P. A. M. DIRAC

AN EYE FOR BEAUTY

BY MICHAEL A. GUILLEN

Leaf through any physics textbook and you'll encounter a handful of names attached to the various equations, laws and theories that constitute modern science: names like Newton, Einstein, Planck, Heisenberg, Schrödinger, Maxwell, Fermi and Dirac.

Many of these formidable scientists had their heyday some fifty to sixty years ago, and of those cited only Dirac still lives. More than that, he remains active as professor of physics at Florida State University at Tallahassee. So there was plenty of significance to a three-day symposium held last month at Loyola University in New Orleans to honor the 78-year-old eminent figure of the Golden Age of physics (see p. 397). Science News took this rare opportunity to perceive Dirac and his times through his own commentary and through the recollections of some of his closest associates and friends.

Dirac grew up during a revolutionary period in science — the first several decades of the twentieth century — when some of the greatest scientists who have ever lived collaborated to construct the theoretical underpinnings of modern physics, chemistry and astronomy. Their principal legacy lies in three magnificent theories: those of special relativity, general relativity and quantum mechanics. As part of this, Dirac can be seen as both a product and an innovator of his times, caught up in a worldwide tempest of radically new ideas.

Dirac will be the first to tell you, in fact, of his debt to his contemporaries — most of whom were his elders. This modesty is a major facet of Dirac's personality. "You have interviewed many great scientists," says Leopold Halpern of Florida State, Dirac's research associate and close personal friend of six years, but "Dirac is probably the most modest of them all." That might be true, but one mustn't be misled. Dirac has never been a follower. Quite the contrary, he is a loner, a maverick by nature and though in his time he refined, embellished and clarified ideas already in the air, he also discovered many wholly new and fundamental areas of research.

"My famous brother-in-law," physicist Eugene Wigner of Louisiana State University, fondly recalls Dirac. "I don't know what [in physics] he did *not* discover," he says quite sincerely. "Oh, I could think of a few things, but *very* few things and perhaps not of basic importance." He sizes up Dirac's importance to the history of science this way: "Of course the development of science is not dependent on single people, but I almost think that if the others had not been around, he would have created most of what exists... perhaps not so much the practical applications, but the basic theories, yes."

John A. Wheeler, physicist at the University of Texas at Austin and longtime colleague of Dirac, puts it this way: "Dirac's contribution showed that Einstein's revelation was not ended in the sense that considerations of simplicity and beauty could be made to yield definite equations which then could be compared to the experiment and checked out."

Dirac was only a baby in 1905 when Einstein published his theory of special relativity and barely a teenager in 1915, when general relativity threatened to supersede the old and venerated Newtonian theory of gravity. These years were formative ones for the young Dirac, who grew up with an immense admiration for Einstein (even though, Halpern says, Dirac often stresses that he and Einstein thought differently about certain theoretical matters). Einstein is unquestionably Dirac's principal hero, says Halpern: "His wife says that Dirac wept at Einstein's death... the only time she has ever seen him cry." Halpern also tells how "[Dirac went] from one Einstein centennial meeting to another all over the world. He didn't want to miss a single one."

Dirac admires especially the manner in which Einstein was able to explain so much about nature on the basis of so few presumptions. He admires, too, the elegant mathematics Einstein created to communicate concisely his insights into nature. Dirac calls this "beautiful mathematics."

And the concept of beautiful mathematics is so important to Dirac that he has advocated and pursued it with religious tenacity. It has become the very theme of his entire career.

In the past, Dirac has explained "beautiful mathematics" by calling attention to specific historical examples that have realized the concept's meaning, most notably Einstein's work. More generally, though, he describes it in terms of certain essentially related qualities: exactness, quintessence, simplicity and an economy of thought.

Dirac argues the unambiguity of what is meant by beautiful mathematics, a concept, he told Science News: "I guess I was just born with." To those critics who attempt to dismiss the concept as laudable but vague and entirely subjective, Dirac says: "I think if they don't understand it, they should just abandon their [theoretical] efforts."

