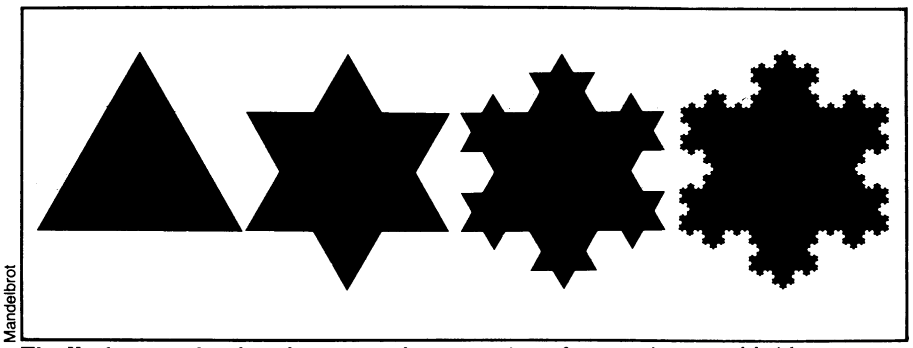
MAKING MUSIC–FRACTALLY

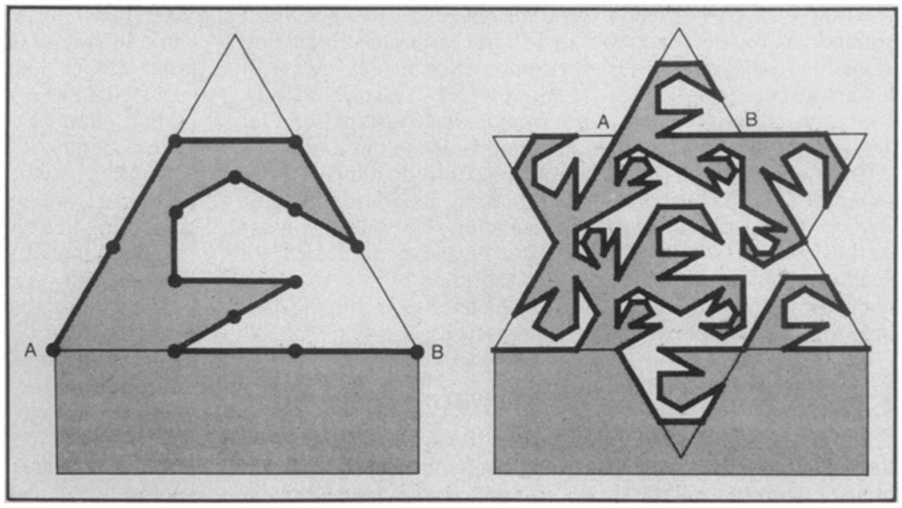
Music hath charms to inflame the savage mind of a mathematician. It makes him think of 1/*f* noise.

BY DIETRICK E. THOMSEN



Mandelbrot

The Koch curve develops by repeated construction of new points on old sides.



The first steps in making the Peano curve. It gets rapidly more complicated.

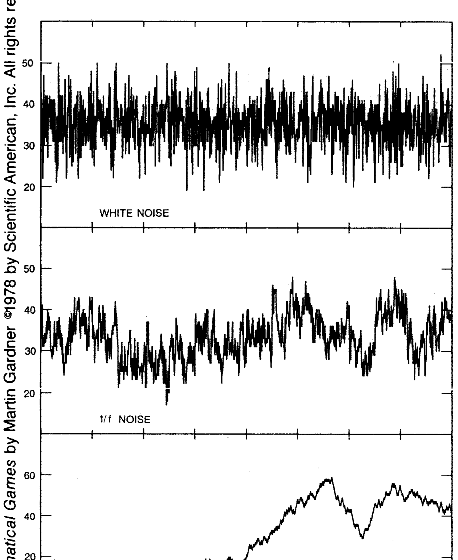
A certain professor of music, a specialist in the life and work of J. S. Bach, used to take pains to combat the notion that Bach and his contemporaries wrote "mathematicians' music," that you had to be a mathematician to appreciate a toccata or fugue. He insisted that there was an esthetic quality to a Bach fugue that was quite apart from any ability to analyze the numerical relations in the rhythmic and melodic progressions.

