Pathways to Chaos

The mathematics of 'chaos' may aid in understanding epileptic seizures, computer failures and other sudden transitions to disorder

By IVARS PETERSON

With very little, if any, warning, the human brain can fail. During such failures, a victim may spend seconds staring blankly into space or, in extreme cases, may lose consciousness and fall stiffly to the ground while his whole body jerks. These seizures, termed epilepsies, are symptoms of uncontrolled overactivity among the brain's nerve cells. In some way, an electrical disturbance originating in a few neurons temporarily takes over the brain, and no other messages get through.

Neural systems fail frequently although they usually fail "soft." Typically, this type of failure affects only a particular, local segment of the brain's elaborate processing network, resulting in something as simple as a misread word. Nevertheless, "hard" failures like epileptic seizures are not uncommon. According to the Epilepsy Foundation of America, about 1 percent of the population suffers from some form of epilepsy.

Now mathematical techniques are developing that may provide some insight into the causes of epilepsies. Curiously, the same mathematical methods can also apply to complex computer systems and imply that computers can suffer "convulsions" too.

Paul E. Rapp of the Medical College of Pennsylvania in Philadelphia observes that computer-based electronic control systems are becoming "more biological." Increasingly complex computer networks, with numerous interconnections and a wide range of independent but coordinated functions, are evolving. Rapp says, "It is therefore possible to ask if future generations of control networks will be vulnerable to forms of failure previously observed only in biological systems."

Rapp presented his ideas at the Second International Workshop on Molecular Electronic Devices held earlier this year at the Naval Research Laboratory in Washington, D.C. (SN: 6/11/83, p. 378). Rapp is one of only a few medical researchers who study the application of sophisticated mathematical techniques to biological systems.

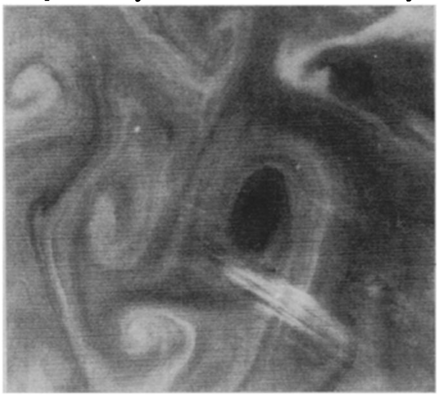
Rapp's concern is that complex electronic networks may "display a failure mode analogous to a convulsion." He says, "Available evidence is indirect and inconclusive. However, the consequences of a convulsive failure in a military control system could be grave. For this reason, the possibility, even if seemingly remote, merits examination." Already there is evidence that computer systems have suffered mysterious breakdowns during military exercises and even in aircraft control systems.

The still-tenuous thread that ties together computer networks and physiological systems is an emerging field of mathematics called topological dynamics that describes the way in which systems change with time. This mathematical approach suggests that systems governed by physical laws can undergo transitions to a highly irregular form of behavior called "chaos." Although chaotic behavior appears random, it is governed by strict mathematical conditions. The theory predicts that just as a small group of neurons may suddenly generate a disruptive electrical signal, so too can electronic components within a computer.

Rapp says, "They are both dynamical systems, they are both subject to instabilities, and they are both subject to analysis using the technique of topological dynamics."

Such analyses may potentially provide insights into the origin of nervous system failures, especially epilepsies, and suggest possible control methods or "cures." They may also provide a better understanding of what properties predispose an electronic control system to a major convulsive failure.

David Ruelle of the Institut des Hautes Etudes Scientifiques in France suggests that systems showing "chaotic" behavior are frequently encountered in physics, chemistry and biology. One example is smoke rising in still air from a cigarette. "Oscillations appear at a certain height in the smoke column, and they are so complicated as to apparently defy understanding," he says. "Although the time evolution obeys strictly deterministic laws, the system seems to behave according to its own free will."



These huge ocean eddies may be examples of "chaotic" water flows.

