ANTS IN LABYRINTHS and Other Fractal Excursions

Researchers are using an increasingly rich palette of fractal shapes to describe clouds, fractured metal surfaces and processes like diffusion

By IVARS PETERSON

When the delicate fragrance of a perfume weaves its way through the air, individual perfume molecules, jostled about haphazardly in the hurly-burly of molecular collisions, follow a jagged path. Air currents further tangle these tortuous paths into "monstrous" trails that until a few years ago could only be vaguely described as "complicated." The same lack of a compact, precise, mathematical way to describe irregular forms also applied to the crinkly roughness of fractured metal surfaces, the shapes of mountains and clouds, and many other naturally occurring patterns. But a new geometry has made these fragmented forms describable and at the same time has encouraged scientists to look at old, seemingly inexplicable experimental results in a new way.

About a decade ago, Benoit B. Mandelbrot, working at the IBM Thomas J. Watson Research Center in Yorktown Heights, N.Y., invented the geometrical concept of a "fractal" to describe nature's irregularities (SN: 8/20/77, p. 122). Fractals represent objects or patterns that appear "self-similar": No matter what scale is used, the pattern looks the same. The new detail that appears in a magnified portion of a fractal shape looks just like the original pattern. And no matter how grainy, tangled or wrinkled they are, the irregularities are still subject to strict rules.

The scaling property of fractals is summarized by a number called the "fractal dimension," which introduces geometrical dimensions that are not whole numbers. While a straight line has one dimension, a wriggly fractal curve can have a dimension anywhere between one and two, depending on how much space the curve fills in following its meandering course. Similarly, a hilly fractal scene can lie somewhere between the second and third dimensions of classical geometry. A landscape with a fractal dimension close to two may show a huge hill with tiny projecting bumps while one with a fractal dimension close to three would feature a rough surface with many medium-sized hills and few large ones. Theoretically, fractal dimensions can also go above three.

Mandelbrot says, "The importance of fractals lies in their ability to capture the essential features of very complicated and irregular objects and processes, in a way that is susceptible to mathematical analysis."

One recent application of these ideas is to the irregular surface of a fractured piece of metal. Mandelbrot worked with some metallurgists to come up with a method that would specify the roughness of a given surface. "We found that a large variety of surfaces, although not all, have a roughness that is very systematic," he says. It can be represented by the fractal dimension. In a paper that will soon appear in NATURE, the researchers note that the measured fractal dimension took on the same value for different specimens of identically treated samples of the same metal. They found that different heat treatments not only affected the toughness of a metal but also changed its fractal dimension. They conclude that this fractal dimension may itself be a useful measure of a metal's toughness or strength, providing metallurgists with a new tool for characterizing metals.

Fractals have also had a great impact on computer graphics (SN: 11/20/82, p. 328). One of Mandelbrot's pastimes is to create fractal objects that look like natural patterns. Using a random model for generating fractals, he found that it was possible to draw pictures of mountains that looked remarkably realistic. Most recently, he and Richard F. Voss of IBM successfully simulated the appearance of clouds. Such pictures, generated by computers following carefully defined mathematical routines, "always look right," says Mandelbrot. But the underlying reason why these random fractals work is still a puzzle.

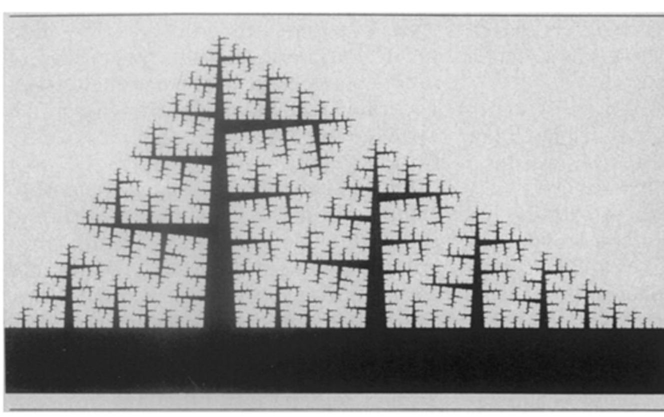
Attempts to generate realistic, random models of trees, however, have not been successful so far. Mandelbrot says, "Trees are one of our biggest failures." As trees grow, twigs and branches tend to avoid one another and to die off when severely overshadowed. "It's a randomness combined with self-interaction to a strong degree," says Mandelbrot. "When we have to take account of things that interfere with each other, it becomes complicated."