This professor's esthetic sense might have been gratified — or maybe not — to learn that the structure of music is based on fractals, those lacy, snowflaky figures with dimensions somewhere between one and two or two and three dimensions (SN: 8/20/77, p. 122). He is likely to have been unhappy with the statement that the common element of all music is what physicists call 1/*f* noise. To anyone who composes very dissonant modern music and has to contend with traditionalist critics "noise" is a touchy word indeed.

Whether musicians like it or not, those two propositions are made by Richard F. Voss of the IBM Watson Research Center, who made an analysis of such things and reported his conclusions at the recent meeting of the American Physical Society in Chicago. Voss's basic premises are that the "time correlations found in music are the same as 1/*f* noise, and the structure of music is fractal."

Besides the strange quality of having dimensions like 1.25 instead of 1 or 2, the most important characteristic of fractals is what is called selfsymmetry. A small piece of the figure mirrors the shape of a larger piece.

The classic example of selfsymmetry is the figure known as the Koch curve, after the German mathematician Helge von Koch, who pointed out its unique qualities. The Koch curve starts with an equilateral triangle. Divide each side of the triangle into three equal parts. Remove the middle third and erect on it two sides of an equilateral triangle based on its length. This makes a six pointed star. Continue the process of subdividing sides and erecting new points. The figure becomes a very complicated snowflake. Continue the subdividing ad infinitum, and the length of the Koch curve, which has increased by $^4/_3$ with every new complication, also becomes infinite. But it still encloses a finite amount of the plane in which it is drawn.



Intensity vs. frequency relations distinguish the three kinds of noise spectrum.

Mathematicians found that absurd. A one-dimensional curve, a circle, a square, enclosing a finite segment of the plane should have a finite length. A curve of infinite length in a finite part of the plane is in effect a continuous surface of dimension 2; it covers all the available space. But the Koch curve doesn't. There is space inside it that is not of it. It is a hybrid between one and two dimensions. Contemplating it, mathematicians had to redefine the term "dimension" so that fractal dimensions could be calculated for it and similar hybrids. The dimension of the Koch curve comes out about 1.26.

Another fractal, the Peano curve, is almost a continuous surface. It is a line so drawn that absolutely every point can come to fall on a boundary between black and white. Its dimension is almost two. "It comes almost to the dimension of the space that contains it," Voss says. Fractals between two and three are also possible. Dimension 2.25 looks like a relief map of mountains and valleys. That's a useful image. One of the main practical applications of fractals is in mapping problems.

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growth within the transparent cornea. Again they found that the vitreous fluid chemical, like the cartilage chemical, was able to keep the blood vessels from contacting the tumor. Allan Fenselau, a biochemist at Hopkins, meanwhile had developed a cell culture system with aortic endothelial cells, the major cell type in blood vessels. The vitreous inhibitor was found to prevent the growth of the endothelial cells in this system without damaging them.

But now still another provocative question surfaced: Is the vitreous inhibitor the same as the cartilage inhibitor? To date, Folkman, Henry Brem and Langer are still not sure whether their inhibitor (taken from calves) is a protein, although they have a hunch it is. The Hopkins team is relatively certain that their inhibitor is a large protein that is very stable, since it resists boiling. How long will it take the Harvard and Hopkins scientists to thoroughly purify and identify their inhibitors? Langer replies: "That is a hard question on which to make a prediction. The animal assays we use are difficult and also take a lot of material." Fenselau concurs. So does Brem: "We never know what roadblocks lie ahead. Every one of the steps so far could have taken 10 years."

Even assuming that the two inhibitors are purified and identified within the next five years, the challenge remains of getting enough of the chemicals for animal and clinical tests. (It now takes literally tons of cartilage to test the cartilage inhibitor in a few rabbits.) But, assuming that's possible, the ultimate challenge lies in seeing whether the inhibitors can alter blood vessel proliferation not only in solid tumors but in the retinas of patients with diabetic retinopathy. The vitreous inhibitor might also be tested on brain cancer patients, because brain tumors are the most vascularized of all tumors. And while laser photocoagulation can now seal off damaged, proliferated blood vessels in the retina and prolong the sight of diabetic retinopathy victims (SN: 4/10/76, p. 232), the inhibitors might turn out to be superior to photocoagulation treatment if they prevent blood vessel proliferation before it actually takes place.

