OFF THE BEAT

Dimensional analysis: Miracle worker of rapid calculation

Unbelievably, this particular statistic had not been among the myriad others with which any tourist to this floating attraction is bombarded, so I expected the question. "I wonder how much water the *Queen Mary* displaces," he asked. We — a cousin and I — were touring the massive British steamship, which now is permanently moored at Long Beach, Calif., and like any worthy scientist, I immediately seized this opportunity to indulge in a rather routine back-of-the-envelope calculation (BOTEC).

The swiftness of my extemporaneous calculations seemed to impress him as much as the final result did. I, however, responded to his admiring effusion with veteran poise, "It's a cheap enough thrill."

Indeed, no scientific trademark is more universally renowned or certain to elicit a cloud of ooohs and aaahs than the BOTEC. It is, quite simply put, a beautiful art form that bears the same relation to formal scientific analysis as a pencil sketch does to an oil painting. Accordingly, not all scientists are adept at BOTEC's — it is a skill acquired separately from classroom training.

Compared to the tedium and protracted suspense of a rigorous theoretical analysis, the BOTEC is a quick charge, like a heart-fluttering ride on the roller coaster. Ask most graduate students, in fact, and they will recall for you their first BOTEC with as much self-satisfaction as any pilot describing a first solo flight.

Actually, BOTEC is a misnomer. Often as not, the hastily wrought computations are scribbled on a variety of makeshift writing surfaces, including napkins, grocery bags, clothes (not necessarily one's own), newspaper margins, restroom walls and, yes, HMS *Queen Mary* brochures. After all, an impatient stroke of inspiration, a scientific squabble or a trivia question cannot always wait for acquisition of the orthodox stationery.

Enrico Fermi was wont to decorate tablecloths with his cryptic formulae whenever he dined out and envelopes were unavailable. And barring all else, any scientist knows that flesh is not an unreasonable last resort. In sum, the urge to do a BOTEC can compel a god-fearing person to deface just about anything within arm's reach.

At a gathering of scientists, BOTEC's are an invaluable surrogate language which, among other things, defies comprehension by the laity. Yet one of the favorite techniques that underlies a sizable fraction of these arcane exchanges is a surprisingly elementary reasoning process, which its founder, Sir Isaac Newton, ostentatiously dubbed the "principle of similitude." Originally described in his *Principia* (1687), the idea has evolved to a modern, expanded version called "dimensional analysis." Before finally demonstrating it using the *Queen Mary* problem, I would like to extoll the technique's virtues and distinguished history.

Dimensional analysis is the St. Christopher of many physics students, who during an exam find themselves unable to recall precisely a crucial mathematical equation; use of the technique usually resolves the confusion with ease and dispatch. So unusually straightforward is the method, in fact, that novices often manage to discover it for themselves under such pressure conditions and, having done so, usually come away from the exam feeling proud of their resourcefulness, but also guilty for having somehow cheated.

The technique's peerless blend of leanness and utility has earned it a noble and enduring niche in science. Still, it is uncertain whether dimensional analysis has been and is being used to its full potential. This doubt was once expressed by the great physicist and Nobel laureate Lord Rayleigh in a 1915 NATURE article: *"I have often been impressed by the scanty attention paid even by original workers in physics to the great principle of similitude. It happens not infrequently that results in the form of 'laws' are put forward as novelties on the basis of elaborate experiments, which might have been predicted* a priori *after a few minutes' consideration."*

His final assertion is not an exaggeration, but an apt testimonial of the extraordinary implications of this method. Typically, a physical law is fruit born by the toil and perceptive genius of many men and women:

- Newton's universal law of gravitation depended for its final presentation on the solution of a stubborn mathematical problem, which took the great genius more than two decades to accomplish. The publication of his *Principia* is thought to have been delayed as a consequence.
- Johannes Kepler's venerable three laws were a result of the astronomer's painstaking appraisal of Tycho Brahe's assiduous observations of planetary motions through a telescope.
- Einstein was bedridden for several weeks following the finalization of his celebrated theory of general relativity. The monumental, decade-long effort had totally exhausted him.