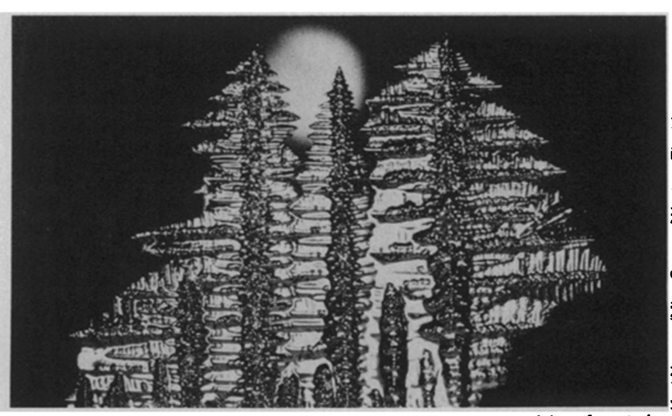
Fractals are rapidly becoming an important scientific tool. The first paper in PHYSCIAL REVIEW LETTERS using the word "fractal" appeared in 1980. Now, fractal articles show up in almost every issue, Mandelbrot says, illustrating the explosive growth of the field.

Late last year, Mandelbrot and Michael F. Shlesinger of the Office of Naval Research in Arlington, Va., organized "Fractals in the Physical Sciences," the first such North American conference, held at the National Bureau of Standards in Gaithersburg, Md. The meeting brought together a diverse group of scientists working on applying fractal ideas to a wide range of physical processes. Shlesinger says, "We thought that if we all got together it would help us standardize notation, meet each other and facilitate further contacts between us."

Shlesinger says that fractal ideas are allowing physicists to reexamine problems that they once ignored. In experiments, scientists usually look for relationships between variables; for example, how the intensity of sound waves scattered from a metal surface depends on the waves' frequency. A theory may predict that doubl-

Mandelbrot/*The Fractal Geometry of Nature*, c 1982 Benoit B. Mandelbrot, W. H. Freeman & Co.

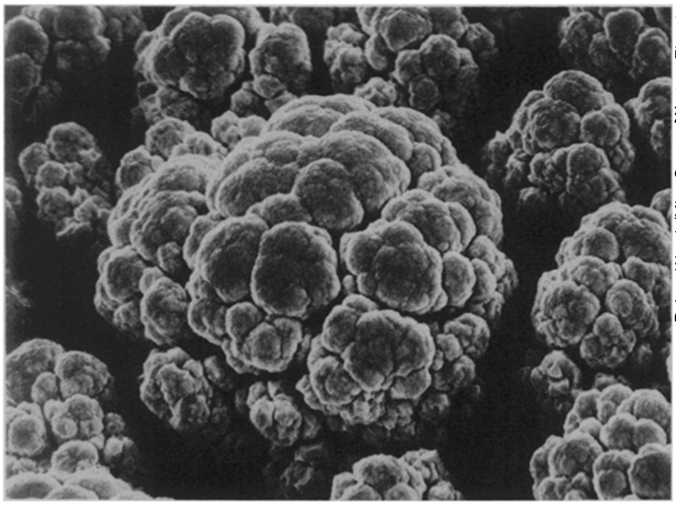




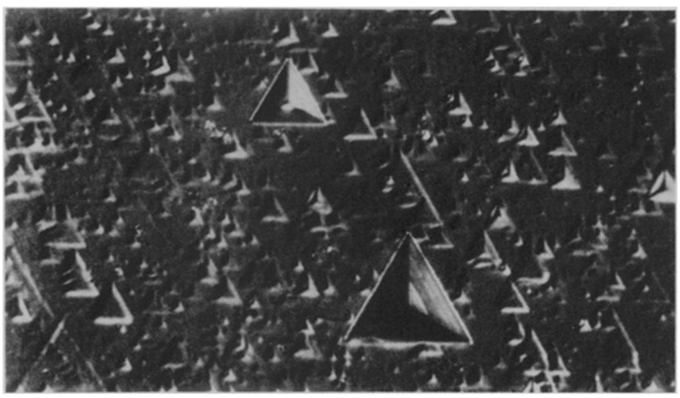
Kurt Nassau/MicroScapes/Western Electric

Fractals are often useful for characterizing the "landscapes" that show up in microscopic views of surfaces. This branching fractal pattern (left) appears to reflect the characteristics of clusters of dendritic tin crystals (right).

These cauliflower-like jumbles are really clusters of electrodeposited gold with the "self-similar" appearance that identifies a fractal pattern.