At the symposium, one heard the phrases "pretty mathematics" and "beautiful mathematics" repeatedly invoked, principally in homage to Dirac's own work. Satisfactory definitions, however, were hard to come by. In the end, perhaps Wheeler's was the best effort; he told Science News: "You can call it art, you can call it beauty ... but I think it's close to rectitude and justice in some of its connotations."

Rectitude and *justice* seem especially appropriate words to use in connection with Dirac's long and fruitful career as a theoretical physicist. Although some of his most creative ideas engendered controversy, they were in the end vindicated by experimental results.

That is not to say that Dirac has never been wrong. As Wigner put it: "Everybody

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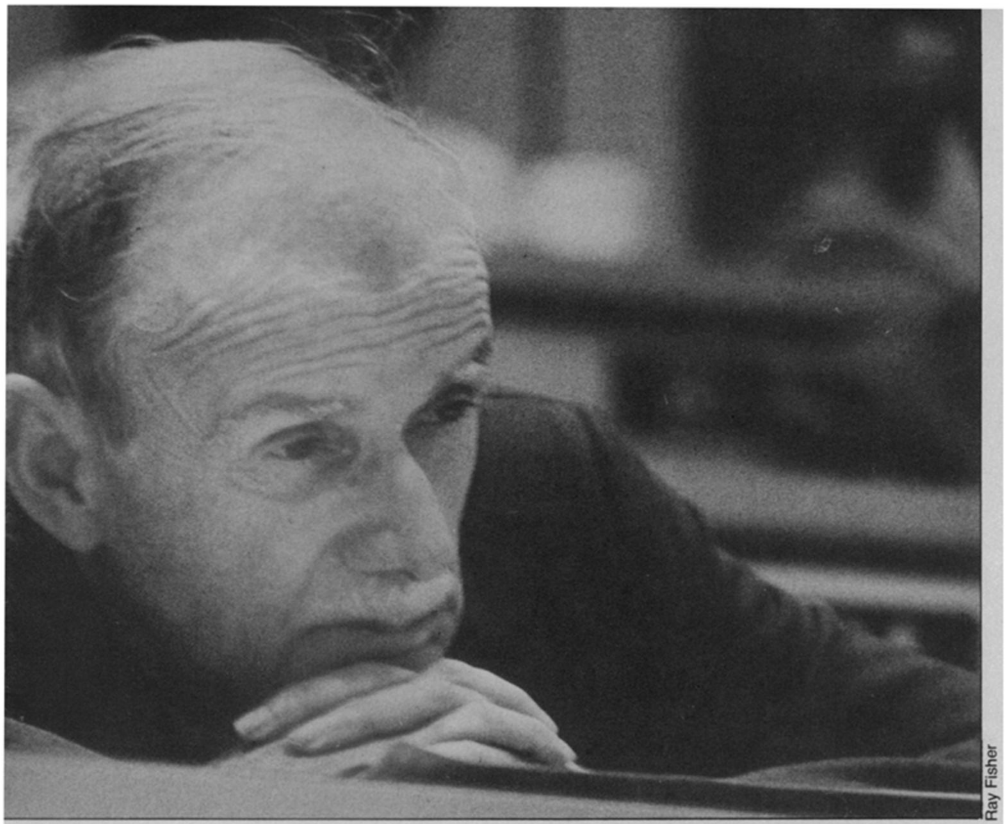
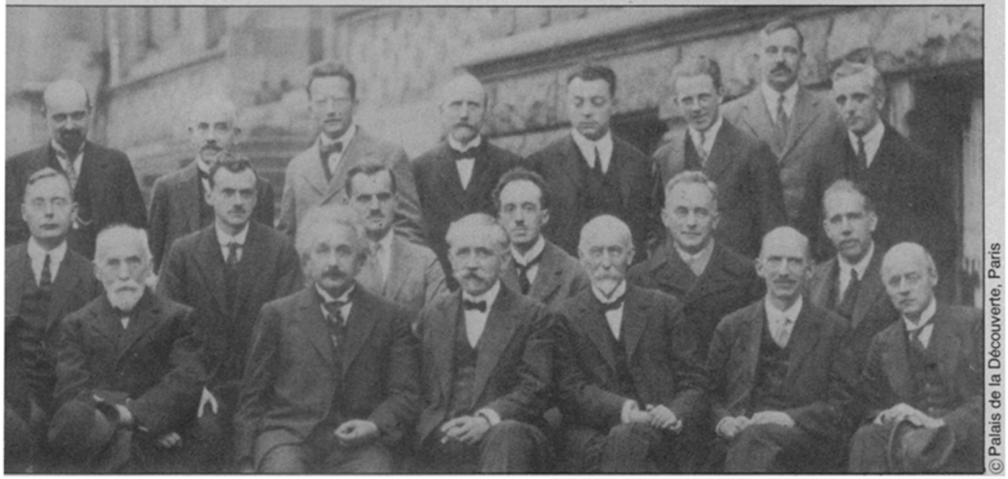


