Shadows from a Higher Dimension

The journey into higher dimensions begins in Flatland

By IVARS PETERSON

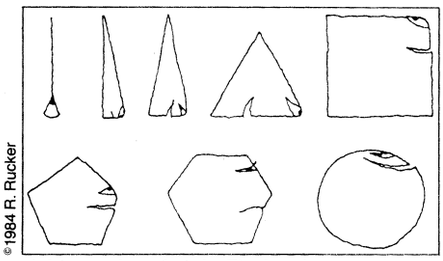
Flatland is a thin book — about 100 pages of Victorian prose that now seems somewhat quaint and old-fashioned. Yet, for a century, the book's central figure and narrator, "A Square," has enticed innumerable readers into a two-dimensional world where a race of rigid geometric forms live and love, work and play.

Edwin A. Abbott's narrative begins: "Imagine a vast sheet of paper on which straight Lines, Triangles, Squares, Pentagons, Hexagons, and other figures, instead of remaining fixed in their places, move freely about, on or in the surface, but without the power of rising above or sinking below it, very much like shadows — only hard and with luminous edges — and you will then have a pretty correct notion of my country and countrymen."

Over the years, this imaginary world has inspired students to take up geometry, mathematicians to conceive of higher dimensions, writers to build on Abbott's themes and artists to explore the nature of space and time. Last month, to celebrate the original publication of *Flatland* in October 1884, mathematician Thomas F. Banchoff of Brown University in Providence, R.I., organized a symposium that brought together many artists and scholars to consider the question of "Visualizing Higher Dimensions" and to pay tribute to Abbott and his playful fantasy.

On the surface, *Flatland* appears simply to be a good story, a clever mathematics lesson and a pointed social satire. But the book also reflects important, widely debated social and religious issues in Victorian Britain, including Abbott's advocacy of women's rights. In Flatland's structured society, for instance, women are merely "straight Lines" — beneath even the lower-class Isosceles Triangles. Squares make up the professional class, Nobles are polygons with six or more sides, and the highest ranking Priests are perfect Circles.

"[A] Woman is a needle; being, so to speak, *all* point, at least at the two extremities," says "A Square," the scholarly



A hierarchy of eight Flatlanders

commentator. "Add to this the power of making herself invisible at will, and you will perceive that a Female, in Flatland, is a creature by no means to be trifled with." Nevertheless, Flatland women also are "devoid of brain-power, and have neither reflection, judgment nor forethought, and hardly any memory." In this world, men believe that educating women is wasted effort, and that communication with women must be in a separate language that contains "irrational and emotional conceptions" not otherwise found in male vocabulary.

These satirical images sprang from the mind of a headmaster at the City of London School, an institution that prepared middle-class boys for professional careers or places at universities like Cambridge. Abbott wrote dozens of books, including texts on school subjects, historical and biblical studies, theological novels and a well-regarded Shakespearean grammar that strongly influenced the study of the Bard's plays. At first glance, *Flatland* seems out of place within this collection, but a closer look shows that it combines elements from all of Abbott's broad range of interests and concerns — from the nature of miracles to the promotion of women's issues and the reform of mathematics education.

Abbott was part of a group of progressive educators seeking changes in the mathematics requirements for university entrance, which at that time included the memorization of lengthy proofs in euclidean geometry. This, the educators felt, was a waste of time and narrowed the study of geometry unnecessarily. However, the conservative mathematics establishment in Britain resisted the attack.

Moreover, while popular interest in the existence of a fourth and higher dimensions grew, establishment mathematicians also refused to consider that such geometries were even conceivable. The concept of higher dimensions would call into question the very existence and permanence of "mathematical truth," as represented by three-dimensional euclidean geometry, they argued. *Flatland* challenged that position strongly. Abbott himself couldn't resist calling Flatland's university "Wentbridge."

