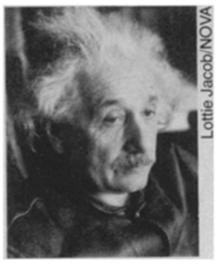
Many Dimensions in Gravity Theory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Quantizing general relativity means rolling dice on Albert Einstein's field

By DIETRICK E. THOMSEN

Albert Einstein didn't like quantum mechanics. Spending some time with the theorists who are trying to quantize his theory of general relativity can lead to the impression that the antipathy was mutual. Although Einstein made some important early contributions to quantum physics, he became dismayed as the philosphical implications of quantum mechanics revealed themselves. Classical physics is rigidly deterministic: a given cause leads inexorably to a given result, always and everywhere. In quantum mechanics a given cause can lead to any of several results. Individual cases cannot be determined. The only laws are statistical ones for large numbers of instances. Growling his famous remark about God not throwing dice, Einstein withdrew mostly to the contemplation of classical field theory.

In order to save classical field theory Einstein destroyed it. The general relativity theory that he developed wrought a revolution in our concepts of space, time and force, giving them a radically different form from what had gone before. And Einstein destroyed the absolutism that had characterized the physics of the past: the notion that there is an absolute evenly flowing time, which is the same for all observers, and that there is a special spatial frame of reference that is absolutely at rest, against which motions can be measured absolutely. Now time differs for different observers; all spatial reference frames are of equal rank; there is no absolute rest, and forces and motions are relative, artifacts of geometry.

Nevertheless, general relativity is still a classical field theory in the sense that it is deterministic. The uncertainties and statistical analyses that are fundamental to quantum mechanics do not play a role in it. But, somehow, eventually it must be mated with them. To do this, to quantize general relativity, has long been seen as a necessary step in the completion of theoretical physics. In past decades it did not seem too urgent. Quantum mechanics deals with behavior on the atomic and subatomic level. General relativity is primarily a theory of gravitation. Practically, the effects of gravity in ordinary particle-physics and atomic-physics experiments are so minuscule as virtually to vanish. There was no clamoring market for a theory of gravity compatible with particle physics.

Now there is. The theory of particle physics, at least, has now moved into realms where gravity is important. These are domains of very high energy, which may actually have existed in the early moments of the history of the universe. Particle physicists have begun to theorize like cosmologists and to act like astronomers, scrutinizing such relics of the early universe as are available for some evidence of the things they have theorized. They need a theory of gravity compatible with particle physics to aid them in what they are doing. The quest was evident recently in New Orleans at the Second New Orleans Conference on Quantum Theory and Gravitation.

It seems clear from the discussion at the meeting that to get such a theory will require some radical changes in some of our present basic ideas about space, time and matter and possibly some spectacular violations of "common sense." (Much in modern physics violates "common sense.") It could require the quantization of space and time, a shift from the continuous space and time, used by geometers from Euclid to Einstein, in which one location or instant shades imperceptibly and indivisibly into the next, to some conception that is bumpy and jumpy like the processes of quantum physics. It could mean a further geometrization of the properties of matter. It will probably mean something of both. One thing it surely needs is more dimensions.

In the words of John William Moffatt of the University of Toronto, "General relativity is based on the algebra of real numbers in four-dimensional space." Real numbers are the ordinary ones we count with, and four-dimensional space is the space-time of ordinary perception — "real" space-time it is often called. It has the three space dimensions in which we see ourselves free to move in any direction we like (assuming there are no physical restraints on us) and time, in which we can move in only one direction. This difference between space and time is by no means scanted in general relativity, but as the theory is formulated, it is nevertheless possible to treat time as a dimension. The only forces in the theory are gravitational ones. (Einstein tried for 40 years to work electromagnetic forces in but could not). Gravity is seen as an effect of the curvature of space-time, and that curvature is determined by a quantity that represents the amount of matter and energy in a given neighborhood. Thus force and matter are in a sense geometrized.