Ray Fisher



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Solvay Conference of 1927 with Dirac and Einstein in attendance.

First Row (from left): I. Langmeír, M. Planck, Madame Curie, H. A. Lorentz, A. Einstein, P. Langevin, Ch. E. Guye, C. T. R. Wilson, O. W. Richardson. Second Row: P. Debye, M. Knudsen, W. L. Bragg, H. A. Kramers, P. A. M. Dirac, A. H. Compton, L. V. de Broglie, M. Born, N. Bohr. Third Row: A. Piccard, E. Henriot, P. Ehrenfest, Ep. Herzen, Th. de Donder, E. Schroedinger, E. Verschaffelt, W. Pauli, W. Heisenberg, R. H. Fowler, L. Brillouin.

makes mistakes, even Paul." And over the years Dirac's imagination has often taken him in farflung directions that Dirac himself candidly terms "failures."

But Dirac is of course remembered for his major theoretical triumphs, the first of which came in 1926, when he was in his final year of graduate studies at St. John's College, Cambridge. At that time he elaborated a seminal version of quantum mechanics, the theory that replaces the tenet that natural processes can be described scientifically with unlimited exactitude with one that sees the universe as ever so

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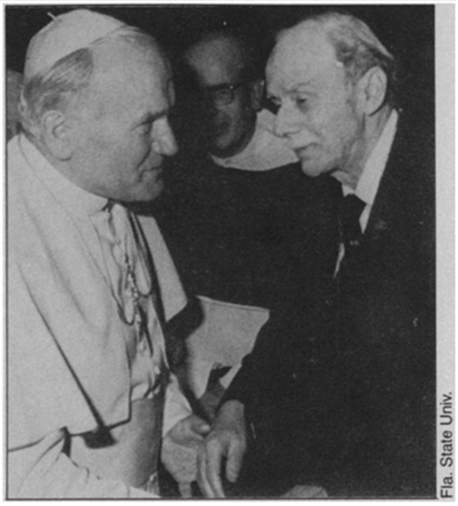


Fla. State Univ.

Is there an affinity between phenomenology and quantum physics? Pope John Paul II and Dirac might have discussed it, but it was a ceremonial occasion and they probably just exchanged pleasantries.

AIP Niels Bohr Library

Paul Adrien Máurice Dirac was born in Bristol, England, on August 8, 1902. In school, he demonstrated a precocity in mathematics that was encouraged by his father, a teacher of the French language. In 1921 he earned his B.Sc. in electrical engineering from the University of Bristol and, failing to find a job in that field, matriculated to graduate school at St. John's College, Cambridge. In 1926 he received his Ph.D. and was appointed a Fellow of the College.

In 1932, at the age of thirty, Dirac was made Lucasian Professor of Mathematics, the same endowed chair once held by Isaac Newton. In 1933 he shared the Nobel Prize in physics with Erwin Schrödinger for his fundamental contributions to atomic theory. In 1937 Dirac married Margit Wigner, sister of Eugene Wigner, himself a well-known physicist of considerable achievement and 1963 Nobel laureate.

Dirac was a visiting scientist at the Institute for Advanced Study in Princeton, N.J., during the academic years 1947/48 and 1958/59. Since 1969, he has been professor emeritus at Cambridge and since 1971 has been at Florida State University at Tallahassee as professor of physics.