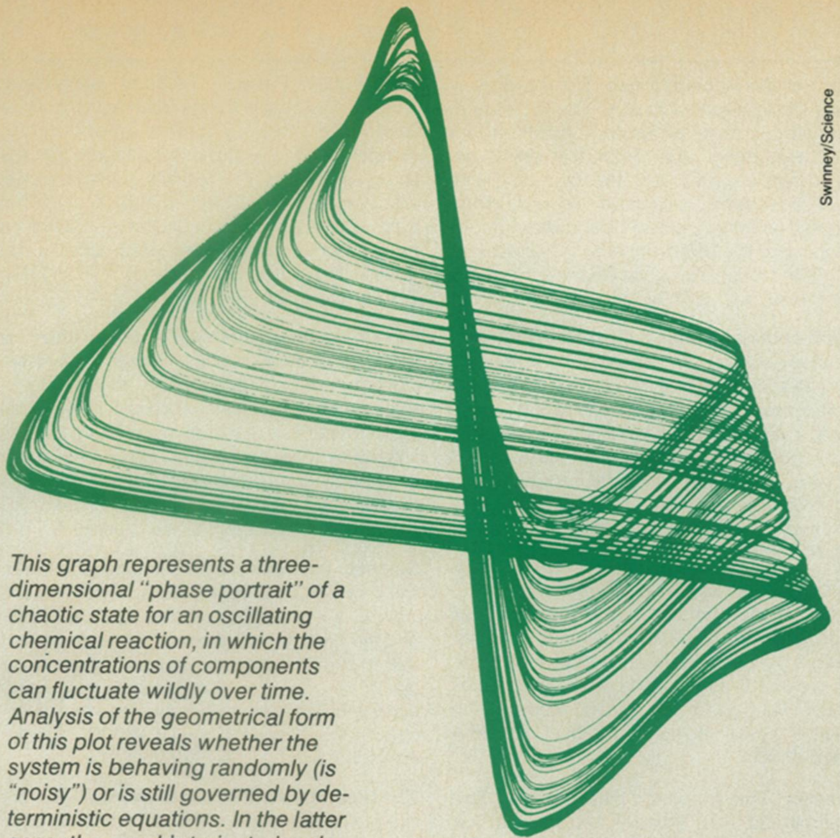
Understanding chaotic behavior involves closely examining the solutions of differential equations that describe and model the phenomena that scientists encounter in nature. They select certain parameters as variables and, to keep things simple, ignore others. In mathematical models of neural systems, for example, the variables can represent the electrical potential across cell membranes, membrane currents and the concentrations of several chemicals present. Some parameters, such as temperature or a particular drug concentration, assume a constant value for a given case but may vary from case to case.

A differential equation links the rate of change over time of one of the variables with the variable's current size and the current size of other variables. Descriptions of complicated systems may require a large set of such equations. In one sense, Rapp notes, a set of differential equations is like a machine that takes in a set of values for all the variables and then generates the new values at some later time. Often, the relationship expressed in the equations is nonlinear; that is, input and output are not proportional.

What some mathematicians have learned is that, under the right conditions, even simple sets of nonlinear differential equations can yield numbers that appear to follow no pattern. Although the equations express direct cause and effect relationships, the numerical results predict that modeled systems can show irregular motion or randomlike, chaotic behavior. In fact, this class of solutions displays a sensitive dependence on initial conditions. A slightly different starting point produces a radically different result.

This can have startling consequences. If weather systems can be described by mathematical equations that shift into chaotic behavior, then a change as slight as a butterfly flapping its wings near a weather station makes long-term weather predictions impossible. Ruelle comments that this work "gives some theoretical excuse to the well-known unreliability of weather forecasts."

Rapp says, "The movement of information can also be described by differential equations." Thus, information flow within a network may potentially become chaotic under the right conditions. Rapp cites a 1979 military exercise that simulated the communications traffic that would result



Swinney/Science

This graph represents a three-dimensional "phase portrait" of a chaotic state for an oscillating chemical reaction, in which the concentrations of components can fluctuate wildly over time. Analysis of the geometrical form of this plot reveals whether the system is behaving randomly (is "noisy") or is still governed by deterministic equations. In the latter case, the graph's trajectories define a "strange attractor," and the system's behavior is termed "chaotic" in a mathematical sense.

from a conventional war. The exercise quickly lost coherence, and later, officials reported that at times during the exercise they wondered whether they were playing the same game. Other reports of possible "computer convulsions" have surfaced more recently, including an accidental, computer-activated missile firing from an airplane.

