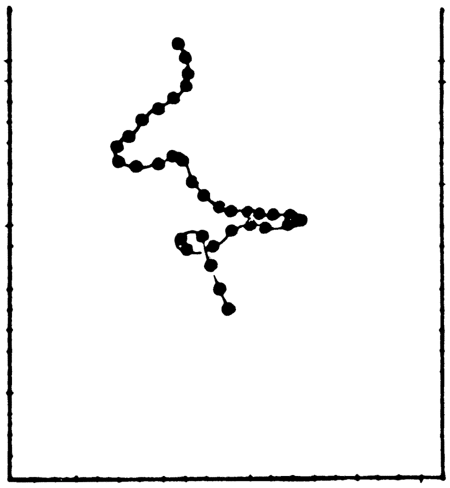
The advent of modern computers adds an even bigger problem. Computers cannot deal in continuous mathematics; they must have the discreteness of arithmetic. So to use computers in physics, arithmetic versions of the equations must be devised, even though these may be only numerical approximations to the elegant continuities of calculus.

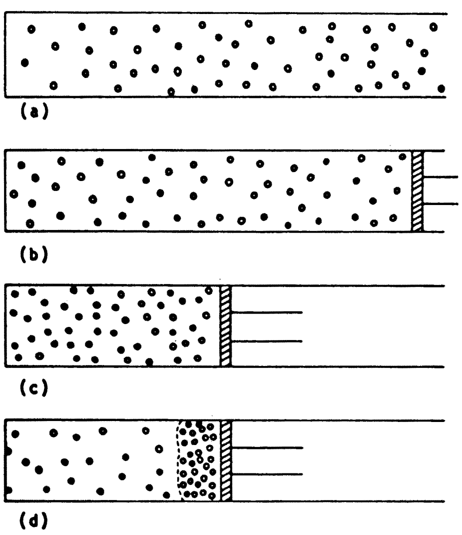
There is also a pedagogical difficulty. Children start out in school with arithmetic. They learn to add discrete oranges to discrete oranges and subtract discrete pears from discrete pears (if they're doing it right). The transition to calculus with its continuities, its infinitesimals, its limits, its instantaneous derivatives is an intellectual leap. It is the traditional wisdom of mathematical pedagogues that this leap is not taken until the last years of secondary instruction or the first years of college. This puts secondary school physics instructors in a bind, because without calculus they are not in a position to justify many of the laws and formulas they present, but must wave their hands and appeal to the elders. This is a procedure that has always been likely to generate student resistance.



Illustrations: Greenspan

Computer generated model of molecules in a heavy gas. Diagram below shows motion of a light molecule introduced to the gas and illustrates the property of buoyancy.