DIRAC: FEW WORDS AND MANY IDEAS

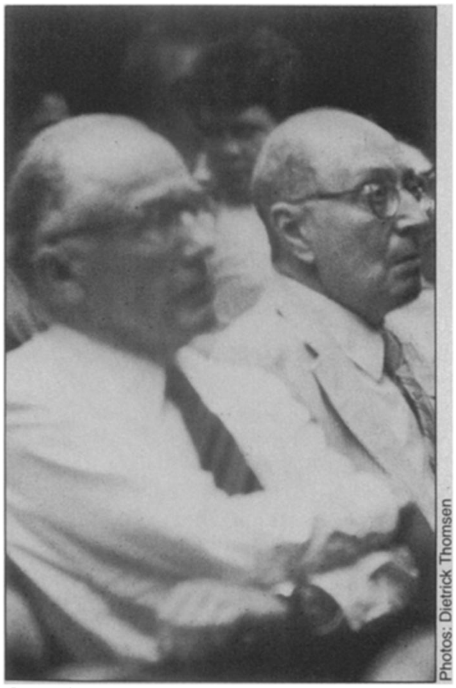
Over the past six decades, theoretical physicist P.A.M. Dirac has published several books and hundreds of professional papers. But as communicative as he is mathematically, Dirac is a man of few words. This is consistent with his life-long reputation of being inwardly directed, a man more at ease in the company of physics and its demands, it would seem, than of most people and theirs.

The recent symposium honoring Dirac (see pp. 394 and 397) gave me an opportunity to observe first-hand Dirac's (in) famous taciturnity and in the end understand it better.

The experience at first wasn't altogether pleasant. I had traveled to New Orleans with the anticipation of a scientist who, like so many others, had acquired a feeling of friendly intimacy with Dirac through studying his seminal contributions to modern physics and the expectations of a writer who was to interview him for a profile story. In both roles I was terribly disappointed when I was introduced to what initially struck me, with all due respect, as a cranky or just downright ornery old man.

Through the first two days Dirac threatened to renege on the interview he had agreed to a week before the symposium. As it was, he had already specified certain restrictions concerning the types of questions he would be willing to answer.

When the moment came to arrange a specific time for the interview, he was very uncooperative. At one point he peevishly protested to one of my queries: "I don't know anything about [the philosophy of science]." When I offered other choices of questions, including ones of history, Dirac eschewed them all and suggested that I interview his research assistant, Leopold E. Halpern, instead. An apologetic Halpern later explained that Dirac "does not like to deal with such subjects," thinking them too vague to bother with. Furthermore, Halpern counseled, "he likes his questions short and precise."



Photos: Dietrick Thomsen

John A. Wheeler and Eugene P. Wigner. Friends and relatives came from all over.

Even with the appropriate assurances delivered, however, Dirac persisted that I speak to someone else, this time: "You should talk to Wigner; he knows about such things, and he has so much energy, you know." (Eugene Wigner, Dirac's brother-in-law and Nobel physicist at Louisiana State University, is the same age as Dirac but exudes the vigor of a man 20 years younger.)

In the end, I obtained my interview, such as it was, in hurried little sessions. One rushed fragment had to be recorded while walking with Dirac the short distance between auditorium and cafeteria. Another brief interview was quickly and unceremoniously carried off during an afternoon coffee break on the very last day of the symposium.

In practically each case, Dirac's response to a question was terse. No words were wasted. But I quickly learned that what appeared to me as Dirac's severe frugality of words seemed to others who have known him for many years as positively a paroxysm of prolixity. "I have never seen him so talkative," one longtime colleague of Dirac put it to me later. Dirac was apparently pleased with the questions.

Had he acquired this behavior only recently, I wondered out loud over breakfast with Wigner, Halpern and John A. Wheeler of the University of Texas at Austin. No, not at all, Halpern assured me. In fact, "for awhile it was said about him [that] he says only three phrases: yes, no and I don't know." With Dirac, there is "an absence of small talk."