And of course, assuming that the inhibitors turn out to be valuable treatments for cancer and diabetic retinopathy, they would still have to be thoroughly tested before the Food and Drug Administration would put them on the market. But would there be enough of the inhibitors for marketing? If they prove to be proteins, perhaps protein chemists could synthesize copies of the real stuff, or the protein could be mass-produced with recombinant DNA techniques — altogether new challenges in their own right. So the road is still a long one, but the researchers involved are optimistic about TAF inhibitors. Says Patz: "I'm encouraged about the possibility that they might ultimately have an application." □

. . . Fractals

Coastlines, river systems, islands and seas, mountains and lakes are all well modeled by different kinds of fractals. In Voss's analysis a musical composition becomes something of a linear mapping problem.

The other element in Voss's analysis is $1/f$ noise. This is a phenomenon long familiar to electronics engineers, although they are not likely to have considered it music. Information theory distinguishes three kinds of noise according to how the intensity of the sound at any given frequency depends upon the value of the frequency. They are white noise, in which there is no frequency dependence, a totally uncorrelated random spectrum, $1/f^2$ noise, which is related to Brownian motion and is well correlated and well understood, and $1/f$ noise, which is intermediate between the two, more correlated than white noise, less correlated than Brownian motion.

$1/f$ noise appears in numerous and varied natural phenomena. It first came to attention in the output of old-fashioned carbon microphones, but it occurs in all electronic components. It limits the accuracy with which time can be measured in instruments from hourglasses to crystal oscillators. Time correlations of this sort are found in undersea currents, Nile flood levels for a thousand years, ionic transport across nerve membranes. There is "no universal explanation for its occurrence," says Voss. And he poses a philosophical question about how music interacts with nature, whether "music imitates the way our world is changing in time."

An analysis of the sound spectrum of J. S. Bach's First Brandenburg Concerto fails to show any $1/f$ correlations. Yet you would expect them somehow in music, says Voss, because the structure of music is fractal, being divided into bars and sections hierarchically organized, being often selfsymmetric. $1/f$ noise has fractal relations, too. To get the formula for $1/f$ noise mathematicians divide the frequency range into short sections like musical bars and relate the sound intensity over each bar to the place of that bar in the succession of bars.

Examination of both loudness fluctuations and melodic variations yields success. Voss says he listened to a classical station, a rock station and even a news and talk show. Melodies from all over the world were checked, too. "There is really no difference between the Beatles and Beethoven," Voss says. "$1/f$ is the common element in music." Music from all over the world.

If this is so then it should be possible to take $1/f$ and map it back into recognizable music. First Voss and his co-workers checked music written by a random number generator. This sounds passing strange and looks weird on a score. It sounds strange because it has no time correlations. "Like monkeys banging on pianos," Voss says.

Science

Machine-made music based on random tone generation (top), $1/f^2$ noise and $1/f$ noise. $1/f$ seems most like real music.

For a second trial they decided to give it some real correlation and used what mathematicians call a Markoff process to generate imitations of a set model. The model was a bit of melody written by the sentimental composer Stephen Foster. The idea was to generate a song Foster might have written. The process did all too well. It kept coming up with songs Foster actually had written.

The monkeys were totally uncorrelated, and the Markoff process altogether too correlated. That convinced Voss that down the middle with $1/f$ was the way to go. He chose Gregorian chant for the first try because it has a very clean fractal structure, a triadic structure based on the Trinity. The idea was to generate short segments with $1/f$ time correlations and then map them into themselves with this triadic structure. The process generated a fake Gregorian chant that sounds like the real thing. It sounded surprisingly similar to a real one also exhibited.

Voss remarks that he did a similar thing with a piece of Chinese music and then played it for some Chinese students at Berkeley. They declared that it sounded familiar but not quite like what they heard at home — a little Vietnamese, perhaps. Voss thinks this is not bad. The fake music sounded not quite like the home product, but very close. So it seems to be upward and onward for composition by the $1/f$ plus fractal method: to multiple voices, polyphony, counterpoint, someday perhaps a tone poem on the fluctuations of the Nile. □