Because nonlinear differential equations approximately model natural phenomena, scientists are taking a closer look at observations of irregular behavior that previously may have been ignored (the kind of results that often ended up in waste baskets). Harry L. Swinney of the University of Texas at Austin, who has demonstrated the transition of oscillating chemical reactions (SN: 9/19/83, p. 188) from smooth, well-ordered behavior into chaos, says, "Everybody's done experiments and gotten noisy data." The kind of noise that he demonstrated, however, is "intrinsic to the reaction. It depends on the dynamics of the system," he adds. "Our feeling is that there are many situations in nature and in industry where you have systems that, no matter how well you control them, exhibit nonperiodic [chaotic] behavior."

The work of people like Swinney and Ruelle has inspired a small group of medical researchers, including Rapp, to apply similar mathematical techniques to the analysis of biological behavior. One physiologist predicts, "Chaotic dynamics will be the appropriate language to describe what is happening in many pathological circumstances, just as the calculus is an appropriate language to describe what happens when an apple falls from a tree."

One important insight, Rapp comments, is that systems may behave "normally" for a wide range of initial conditions and then suddenly shift into a chaotic mode when a parameter, such as the concentration of an administered drug in the case of a neural system, moves through a critical value. Thus a tiny change in a parameter can result in dramatically altered behavior. This abrupt change in behavior characteristics, obtained reproducibly in response to a small change in the value of a system parameter, is called a bifurcation.

The question is whether an epileptic episode is an example of "the consequences of a bifurcation to chaotic behavior," says Rapp. Indirect theoretical and experimental evidence suggests that transitions from normal behavior to convulsions are probably the result of bifurcations and that neurons and neural networks are capable of shifting into chaotic behavior, he says. However, the question cannot be answered yet with certainty.

Rapp says, "This has some very exciting implications from the point of view of treatment because it suggests that [a more] rational design of chemical therapy of convulsive disorders could be possible." A reliable mathematical model would allow a physician to identify which drugs "reset parameters" so that a convulsion stops. "One can view this form of clinical intervention as essentially an exercise in parameter resetting," says Rapp.

These mathematical methods "are now available for the first time to permit the rigorous analysis of dramatic irregular behavior," says Rapp. "You no longer have to look at highly disordered behavior and simply regard it as inevitable, as an act of fate or something you can do nothing about. It can be analyzed."

Physiologist Leon Glass of McGill University in Montreal has applied the notions of topological dynamics to heart cells and the processes that lead to irregular heartbeats. He has observed the onset of chaotic dynamics in chicken heart cells as the result of electrical stimulation at particular frequencies and amplitudes.

Glass suggests that these mathematical ideas may apply to a variety of diseases (which he calls "dynamical diseases") and other physiological problems. He notes, for example, that when mechanical ventilators are used to help patients breathe, depending on the machine's frequency and amplitude, situations often arise when a patient "fights the ventilator" and receives insufficient air. Glass says that practitioners are rarely sensitive to the fact that proper breathing will occur at some frequencies and amplitudes but not at others.

"What may be happening is that this reflects some kind of irregular, perhaps chaotic, dynamics that arises just because of the nature of the frequency," says Glass. "Maybe it's a little bit analogous to the irregular dynamics of the fibrillating [uncoordinated twitching] heart."

However, Glass warns that because so many biological behaviors are irregular, "there's a tendency for people to say everything is chaotic," in the mathematical sense of the word. Careful experiments are necessary to show that the behavior results from bifurcations because of changes in parameters, he says.

Rapp argues that chaotic behavior is not limited to biological systems. "In particular, it is observed in the kinds of physical devices and networks that are used to construct the hardware of electronic and optical systems," he says. These include laser discharges and the sudden appearance of background noise in electronic devices like some semiconductor oscillators.

Rapp concludes, "At present, dynamical theorists are confronted by a *terra incognita*. Present ignorance makes it impossible to confidently identify which systems are, or are not, robust against parameter-dependent failures." In the case of computer systems, he suggests that these failures may become increasingly probable as electronic hardware becomes more sophisticated. In many situations, a computer that fails as often as the human brain could produce disastrous results. □