Offered Wigner: "He wishes to be nice to people ... [but] he is a loner." For most of his life, Wigner continued, Dirac "was very restrained," adding, almost as if he just then realized it: "You know, he published fewer articles with other people than anybody else I know. Most people publish about one-half of their articles in collaboration with others."

His self-imposed isolation over the years even apparently affected Dirac's pedagogical role, or lack of it. "He was at Princeton several years," Wigner recalled, "and he never had to my knowledge any contact with students." In fact, Wigner claimed, "I never saw him collaborating with a student."

Yes, "he talks very rarely at all," said Halpern, who has worked more closely with Dirac during the past six years than anyone else. And when working, Halpern added, Dirac "doesn't need anybody. He needs me [only] occasionally for some information."

In listening to recollections and commentary such as this and in conversing with Dirac himself, to the extent that's possible, one gets the feeling that there's more to Dirac's taciturnity than mere quirkiness. Rather, it seems to be a natural accompaniment to his conception of the physicist's ultimate reliance on the mathematical language: In a passage of his 1930 book on quantum mechanics, he spoke about the fundamental laws of nature, which, he maintains, "control a substratum of which we cannot form a mental picture without introducing irrelevancies."

Any scientist with such a viewpoint might be expected to abstain from graphic and, it would seem to follow, verbal communication, both of these being seen as obfuscatory. This abstinence might be doubly expected of someone who, as Wheeler said of Dirac, "has a passion for rectitude and justice in physics" and in life.

One senses in Dirac an almost inhumanly dedicated striving toward the austerity that is required of a scientist who is to comprehend the fundamental laws of nature without the attendant "irrelevancies." It is this no-nonsense, no-frills conciseness that so characterizes Dirac's speech and publications. (As physics students are well aware, his books on special and general relativity have so few pages as to make them cheaper to photocopy than to purchase.) And in the end, it is what accounts for his articulateness, the level of which could be only diminished with greater verbosity. Because, as Halpern said, Dirac speaks few words, "but he says a lot. . . ."

—Michael A. Guillen

Leopold Halpern: Assistant and buffer against the world for Dirac.

Fla. State Univ.

MATHEMATICAL PHYSICAL BIRTHDAY PARTY

THE 'BEAUTIFUL MATH' PEOPLE HONOR ONE OF THEIR LEADERS

BY DIETRICK E. THOMSEN

"I was not trying to solve directly some physical problem but to look for some pretty mathematics." So P. A. M. Dirac describes the beginning of the work that led to the famous Dirac equation, which is basic to the modern understanding of the behavior of electrons and positrons, of matter and antimatter, if you will. The statement is typical of this brief, soft-spoken man, succinct and somewhat astounding. There is the possibly somewhat disingenuous disclaimer of physical intent and the by now familiar creedal statement in praise of the beauty of mathematics.

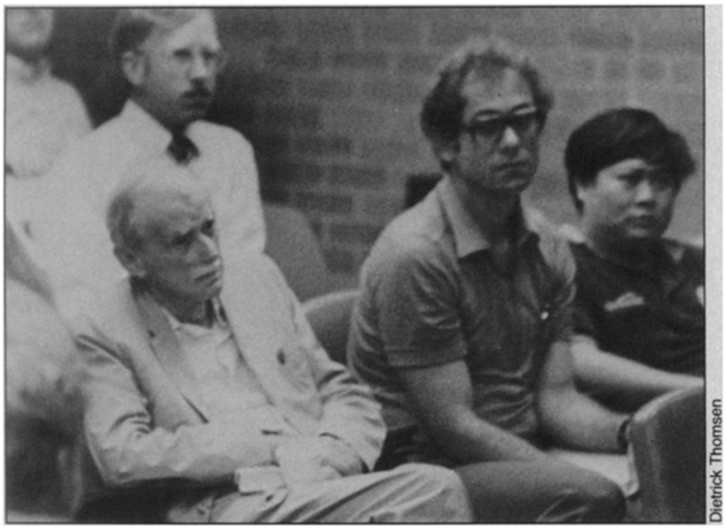
Dirac was making the first speech at a symposium celebrating his eightieth birthday. (It was a somewhat premature symposium. His eightieth anniversary isn't until 1982, but the organizers had wanted to get the proceedings of the symposium published in time for the actual date.) Physicists whose work somehow connected to Dirac's gathered at Loyola University in New Orleans to talk about those connections. The phrase "pretty mathematics" became a kind of leitmotiv of the meeting.