It is only within this context that dimensional analysis can fully be appreciated. For, in many instances, this humble theoretical yeoman is capable of "rediscovering" aristocratic laws with embarrassing and miraculous ease. In some cases, it also reveals something else, something quite remarkable and often unexpected: Based on rudimentary, logical considerations alone, the form of a particular law is uniquely predetermined, so that its dis-

covery in that form is inevitable and, therefore, readily foreseeable — dimensional analysis essentially short-circuits the formal process. Thus, one needn't understand the complicated physics behind Kepler's third law in order to formulate it and assert, furthermore, that it couldn't have been said in any other way.

Philosophers of science, therefore, regard the success of dimensional analysis as evidence that human reason, unassisted by experiment, can deduce the laws of nature. The modern concept, however, which derives from the Renaissance, has it that empiricism is ultimately and necessarily the arbiter of scientific truth.

One notable advocate of the contending philosophy was the late, great Sir Arthur Eddington, who in 1919 directed the experiment that originally verified Einstein's exciting prediction that light rays were bent by a gravitational field. He wrote:

"I believe that... all the laws of nature that are usually classed as fundamental can be foreseen wholly from epistemological considerations... [thus] all the fundamental laws and constants of physics can be deduced unambiguously from a priori *considerations, and are therefore subjective."*

This remarkable point of view — often called "selective subjectivism" — is as provocative today as it ever has been. And though it clearly deserves it, further elaboration here would be too great a digression. It is, instead, time now to address the *Queen Mary* problem using this marvel of human reasoning. As preparation, you shall need one regular-sized envelope.

In order to obtain the appropriate equation, follow this general recipe, which is the heart of dimensional analysis:

(1) Decide which factors in the problem are likely to influence the unknown quantity.

(2) In any combination, multiply and divide these factors so that the outcome has the same units of measurement as the unknown quantity. This "winning" combination, which may include a multiplicity of any factor, is set equal to the unknown quantity.

For step (1), I suggest you guess that the amount of displaced water is determined by the *Queen*'s bulk-mass and the density of water (since, for example, if it were floating on a denser liquid, like maple syrup, the ship would have a shallower draft).

Fine. Now proceed by trial and error to fulfill step (2). Remarkably, you will find there is only one possible combination of mass and density (which is measured as mass per volume) that has the units of volume. It is "mass divided by density," and according to dimensional analysis this quotient is equal to the unknown quantity.

With the equation now in hand, you only need to assign correct values to the factors and thereby obtain an answer for the unknown quantity. When I did this for my cousin, I recalled that the density of water is one gram per cubic centimeter (a handy number worth remembering) and, being where I was, it wasn't difficult to find out that the gigantic steamer weighs 81,237 tons, which converts to about 74 billion grams.

Doing the requisite division, you will find that the Cunard liner displaces 74 billion cubic centimeters, or about 18 million gallons — enough to fill a small lake!

You might be surprised to learn that the equation we derived so effortlessly is none-other than Archimedes' famous principle of buoyancy, which states that any floating object displaces a weight of fluid equal to its own. Discovered in the third century B.C., the idea reportedly occurred to the Greek genius one day while he sat in the bathtub.

For those of you who are now suitably impressed with dimensional analysis and anxious to "solo" on a BOTEC, I offer the following question: "About how fast will a penny dropped from the Empire State Building be falling only four seconds after its release (ignore air resistance)?" The answer will amaze you, especially when expressed in miles-per-hour. Bon BOTEC!

—Michael A. Guillen

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...Laetrile

retrospectively review human case records to see whether there is any hard clinical evidence that Laetrile can help cancer patients. It there is, the NCI may then undertake a clinical trial to further document and clarify Laetrile's effects on cancer patients.

A clinical trial, of course, would be the quickest and most scientific way to determine whether Laetrile can help cancer patients or not. But the NCI is not anxious to rush into one because of the ethical question of whether it is right to give some cancer patients Laetrile only. An editorial in the Jan. 26 NEW ENGLAND JOURNAL OF MEDICINE points out that this ethical question might be solved "if all patients were fully informed and if the study were designed so that under no circumstances would a patient with a tumor known to be treatable by conventional therapy be assigned Laetrile treatment alone." □

EPA attacks organics

The first proposed regulations to reduce human exposure to organic chemicals in drinking water (SN: 12/31/77, p. 428) were issued last week by the Environmental Protection Agency. Regulations on trihalomethanes — formed when chlorine combines with decaying plant matter — would restrict their concentration to less than 100 parts per billion. Other organics would be caught with activated-carbon filtration systems. □