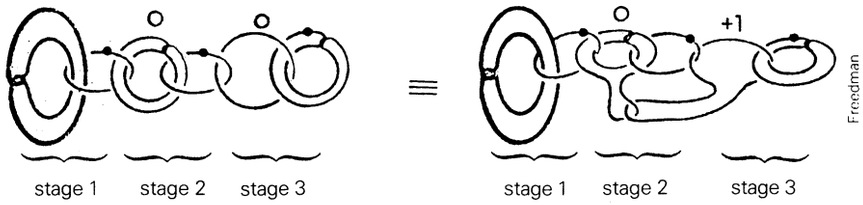


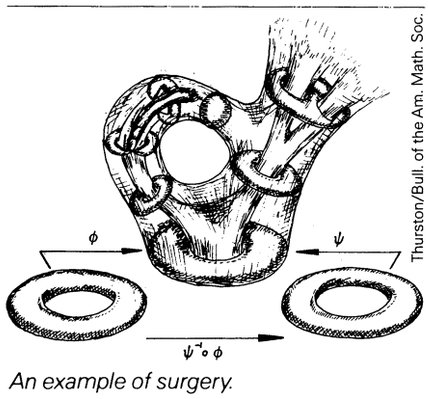
An example of handle body calculus.

than did those of dimensions three and four; however, even the simplest manifolds — spheres — would sometimes exhibit certain bizarre behaviors in high dimensions. For example, in 1956 John Milnor of Princeton University stunned the mathematical world by showing that the seven-dimensional sphere can be made into a "differentiable" manifold in 28 different ways. In other words, Poincaré's scheme of representing the solutions to differential equations on manifolds would, in dimension seven, encounter 28 different versions of the sphere.

Milnor constructed these exotic spheres by multidimensional plumbing — cutting holes here and there, connecting them by various tubes, and then deforming what resulted into a new kind of sphere. This procedure, called surgery, has proved as valuable as cobordism in the general task of classifying manifolds. And its modern form leads to what is called "handle-body calculus," a scheme for computing the effects of adding handles to manifolds. (If the handles are interlocked, or tied in knots, things can get rather complicated.)

Since the 1960s, efforts toward proving Poincaré's conjecture have been stalled — at the point where all but the last two cases had been verified. Now Michael Freedman of the University of California at San Diego has completed work on proving the next to last case — the fourth dimension. Strangely, the last unproved case is the third dimension — the same one that stumped Poincaré.

Freedman's work concerns four-dimensional manifolds and is based largely on handle calculus. "The dream remained since my graduate school days," writes Freedman, "that some key principle from the high dimensional theory would extend, at least to dimension four, and bring with it the beautiful adherence of topology to algebra familiar in dimensions greater than or equal to five."



An example of surgery.

"There is such a principle," continues Freedman. And in the parlance of mathematics he goes on to describe it as "a homotopy-theoretic criterion for imbedding a topological 2-handle in a smooth four-dimensional manifold with boundary." Freedman blends homotopy, handles and boundaries for a comprehensive analysis of the topology of four-dimensional manifolds that yields the first complete classification of manifolds in this dimension. Although Freedman's paper has not yet been published, it has been checked carefully throughout the past year by topologists at several major universities. Now, only the third dimension remains without a satisfactory classification, and without a proof of the Poincaré conjecture.

But this case, too, appears to be nearing completion. William P. Thurston of the University of Colorado, writing in the May 1982 BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, sets forth a program for classification of three-dimensional manifolds, including resolution of the original Poincaré conjecture. Thurston's program uses surgery to reduce complex manifolds to simple cases in much the same way factoring reduces large composite numbers to products of primes.

Thurston's paper offers a direction for research, not a complete theory. However, the mathematician marshals a lot of evidence in support of this program, not least new and beautiful computer realizations of many of the algebraic and geometric constructs needed to model three-dimensional manifolds.

He uses the computer, for example, to portray the geometric structures resulting from surgery. "The geometric structures turn out to be very beautiful when you learn to see them," writes Thurston. "Often the information which determines a geometric structure can be expressed in terms of some construction in plane Euclidean geometry."

Two years ago mathematicians celebrated the successful completion of a century-long effort to classify "finite simple groups" (SN: 9/27/80, p. 204). Now they are nearing the end of a similar research effort concerning the classification of manifolds. The end may come next year. Then again, it may not be until the end of the century that the case for the third dimension is finally verified. When it is completed, however, both algebra and geometry — the two traditional branches of mathematics — will be governed by grand theories of classification and evolution, theories that relate fundamental objects to each other in a way that imposes useful structure where formerly only chaos reigned. □

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THE CONSUMER'S BOOK OF HEALTH: How to Stretch Your Health Care Dollar — Jordan Braverman. The purpose of this book, says the preface, is to give you, the consumer-patient, a basic awareness and understanding of the major health care programs, institutions and services that exist today so that you will know how to find, evaluate and pay for those services you need. Saunders Pr (HR&W), 305 p., paper, $7.95.

DIET, NUTRITION, AND CANCER — National Research Council, Committee on Diet, Nutrition, and Cancer, Clifford Grobstein, Chairman. Summarizes the most relevant scientific information on diet and cancer. Recommends dietary guidelines designed to reduce the risks of developing cancer. (See SN: 6/26/82, p. 422.) Natl Acad Pr, 1982, 496 p., paper, $13.50.

THE ENERGY ANSWER 1982-2000 — Richard C. Dorf. Discusses the energy dilemma in the United States, the effects of technological innovation, the development of existing energy sources and prospects for the next two decades. Brick Hse Pub, 1982, 115 p., charts & graphs, paper, $8.95.

THE FOSSIL RECORD AND EVOLUTION: Readings from *Scientific American* — Introductions by Léo F. Laporte. Provides an overview of evolution and history of life as recorded by the sequence of fossils preserved in the earth's crust. W H Freeman, 1982, 225 p., color/b&w illus., paper, $11.95.

PLANETS OF ROCK AND ICE: From Mercury to the Moons of Saturn — Clark R. Chapman. Written to show how fascinating and important the planets are when we examine them closely. Explains for the general reader the important things we have learned thus far from planetary exploration about the worlds from Mercury to Saturn's moons. Seen through the eyes of scientists, after years of analysis and creative thought, the planets, says the author, play a central role in understanding our own world. This book is a completely revised and expanded edition of *The Inner Planets.* Scribner, 1982, 222 p., illus., $13.95.

RESEARCH & DEVELOPMENT: Federal Budget — FY 1983/Impact and Challenge, AAAS Report VII — Willis H. Shapley, Albert H. Teich and Jill P. Weinberg. An examination of the proposals in the 1983 budget from the perspective of their effect on research and development in the public and private sectors. In the final chapter the budget process is explained and the steps in the process are outlined. (See SN: 7/3/82, p. 6.) AAAS, 1982, 159 p., paper, $8.

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