Dirac wants to tell us something about his attitude—following Flaubert's dictum that seems to be the *mot juste*—while he was deriving the equation now called by his name, the one that describes the behavior of electrons and positrons. He was fascinated, he says, by a number of mathematical elements of a certain kind, certain matrices (called after the name of Wolfgang Pauli). These matrices have the property that if taken in pairs and multiplied, they yield +1 for a result if the multiplication goes in one order (say AB) and −1 if it goes in the opposite order (that is, BA). The technical term for this is "anticommutation." It means that there is an exciting (to the mathematical mind) kind of upside down symmetry among whatever it is that these matrices represent.

Dirac wants to try to relate them to the momentum of a particle. He multiplies them by the components of a momentum and comes up with equations describing the behavior of a particle of zero rest mass. This did not seem of much use at the time. He wanted to see if the equation could be extended to the behavior of electrons. To do that meant bringing in terms representing the mass of the electron. That meant modifying those fascinating matrices. After some struggle it proved possible. The anticommutation symmetry was preserved. The resulting equations are mathematically consistent, and they contain the famous prediction of antimatter and give the electron's spin and magnetic moment a theoretical context they had not had before.

But that's 50 years in the past, and what's really at issue is a more modern development. One can set up an equation similar in form to the previous one, with analogous anticommutation properties. This equation predicts only positive energy states (the Dirac equation had surprised physicists by predicting negative ones as well) and it contains what are called "internal degrees of freedom" for the particles it describes. That is, the new equation can take account of internal structure in the particles; the original Dirac equation treats electrons and positrons as if they were structureless geometric points. Hint: The

Dirac may look relaxed but he is probably making mental note of devastating questions for the speaker.



Dietrick Thomsen

new equation would be very useful for describing particles built up out of quarks; it could be a quantum mechanics that takes account of internal constituents. For it to be that it is necessary to get terms describing electromagnetic interactions into the equation. Nearly all these particles are electrically charged, and so a realistic theory has to include electromagnetism.

Dirac couldn't do it. No matter how he tried, the resulting equations came out with algebraic inconsistencies. Some years ago he gave up this line of inquiry. More recently E. C. G. Sudarshan and collaborators have taken it up, and Dirac was pleased to announce at the symposium that they had succeeded in modifying the equation to get electromagnetism in.

What Sudarshan, N. Mukunda (on leave from the Indian Institute of Science in Bangalore) and C. C. Chiang (on leave from the National Taiwan Normal University in Taipeh) have done is to work on those same matrices that have held so much interest and that here particularly concern the internal dynamics of the hypothetical particle. By a slight alteration in the definitions of these terms and more complicated rules about which ones anticommute with which ones, equations describing the motion of a hypothetical particle with no spin but with a finite (that is, nonzero) rest mass can be derived. *These* equations (unlike the ones first found by Dirac) do not become algebraically inconsistent when the effect of an electromagnetic field is inserted into them. (In technical terms what was done was a change from bose statistics, which Dirac's original formulation had, to "parabose" statistics.) So now there is an equation of motion that could represent quantum mechanically a spinless, finite-mass, electrically charged particle with some kind of internal structure. It is not now clear exactly what use this may come to have in the physics of actual particles that fit such a bill, but the potential is certainly interesting.

The pursuit of mathematical beauty tends to engender holistic attitudes. When Dirac writes down mathematics, the formulations have a terseness and an economy of terms that one suspects many others would not dare. The terseness and economy adumbrate great generality: The more you can encompass in little, the better off you are.

This was thus a gathering of people who like the idea of unifying all of physics into a single mathematical framework, a unified field theory, as it's called. But not necessarily the same kind of unified field theory as that being developed by the particle physicists led by Steven Weinberg and Abdus Salam.