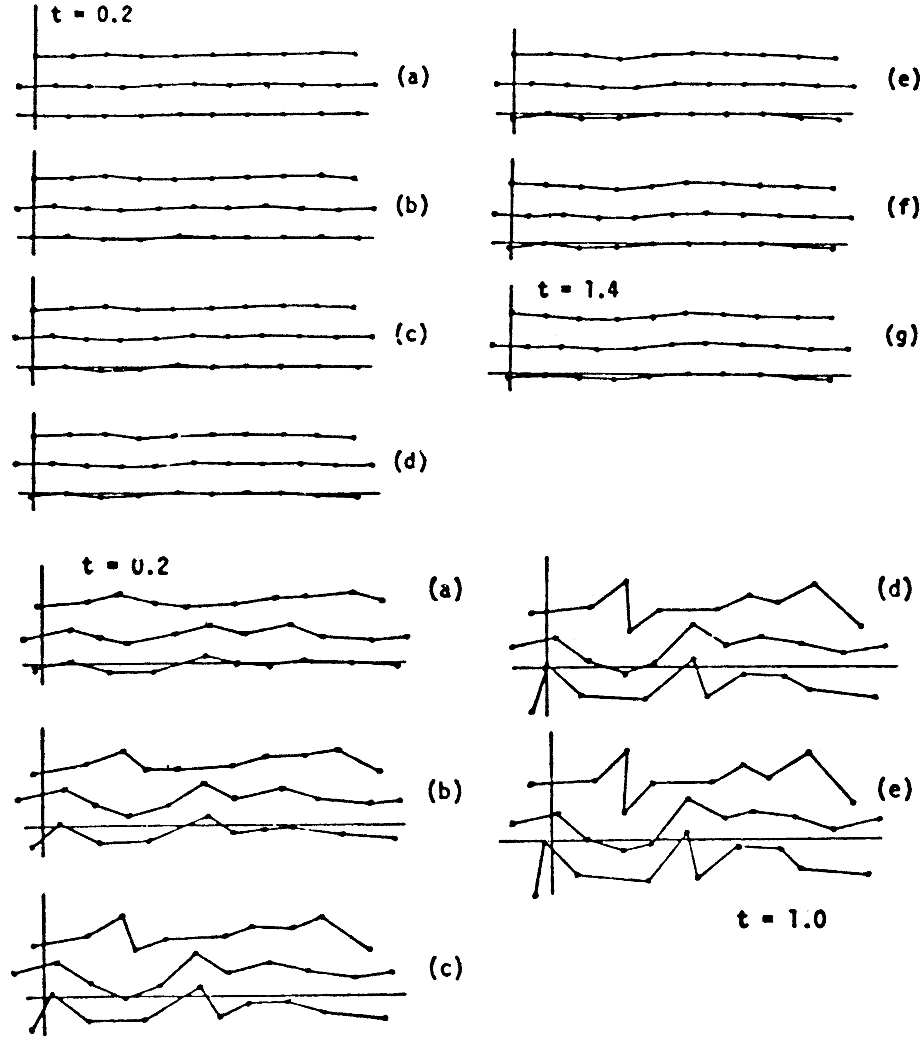
A plunger slowly entering a tube of gas will merely redistribute the molecules uniformly (a, b, c). Above a certain speed, the motion of the plunger forms a shock wave (d).

The necessity for feeding physics to computers in ways that they can digest it started Donald Greenspan of the Department of Computer Sciences at the University of Wisconsin at Madison on a program of reformulating the basic mathematical statements of physics in arithmetical terms. He began with basic Newtonian dynamics, has added fluid dynamics and continuum mechanics and is now into special relativity. He reviews his progress to date in an article in COMPUTERS AND MATHEMATICS WITH APPLICATIONS.

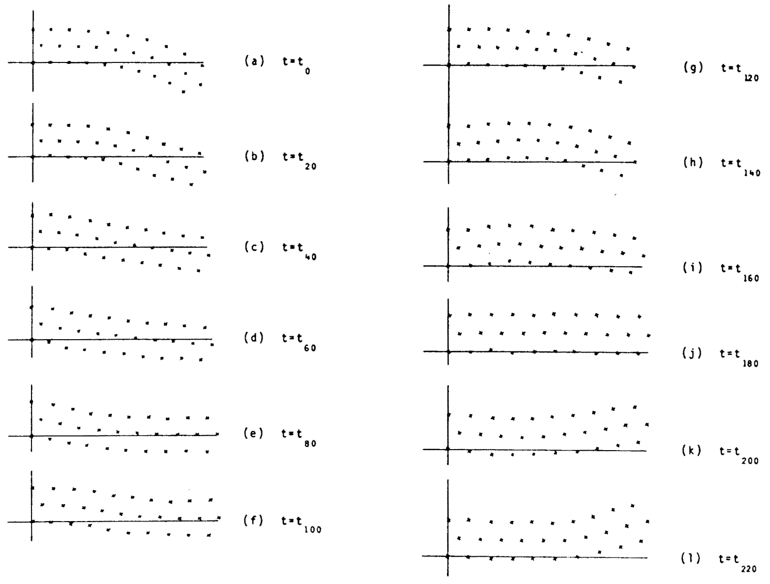
Greenspan's procedure starts with a tabulation of the data from a physical event, for example, snapshots of the position from time to time of a falling object. He uses the data to set up equations for the average velocity of the body over the intervals between snaps and its acceleration. This differs basically from the calculus procedure, which seeks expressions that will give the instantaneous value of the body's velocity at any time. In fact, one of the basic calculus procedures, differentiation, was invented to give meaning and intellectual respectability to such a notion as an instantaneous velocity.