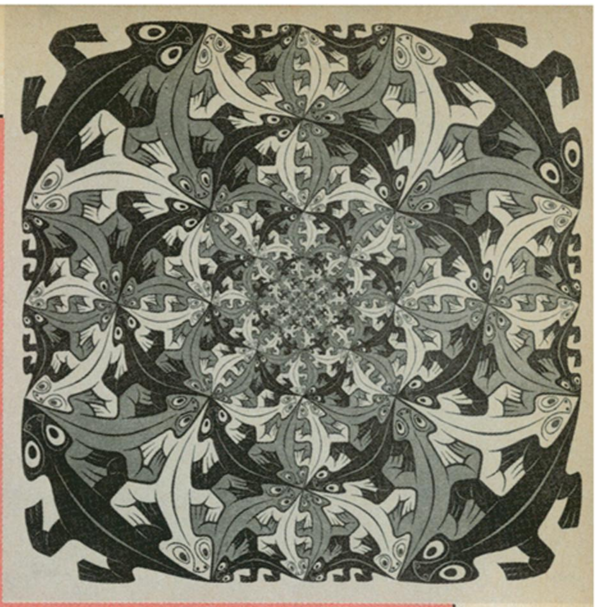
When subatomic particles come into the picture, geometrizing their properties requires more dimensions. The four dimensions of ordinary space-time represent the external "degrees of freedom" of a body, its ability to move in space and the changes in time that come over it. Subatomic particles are more than simple point masses exerting gravitational forces. They have what are known as internal degrees of freedom, intrinsic properties that change by quantum jumps. As these changes occur they alter the physical state and often the identity of the particles to which they occur.

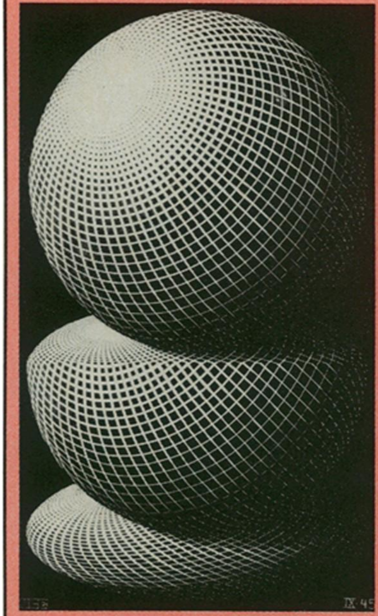
In cases where there are more degrees of freedom, more quantities that can change than the four dimensions of ordinary space-time, physicists have long found it useful to retain geometric imagery by using, as calculational devices, spaces with enough dimensions to accommodate all the degrees of freedom relevant to the problem. Leopold Halpern of Florida State University in Tallahassee quotes an 1837 statement of Bernhard Riemann: "Physics and geometry are complementary in the description of nature."

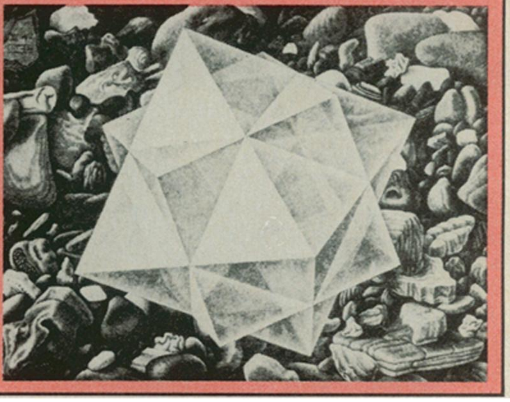
Various spaces are chosen as approaches to the problem according to criteria that seem important to a particular theorist. Moffatt chooses an eight-dimensional space because, he says, it gives him the connection to spin that he wants. Spin is an important property of subatomic particles. It seems also to play a crucial role in the amalgamation of quantum physics and general relativity. Halpern is led to a 10-dimensional space by considerations of the geometry of general relativity theory. Motions in general relativity are described with the aid of particular mathematical groups called de Sitter groups. These groups are 10-dimensional. Other theorists opt for 12 dimensions.

The point is that for these people it is easier to calculate out the physics in these multidimensional spaces than to try to make sense of it in four-dimensional real space-time. The work is done with equations, not by trying to draw 12 dimensions. Much of the mathematics is done

Dimensions of M.C. Escher


The artist can roll up lizards and compact them around a point. Maybe this is a metaphor for the physicists' compacting and rolling of dimensions.


How a sphere can be projected to a disk. Escher does it almost like a mathematician.


For an idea of many dimensions, one must be able to visualize many interconnected axes.