Arthur Wightman of Princeton University asked "contra Weinberg, are there useful nonrenormalizable field theories?" That is, whether the class of quantum theories that practical particle physicists customarily throw away because they contain infinities and divergent series may not have something to teach us about physics.

Wightman spoke as a representative of quantum field theory as a branch of mathematics. Others deplore present-day practical physics because particle physicists tend to formulate their theories as if gravity didn't exist. Quantum effects (that is, particle physics) and gravity (general relativity) are kept in separate compartments. To this attitude, Leopold Halpern of Florida State University, Dirac's assistant for the last several years, says, "It must be possible to describe all physics as one kind of universe."

Halpern's method is to take the four-dimensional space of ordinary experience and consider it as embedded in a mythical multidimensional space. This is a not unusual trick in mathematical physics. Often by looking from the multidimensional point of view the theorist can see connections among phenomena that seem totally disconnected from inside four dimensions. Halpern is working in 10 dimensions to find a theory like general relativity and theories like those used in current unification efforts in particle physics (non-Abelian gauge theories) that seem to be related to each other.

Both Halpern and George Hsieh of the University of Pittsburgh start from the variable force of gravity derived from Dirac's large numbers hypothesis (see p. 395), which Hsieh calls "a kind of Mach's Principle inspiration of general relativity." Ernst Mach was a philosopher of science, and this "Mach's Principle" is his suggestion that the mass of every body in the universe depends on the masses of all the other bodies for its existence. Einstein interpreted this to mean that mass is a fixed quantity and so is the relative strength of gravitational forces, and put this interpretation into his theory as an assumption. Other assumptions are possible without destroying the mathematical beauty of the formulation of space and time in general relativity, and so Dirac has differed with Einstein in proposing the large numbers hypothesis to replace the Einsteinian assumption, which is known as the principle of equivalence.

With its ratios of electric and gravitational constants, the large numbers hypothesis may be suggesting a way to overcome the differences between gravity and electromagnetism, a project on which Einstein worked for a long time without success. Hsieh reports no detailed success, but he does make the point that "perhaps [unified field theories] look unconventional because we are used to looking at physical theories as local. Dirac's is nonlocal. There could be another type of laws, global laws. Physicists are not used to thinking about them." (Local laws deal with things happening in a small volume of space and time. Particle physics laws are a primary example. Global theories deal with the whole universe and encompass distant bodies. In the aspect of being a geometric theory of space and time at least, general relativity is a global theory although the nitty gritty discussions of actual space curvature tend to apply to small areas.)

If all this unification can take place, the next step is an algebra of causation, says David Finkelstein of Georgia Institute of Technology. He attempted to supply it, a sort of grammar of history. Imagine any present event. It has one or more causes in the immediate past. Each of them has causes behind it. Start with the set of immediate causes of the present event. Then go the next generation back. And so on and so on. Generation by generation one can define a sequence of sets, each containing the one before it, each containing causes of the given event. If one can derive the right algebraic rules for manipulating these sets, one might come out with "a theory of the pattern of causal relations" in the history of the universe.

Finkelstein managed to find a proper definition for the sets and proper algebraic rules. As a mathematical exercise it works. As a physical exercise it fails because it disagrees with general relativity. But the beauty of doing it is that out there may be another way that will work. ☐

Come explore a comet.

Halley's Comet returns in 1985. The Europeans, the Soviets, and the Japanese are readying spacecraft to meet it. Our nation plans to watch from afar because NASA has no money for a Halley mission even though only the U. S. has the technology needed to fully probe this comet for the clues it offers on the origin of our solar system. We stand to lose not only knowledge, but also new technologies and a bit of our prestige.

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An Eye for Beauty

Continued from page 395

slightly but uncorrectably out-of-focus.

Dirac's was not the first word on this subject — he was decisively inspired by the pioneering work of Werner Heisenberg. Nonetheless, his version of quantum theory was distinguished by its generality and logical clarity and a tidy mathematical notation he introduced that is still preferred by most scientists.

