

The Arithmetic of Apportionment

"Representatives shall be apportioned among the several States according to their respective numbers . . ."

— The United States Constitution

Every 10 years census data force the United States Congress to reapportion the House of Representatives, to ensure fair representation. Even though apportionment appears to be a simple act of arithmetic, disputes about the proper basis for apportionment have led, among other things, to the first use of a presidential veto in U.S. history. Moreover, malapportionment resulting from these disputes was directly responsible for the election as president of Rutherford B. Hayes, even though his opponent won the popular vote.

Now, in a recently published book (*Fair Representation: Meeting the Ideal of One Man, One Vote*, Yale Univ. Pr., 1982) Michel Balinski and H. Peyton Young argue that the present method of reapportionment is intrinsically biased against large states. It is surprising that after 200 years there is still no consensus on the simple arithmetic of fair apportionment.

The problem is familiar to every school child: When you divide one big number by another, you usually have a remainder. Politically, it is the remainders that have dominated the debate, creating what Ohio's nineteenth century Representative Samuel Vinton termed "a disreputable contest about the unrepresented fractions." The effect of remainders is not insignificant, especially when they are used to round either up or down. The difference between four and five representatives can make a difference of 25 percent in the representation of a small state, and can change by an equal amount the weight of its citizens in national legislation.

Once the census data are known, each state's quota in the House can be determined by simple division of state populations by the number of seats to be apportioned. This will, in each case, yield some whole number plus a remaining fraction. The first apportionment bill, authored by Alexander Hamilton in response to the census of 1790, specified that each state would receive the number of representatives determined by the whole number in its quota. Whatever seats remain are then assigned to those states having the largest remaining fraction.

This seemed fair enough, giving additional seats to those states most deserving on the basis of the left-over fractions. However, George Washington vetoed the bill, because "there is no one proportion or division which, applied to the respective numbers of the states, will yield the number and allotment of representatives proposed by the bill."

Washington preferred a method proposed by Thomas Jefferson, and subsequently passed by Congress, based on a single divisor: knowing the size of the House and the populations of the states, find "that divisor which applied to every state, gives to them such [whole] numbers as, added together, come nearest to [the

desired total]." Jefferson used the divisor 33,000: one representative for every 33,000 persons, excluding fractions.

Washington and Jefferson's preference for this method may not have been based entirely on constitutional principles: Jefferson's method, it turned out, gave Virginia one more seat (at the expense of Delaware) than did Hamilton's (see Table 1). Virginia's quota in the 105-seat House was 18,310, and the remainder of .310 was not high enough to earn Virginia one of the extra seats being apportioned under Hamilton's method. But by selecting a (carefully chosen) single divisor, it was possible to raise Virginia's representation to 19, since its 1790 population of 630,560 is just barely 19 times 33,000.

Jefferson's method was used for nearly half a century, with divisors climbing as population increased. By 1830 it was 47,700. But the bias that led to Virginia's gain at Delaware's expense became more noticeable, and more difficult to sustain politically. After much debate over various alternatives, Daniel Webster proposed in 1832 a new method (actually just a variant of Jefferson's) in order to give each state a representation in the House that is "as near as may be" to its true proportion (or quota).

Webster's method required that the divisor be chosen so that the whole numbers nearest to the quotients add up to the desired total. He was motivated by some compelling anomalies in the proposed apportionment based on the 1830 census. New York, for example, with a quota of 38,593, was assigned 40 seats, whereas Vermont, with a quota of 5,646, received only 5 seats. It was, Webster said, "undeniably true" that taking away New York's 40th representative and giving it to Vermont would bring both states closer to

their true proportion. Similar claims were true of two other pairs of states in the 1830 census.

Despite these arguments, politics prevailed: Jefferson's method continued as the law of the land. Finally, in 1850, Samuel Vinton of Ohio succeeded in persuading Congress to adopt Hamilton's method as the official standard of apportionment. But in subsequent censuses the results were so unsatisfactory that various pretexts were used to alter the actual apportionment, without officially changing the law. This tinkering led in 1876 to the election of Hayes over Samuel Tilden by a margin of one vote in the electoral college, despite the fact that Tilden won the popular vote, and the fact that if the House had been apportioned according to the law (by Hamilton's method), Tilden would have prevailed also in the electoral college.

Real trouble emerged, however, after the 1880 census when it became apparent that Hamilton's method will in some cases diminish the representation of a state when the total size of the House is increased! As Congress debated whether to increase the size of the House, it was pointed out that according to current law, Alabama would receive 8 representatives if the House had 299 members, but only 7 if the House had 300 members.