Greenspan finds that he can start from his equations for average values and derive the basic formulas of Newtonian dynamics without recourse to calculus. This is especially true for the conservation laws, which describe the basic intellectual and philosophical content of the science. Conservation of energy, for example, comes out exactly as it does by calculus methods. The formula is the same, and what is extremely important, it comes out independent of the length of the time interval Greenspan chooses for his averages, just as the derivation by calculus methods comes out independent of the (continuous and instantaneous) time.

This leads Greenspan to suggest that what he has is not merely an approximate method useful for training computers,



Flow of a liquid emerging from a nozzle is laminar at low speed and develops more and more chaotically as the speed of flow increases. Eventually the motion becomes turbulent.



Computer simulation of flutter motion in a vibrating bar as determined by arithmetical methods of deriving physics equations.

but an alternate to the calculus procedure that has an intellectual rigor of its own. It raises the philosophical question whether the notions of continuity and differentiability that lie at the basis of much of physics really need to be there. It may also provide a respectable alternative for secondary-school teachers.

When it comes to the dynamics of aggregates, such as metal bars or tubes of gas, with their many individual molecules, Greenspan finds he can even go the calculus methods a few points better. It seems that the discreteness of arithmetic matches the discreteness of the structures and allows physical factors to be brought into the mathematics that the traditional dynamics had to ignore. Take, for example, the elastic vibration of a flexible bar. The arithmetic methods show, counter to the calculus treatment, that the bar does not swing smoothly, but flutters up because a wave motion that travels through the bar is superimposed on the upward swing. "Engineers have been aware, for some time, of such waves on the surfaces of vibrating materials," Greenspan writes. It is also possible to treat the physics of bars without imagining them to be of infinite length, as traditional continuous formulas often require.

The arithmetic methods can also provide more realistic models of fluid behavior, such as liquid spraying through a nozzle, laminar, vortical and turbulent flows generally, and even some unusual splash effects in melting metals.

From classical mechanics, arithmetical methods can move on into special relativity. Special relativity represents both an intellectual and mathematical complication of classical mechanics. One must deal with mindbenders like the constancy of the speed of light and the conceptual equivalence of mass and energy. In the traditional treatment, not only must the dynamical equations be continuous, but so must the transformation equations by which the dynamical equations are translated from terms appropriate to one frame of reference to those appropriate to another. And the constancy of the speed of light in all special-relativistic frames insures that the transformation equations are not a relatively trivial part of the theory as they are in Newtonian dynamics.

Yet Greenspan finds that one can go from the second grade to the Lorentz transformation without stopping at Grandpa Leibniz's house on the way. He finds he can derive the basic conservation laws in the forms appropriate to special relativity. Using arithmetical methods he can also obtain such relativistic exotica as $E = mc^2$, the Einsteinian law relating force to rate of change of momentum and the formula for the increase of mass with velocity. And he concludes that if you anchor one frame of reference to the ground and another in a rocket and install identical computers in both frames, you can compute with his statement of the Einsteinian force law and all the resultant computations would be related by the Lorentz transformation.

So what appears to be developing here is not only a method for the care and feeding of computers, but a challenge to some of the basic assumptions of traditional mathematical physics. In the future, Greenspan hopes to take his methods into general relativity, electromagnetic wave theory and quantum mechanics. □