Robert Woods/MicroScapes/Western Electric



The array of triangles represents vaporization pits on the surface of an arsenic crystal. This pattern, too, seems to have a fractal structure.

Clifton Draper/MicroScapes/Western Electric

ing the frequency will quadruple the intensity. Thus, the intensity should be proportional to the frequency squared. However, in many experiments, the exponents that express the proportionality turn out to be numbers like 2.79 instead of integers like two.

We were brought up to think of integers as the natural way of representing physical processes, says Shlesinger. "What we see now is that that's not true. The noninteger exponents are the physical ones." Physicists now realize that all these things that they've been putting on the back burner because they didn't understand them are what's natural, says Shlesinger. "There is something geometric hidden in these exponents. They are the dimensions of fractal geometric objects."

Shlesinger adds, "Some problems become very, very simple if you look at them in the right way. Now that fractals have come along, some things that were very difficult become easy. It gives you a language in which to describe and measure the amount of structure, rather than just saying the paths are stringy or the surfaces have holes in them. It gives you numbers that you can play with, and maybe you can find relationships between them."

With this new realization, physicists and many other researchers are using fractal patterns to model physical processes like the sudden, temperature-dependent onset of superconductivity in thin films of lead, the adsorption of gaseous krypton atoms on graphite surfaces or the agglomeration of small particles of gold to form larger clusters.

One approach is to measure experimentally some property in a physical system that scales according to a fractal dimension. This property can be something like the current in a metallic film or the pattern of frequencies found in light emitted by molecules. Because the physical process itself is often too complicated for straightforward mathematical analysis, the researcher then looks for a more regular fractal pattern with the same fractal dimension but on which mathematical calculations are easier to perform. Shlesinger says, "It's a sample environment in which to test the effects of dimension or how tortuous paths are." The same fractal pattern may be a crude but useful model for many different physical processes that happen to have the same fractal dimension.

"Percolation clusters" are particularly useful for this kind of modeling. Such a cluster has properties similar to those of a floor covered with a random mixture of copper and vinyl tiles. Current will flow from one side of the floor to the other if there is a continuous copper path, no matter how roundabout. If most of the tiles are vinyl, current isn't likely to go through. Adding more randomly placed copper tiles increases the likelihood of linking patches of copper tiles to form a continuous path. A percolating cluster represents the point at which a conducting path is first created. This construct seems to work as a mathematical metaphor for many different diffusion processes like the ability of oil to seep through porous rock and the spread of plant species in a forest. It also works for abrupt changes in phase like the lining up of atomic spins to create a magnet or the onset of superconductivity in a thin metallic film.

"The problem is that it's hard to understand calculations done on the actual, random percolation cluster," says Mandelbrot. "They take time, and the calculations are only approximate because it's a complicated problem." Researchers interested in the problem set about looking for fractal patterns that were systematic enough to ease mathematical calculations but random enough to be realistic. At the fractal meeting, Mandelbrot proudly unveiled his latest creation: a mazelike pattern of connected rings within rings within rings, and so on (shown on the front cover). "The randomness makes it realistic, and its systematic character makes it workable," he says.

Another intriguing and ultimately scientifically rewarding pursuit is to look at "fractals upon fractals." Diffusion, as exemplified by the peregrinations of the perfume molecules mentioned earlier, is a fractal process. When diffusion occurs along a fractal surface, then the process is something like letting an ant wander in a labyrinth and seeing where it goes. An ant constrained to wander along a straight line always eventually returns to its starting point. On a two-dimensional plane surface, the ant gets lost. But on fractal paths with dimensions between one and two, what happens to the ant is unclear. Depending on the nature of its fractal labyrinth, the ant may keep running into dead ends forever or perhaps return only infrequently to its starting point. This curious analogy, too, is a useful image for many physical processes, says Mandelbrot.

One of the more startling results of current research on fractal-modeled physical processes is that all kinds of very different problems give rise to numbers, fractal dimensions, that are very close to each other. These phenomena range from the distribution of galaxies in the universe to the nature of turbulence in flowing fluids. Mandelbrot says that it is too early yet to tell whether these phenomena are a collection of separate problems with a different explanation for each case or whether there is some underlying principle that will explain many of them simultaneously.

Meanwhile, as the use of fractals as a descriptive tool diffuses into more and more scientific fields, from cosmology to ecology, explanations for why fractals work will begin to emerge. "From the beginning, description went faster then explanation," says Mandelbrot. "At present, it's a theory that works without being fully explained." □