The problem, again, can be found in the behavior of the remainders. Alabama's quota increases from 7.646 to 7.671 as the House size increases from 299 to 300. Indeed, the quotas of all states increase in the same proportion. But the *remainders* of these quotas — the fractions that determine priority for extra seats under Hamilton's method — do not increase in equal proportion. In 1880, the remainders of Illinois and Texas each increased from below that of Alabama to above it (see

Table 1

1790 Congressional Apportionment Options

State	Population	Quota	Jefferson	Hamilton
Virginia	630,560	18.310	19	18
Massachusetts	475,327	13.803	14	14
Pennsylvania	432,879	12.570	13	13
—	—	—	—	—
Rhode Island	68,446	1.988	2	2
Delaware	55,540	1.613	1	2
Total	3,615,920	105.000	105	105

A comparison of Jefferson's divisor method with Hamilton's remainder method for apportionment after the first U. S. census in 1790. Jefferson's method, adopted by Congress, favors Virginia over Delaware in assignment of seats. In general, this method is biased in favor of large states, and for that reason was abandoned by Congress in 1830.

Table 2). Thus, one of them took Alabama's extra seat, the other took the 300th seat, and no extra seat was left for Alabama.

Even graver problems then emerged. In certain cases when population shifts among states, Hamilton's method can force a state that grows relative to another to give up a seat in favor of the other state. A similar problem occurred when Oklahoma entered the Union. Its fair quota would have entitled it to about 5 seats. But if 5 seats were added to the House, Hamilton's apportionment, in addition to giving Oklahoma its 5 seats, would transfer one seat from Maine to New York without any actual change in the population of either Maine or New York!

Washington was right, after all: Hamilton's method does not meet the constitutional requirement of apportionment "according to their respective numbers." The use of remainders to determine priority is simply not a proportional device. Jefferson's method of divisor, or any of numerous other similar divisor methods, escapes most of these paradoxes of population shifts. Indeed, Balinski and Young prove in their book that divisor methods are the *only* methods that avoid these paradoxes.

Congress overthrew Jefferson's method of divisors, however, because it showed a systematic bias toward large states. As a class, large states had, under Jefferson's method, far greater likelihood of receiving representation in excess of their quota than did smaller states. But now Hamilton's method also proved flawed. This dilemma led in the first half of the twentieth century to a major struggle in Congress to find a new method.

First, in 1910 Congress abandoned the Vinton-Hamilton method and adopted instead Webster's divisor method. At the same time, Joseph Hill, chief statistician of the Bureau of the Census, suggested a completely new method, designed to avoid situations that could not be improved upon by the transfer of seats: Hill proposed to give to each state a number of seats so that no transfer of any one seat can reduce the percentage difference in representation between these states.

Hill's proposal was championed by Edward V. Huntington, professor of mathematics at Harvard, chiefly on the grounds that it was the only method that treated large and small states without bias. The challenge posed by Huntington was so powerful that no reapportionment ever passed Congress for the 1920 census. Since rural areas were losing population to cities, representatives of these areas had a vested interest in prolonging the argument, on whatever grounds.

Congress finally asked the National Academy of Sciences for an evaluation of the competing claims. The Academy's report endorsed the Hill/Huntington method because "it occupies mathematically a neutral position with respect to emphasis on larger and smaller states." Sub-

Table 2

The "Alabama Paradox" in the Apportionment of 1880

State	Population	Quota at 299	Quota at 300	Percentage Increase	Absolute Increase
Alabama	1,262,505	7.646	7.671	.0033	.025
Texas	1,591,749	9.640	9.672	.0033	.032
Illinois	3,077,871	18.640	18.702	.0033	.062

In the apportionment of 1880, Alabama would have lost a seat had the House size been increased by 1. The reason, as shown in these figures, can be found in the manner in which the remainders of the quotas change: The remainders of larger states increase more rapidly than those of smaller states, so they overtake Alabama in the contest for unallocated seats under the Hamilton method of apportionment.

Table 3

Difficulty of Staying within Quota

State	Population	Quota	Apportionment
A	70,653	1.552	2
B	117,404	2.579	3
C	210,923	4.633	5
D	1,194,456	26.236	25
Total	1,593,436	35	35

Both the Webster and Hamilton methods agree on the apportionments shown above, in which state D receives fewer than its whole number of representatives. But taking a representative from any smaller state to correct this violation of quota would impose dramatic variations of the representation proportions in whichever state was forced to give up a seat. The seat that D lost violates its quota, but contributes to greater equity in representation among all the states.

sequently, in 1941, Congress adopted this method, which is still in use today.

Despite the claims made at the time of its adoption, Balinski and Young now show that Hill's method, applied retrospectively to all 20 U.S. apportionments, consistently favors small states over large ones. The bias is not as pronounced as the (reverse) bias of the Jefferson method, but it is nonetheless present. They show, moreover, that Webster's is the only unbiased divisor method of apportionment.

Interestingly, one of the more common-sense principles of apportionment—staying near quota—is not satisfied by either Hill's or Webster's method. Most people feel that if the true quota of a state is, say, 5.43, then its number of representatives should be either 5 or 6, never 4 or 7. This principle, called "staying within quota" by Balinski and Young, turns out to be violated rather frequently (see Table 3).

Indeed, they show that there is no method of apportionment that both stays within quota and avoids the population-related paradoxes. The reason for this is that the change of one seat required to force some state to be within quota may cause disproportionate change in the comparative assignments of seats to the remaining states. So there is no perfect method of apportionment—only a collection of imperfect methods, each attempting to define arithmetically what the founding fathers meant by the simple words "according to." □

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