Surface connections for network problems

A seemingly simple mathematical puzzle is sometimes impossible to solve unless its constraints are relaxed slightly. One such puzzle is the problem of finding a way to connect five points marked on a flat surface so that lines join each point to all of the other points yet don't cross each other. No matter how hard one tries, it can't be done on a plane or on the surface of a sphere. There is always one pair of points left unconnected. However, if a "handle" is attached to the sphere to create an object called a torus, then the final line can be drawn without crossing the other lines, and the problem is solved.

This well-known result forms part of the background to some new results in a branch of mathematics called graph theory, which concerns arrays of points and their connections. Graph theory is a useful tool for designing and analyzing a variety of networks. For example, it can be used to help in the complicated task of finding the best routes for laying down microscopic threads of bare metal on the surface of an integrated circuit chip to connect as many as a million circuit elements into an efficient electronic network.

The new mathematical results come from Paul D. Seymour, now at Bell Communications Research, Inc., in Murray Hill, N.J., and G. Neil Robertson of Ohio State University in Columbus. Their work involves broad, general questions concerning when graphs can and cannot be drawn on a given surface.

In 1930, Polish mathematician Kasimir Kuratowski showed that if a graph drawn on a plane or a sphere's surface has hidden within it one of two "minimal" graphs, then there must be at least one place where lines cross. One such minimal graph is a group of five points all connected to one another. The second minimal graph consists of two sets of three points, with lines connecting each point in the first set to each point in the second set. Neither graph can be "embedded" or drawn on a plane surface without introducing an intersection. The presence of either of these minimal graphs within another graph immediately makes the entire graph "nonplanar."

A few years later, Hungarian mathematician Paul Erdös proposed that a similar, finite, complete list of minimal graphs could also be found for more complicated surfaces, such as a sphere with one handle (a torus), a sphere with two handles and so on. This list would provide a way of telling whether a complicated graph could be embedded on a particular surface. Recently, Seymour and Robertson were able to prove that the Erdös conjecture was true. All surfaces, no matter how many handles they sprout, have a finite, complete list of minimal graphs.

Mathematician Ronald L. Graham of AT&T Bell Laboratories in Murray Hill, N.J., says this means that there are only a limited number of "obstructions" that stand in the way of embedding a graph on a particular surface. However, while there are only two minimal graphs for a plane or a sphere, a torus has between 800 and 1,500 such graphs. A two-handled surface may have as many as 80,000 minimal graphs. In addition, there's a different list of these graphs for each different surface.

Graham notes that no one has yet made complete lists of these special graphs. In the future, some systematic way of writing them all down may be developed, he says, "but the fact that the list is finite is a quantum leap in our understanding of the situation."

Seymour says that this result is an offshoot of work on proving a more general conjecture suggested by German mathematician Kurt Wagner about 20 years ago. The Wagner conjecture states that for any infinite list of graphs, at least one of them is contained inside another. There seems to be no reason why this ought to be true, but Seymour and Robertson have already proven the conjecture's validity in several special cases, including the one that led to the proof for the Erdös conjecture.

So far, Seymour and Robertson have published seven long papers showing steps on the way to the general proof. "There are a lot of details," says Seymour. He expects that the complete proof will take at least three more lengthy papers before they are through. "It's like an excavation," says Seymour. "We're digging away, and the hidden shapes begin to appear."

— I. Peterson

Coal burns best in pipes that hum

A coal-burning system with a central tubal chamber that resonates like an organ pipe during combustion has been designed by a Georgia engineer. The system's 70-hertz hum is no accident, but instead a design feature that should catapult its energy-conversion efficiency well above the norm, says Ben Zinn of the Georgia Institute of Technology in Atlanta.

There are two goals in optimizing combustion efficiency: to burn fuel as completely as possible, and to burn it with as little air as possible. Incomplete combustion obviously wastes fuel; less obvious is the fact that use of too much air robs the system of heat. The acoustic waves resonating through the prototype "pulsating combustor" make it possible for Zinn to obtain virtually complete combustion with almost no "excess air." Says Zinn, "I'm not aware of anyone ever getting the results we have. I think we have something unique."

Burning can only take place in the presence of oxygen. Just as an auto engine must breathe in a certain critical ratio of air along with its fuel, so must any other combustion system. Theoretically, the amount of air required to burn a given amount of fuel completely is usually not enough, in practice, to prevent the production of smoke. As a result, combustion engineers must always budget in a certain amount of excess air. The *Dictionary of Energy* (Schocken Books, New York 1983) cites as typical values "50 percent excess air for coal-fired [combustion units], 20 percent for oil-fired and 10 percent for gas-fired installations." Any excess air in the system will be heated by the hot combustion process and eventually be exhausted along with other waste gases. The more excess air used, the more heat robbed from the system.

Zinn can get 92 percent combustion efficiency—a figure many energy managers could live with — using no excess air. By adding six or seven percent excess air, he achieves greater than 97 percent combustion efficiency, a value electric utilities strive for with their better systems. Moreover, Zinn points out, to achieve comparable efficiencies, most of those other systems require use of "at least 20 percent and sometimes 30 percent extra air. That," he emphasizes, "is a huge difference."

Zinn's system taps an acoustic principle formulated during the 19th century by a physicist named Rijke: By heating gases within a tube at a critical point, the resulting excitation of gas molecules will generate acoustic oscillations that make the pipe sing. The heat source in Zinn's device is the combustion process itself, which occurs on a porous metal grid inside the pipe. Fuel entering the chamber from a portal along one wall drops onto the grid where it meets cold air that's been pumped in from below.

In his model, the combustion tube is 9 feet long and 5.5 inches in diameter. Its dimensions determine the oscillation frequency, which for this system "means that the molecules in each direction are moving back and forth 70 times a second," Zinn says. "I have such superior combustion because the acoustics give me much better mixing of fuel and air." But the oscillations have a second benefit: It turns out they increase the transfer of heat from the hot combustion-exhaust gases to the walls of the combustion chamber, increasing the energy available to do work — for example to heat the steam that drives a turbine to generate electricity.

"The implication of these advantages is that for the same energy output, pulsating combustors can be smaller than conventional ones and therefore less expensive," Zinn says. Another advantage of his system is that it doesn't require use of pulverized coal, a slightly more costly form of the fuel. Georgia Tech has a patent pending on Zinn's design.

—J. Raloff