This initial exercise in beautiful mathematics was quickly followed by several others. In 1927 Dirac reformulated in terms of the new quantum mechanics James Clerk Maxwell's original concept of an electromagnetic force field. The result declared that the supposedly intangible force field actually effected its influence through the exchange of electrons—much as the force of a pitcher's toss is transmitted to the catcher and vice versa via an exchanged baseball. This was the first example of a quantum field theory, a subject that is now a theoretical mainstay.

In 1928, Dirac was the first to incorporate the precepts of special relativity into the theory of quantum mechanics as applied to the electron. The principal result was the Dirac equation, and it naturally accounted for certain inherent properties of the electron that hitherto had been only awkwardly accounted for in theoretical calculations.

Most remarkable, however, the equation seemed to imply the existence of particles that could have negative energy. Encountering similar mathematics in the past, physicists had simply ignored the implications as being physically unrealistic or nonsensical. Dirac chose to do otherwise. He figured out a way of arguing persuasively that not only a few such negative-energy particles but, figuratively speaking, an entire "sea" of them might indeed exist. This sea of negative energy states is unobservable directly but its presence implies the existence of a positively charged particle with highly unusual properties that is observable. By 1931, it was realized by Dirac and others that in fact it was a wholly new particle whose existence was being implied. It would probably have the same mass as an electron and in the proximity of an electron would induce the particles' mutual annihilation, creating in their place pure energy.

In 1932, experimentalist Carl D. Anderson confirmed the prediction, and named the new particle "positron." It was the first specimen of "antimatter" ever discovered.

Dirac told Science News that he had had so much confidence in his equation that he was not at all surprised by Anderson's discovery. Dirac contends, besides, that he had been in close communication with the English physicist P. M. S. Blackett, who "had his evidence [for the positron] before Anderson ... [but] was cautious about publishing his results."

Most historians regard Dirac's prediction and its subsequent verification as one of the most spectacular achievements ever in theoretical physics. Yet with characteristic selflessness and his abiding admiration for Einstein, Dirac sees things differently and reappropriates the credit. In a speech given two years ago during an Einstein centennial celebration hosted by the Pontifical Academy of Sciences he said: "Special relativity led to a long line of development.... It led to a square root in the formula for the energy of a moving body, so that mathematically the energy could be negative. This did not matter at first; one could just say that negative energy states do not occur. But with the arrival of quantum theory, one...was forced to look for a meaning for the negative energies. This led to antimatter, which is thus a direct consequence of Einstein's special relativity."

Not all of Dirac's forecasts have been as dramatically vindicated as his positron prediction, and some have yet to be resolved. This is true of a hypothesis he first enunciated in 1937. It's built upon a single, disarmingly simple observation: Certain ratios of physical constants — numbers that repeatedly arise in the equations of physics and thus presumably have special significance in nature — have extremely large numerical values. Furthermore, several of those values are almost equal to one another and to either of two values, 10^{40} and 10^{80}.

Deciding that this state of affairs exists by dint of more than just a chance numerological fluke, Dirac offered an explanation (the Big Numbers Hypothesis) that, almost unbelievably, implicates the history and evolution of our entire universe and raises the possibility that gravity on earth and everywhere else is slowly weakening its grip on things.

Thus far, numerous attempted measurements of this weakening have proved inconclusive. Dirac admits that there is thus far no evidence for his hypothesis's prediction, but hastens to add that "I don't think the last word has been said on the subject."

It is perhaps in part a tribute to Dirac's stature as a theoretical physicist that while expressing their skepticism, so many scientists are nonetheless actively involved in attempting to establish the absence or presence of a universal weakening of gravity.

This entire situation does not lack poetic irony, considering that Einstein's theory of general relativity could not alone account for any such kind of gravitational weakening. There is thus the possibility, albeit remote, that one of the first examples of pretty mathematics admired by a teen-aged Paul Dirac will come up short and be in sore need of some repair. But if that day arrives, and Dirac has anything to say about it, one expects that the necessary modifications would be conceptually fundamental and wrought with precious care to preserve the beauty of the mathematics. ☐