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according to group theory. A group is any collection of entities that has some rule by which one member of the group can be generated from other members. An example is one of the important groups that keeps coming up in this work, the Lorentz group from special relativity. Special relativity is a theory that tells us how to describe motions in any frame of reference — that is, from any point of view — that is moving with uniform velocity. Given a description of a motion in one such frame of reference, the theory gives us means of calculating how to describe it in another. All possible transformations of this kind make up the Lorentz group. The group's theory and structure can tell what transformations are possible and whether they have to be made in a particular order.

The importance of the Lorentz group in attempts to unite general relativity and quantum mechanics is that the transformations it represents are fundamental and so it must appear as a subgroup in any group representation chosen for the overall theory. Other groups of interest are those that particle physicists use to describe the various changes of the internal properties of particles. Putting all this together to get a group representation that encompasses all the things the theory wants to deal with is a formidable task, as Thomas Love of Tulane University in New Orleans pointed out.

Reviewing the ways of uniting the groups of particle physics with those of relativity theory, Love suggested that some changes from what particle physicists are used to may be necessary. Particle physicists have tried to use a group labeled SU(5) to make a grand unified theory of their subject. Love suggests it may be necessary to use one called SU(3,2) instead. This would change some predictions of particle physics: SU(5) theory predicts that protons are unstable and decay radioactively; SU(3,2) predicts protons are stable. (In fact the latter prediction seems more in line with the most recent experimental observations.) SU(3,2) also produces something that looks odd to common-sense preception. Our ordinary four-dimensional space-time is described by ordinary real numbers. SU(3,2) produces four-dimensional space-time in which complex numbers are involved. However, the complex numbers provide a natural way to quantize, Love says.

Once the theorists have made their excursions into many dimensions and worked out a consistent theory there, that result must be projected back onto four dimensions in order to give a description of events in the ordinary space-time we can understand. This is analogous to projecting a three-dimensional object onto a sheet of paper to form a two-dimensional image.

These projection techniques themselves give rise to space with geometries of their own, spaces whose constitutive elements are not points as in Euclidean or Einsteinean spaces, but rays, fibers or bundles of fibers representing the acts of projection. These projection spaces can themselves be an important arena for working out the theory. A.R. Marlow of Loyola University in New Orleans argued for a particular one that he considers to be itself a natural origin for relativistic mathematics and the simplest one involved with spins of half a unit, which are quite important among subatomic particles.

Having gotten safely back to ordinary four-dimensional space-time, what do the theorists do with the extra dimensions? People like Moffatt and Marlow say they are superfluous. They are mathematical artifacts useful in calculation. Afterwards they are thrown away.

Other theorists are now beginning to assign a physical reality to the extra dimensions. (This is one of the strangest of the new ideas.) These theorists "roll the extra dimensions up into balls" of the size of the so-called Planck length, 10^{-33} centimeters. This can be interpreted in several ways: If we could penetrate such a small distance and distinguish things there, we would perceive that space is 8-, 10-, or 12-dimensional rather than 4-dimensional. If there were such a thing as a subatomic particle with consciousness, it would perceive itself to live in such a space. Or, as Halpern put it in a press release keyed to a talk he gave at a recent meeting of the American Physical Society, we can imagine every point in four-dimensional space-time as surrounded by a little ball of 10 dimensions.

An alternative approach is to quantize space itself. James A. Brooke of the University of Toronto proposes a "quantum stochastic space." When physicists want to define the laws of dynamics in ordinary space, they have recourse to hypothetical test particles, which are imagined to occupy no more than a geometric point. It is possible in ordinary space to locate such particles in theory with any precision we care to demand. Brooke makes the suggestion that instead we make the test particles truly quantum mechanical: their location will have the uncertain characteristic of quantum objects; we can no longer say precisely where they are, only that they are within a certain stretch. Out of this kind of quantum test-particle Brooke proceeds to build up his quantum stochastic space.

Perhaps the last word here should be given to the mathematician and logician David Finkelstein of Georgia Institute of Technology. "Quantum mechanics overrules some of classical logic," he says. To accommodate it we must "revise the logical substructure of mathematics." But, "the world is made of quantum particles," so do it we must. He is working on an algebra of such particles that may make sense of it all. ☐