Flatland also represents one of Abbott's attempts to reconcile scientific and religious ideas — the relationship between material proof and religious faith. In the book, "A Square" receives a visit from a ghostly sphere from Spaceland, who eventually demonstrates that Flatland is but one world in a multidimensional universe. Using this analogy, Abbott sought to show that establishing scientific reality also requires a leap of faith. Conversely, miracles could be explained in terms that didn't violate physical laws. Like early scientific theories, couched in terms of things already known, miracles were perhaps just shadows of phenomena outside of everyday experience—perhaps visitations from a higher dimension—whether in Lineland, Flatland or Spaceland.

Banchoff suggests that *Flatland* raises the fundamental question, "How do you react when you come face to face with the truly transcendental, with something which you recognize almost from the very start that you will not be able to comprehend fully? How do you organize your insights, and how do you communicate them to other people? ... The *Flatland* analogy provides one of the best ways to approach the challenge."

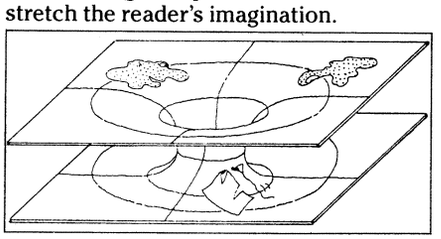
When *Flatland* was first published, the book was an immediate success. That popularity has lasted, as demonstrated by the sale of more than 300,000 copies in the last 30 years and the appearance of six

Flatland *was first published in October 1884. A century later, the book is still one of the best introductions to visualizing higher dimensions and was recently the subject of a symposium.*

Courtesy: Banchoff/Brown Univ.

new editions in the last two years. Several writers have imagined further adventures for "A Square" or created their own analogous worlds, such as Dionys Burger's *Sphereland* (Barnes & Noble Books, 1983), a fantasy about curved spaces.

In *The 4th Dimension: Toward a Geometry of Higher Reality* (Houghton Mifflin, 1984), mathematician and science fiction writer Rudy Rucker sends "A Square" to explore curved spaces, wormholes ("magic doors to other worlds") and other bizarre features suggested by modern theories of space and time. This whimsical approach allows Rucker to introduce a wide range of provocative ideas that stretch the reader's imagination.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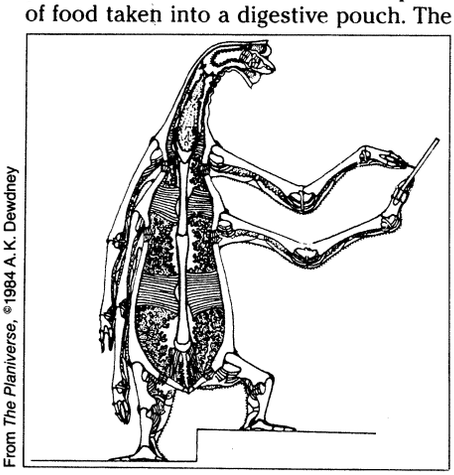
Flatlanders at a space tunnel's mouth

Rucker writes: "No one can point to the fourth dimension, yet it is all around us.... There are, in fact, many higher dimensions. One of these higher dimensions is time, another higher dimension is the direction in which space is curved, and still another higher dimension may lead us to some utterly different universes existing parallel to our own. At the deepest level, our world can be regarded as a pattern in infinite-dimensional space, a space in which we and our minds move like fish in water."

The Planiverse: Computer Contact with a Two-Dimensional World (Poseidon Press, 1984) by Alexander K. Dewdney, a computer scientist at the University of Western Ontario in London, presents a much more sophisticated two-dimensional world than Abbott's Flatland. Dewdney's tale carefully builds a credible, logically consistent astronomy, physics, chemistry, geology, biology and technology for a two-dimensional universe.

In Dewdney's flat world (where the inhabitants roam the edge of the disk-shaped planet Arde, and "down" means toward the center of the disk), living creatures have "zipper organs," which consist of two narrow bands of interlocking teeth, for pumping fluids. Digestion is a curious process that involves the slow absorption of food taken into a digestive pouch. The


From *The Planiverse*, ©1984 A.K. Dewdney

Yendred, transparent denizen of Arde

indigestible portion of the food is eventually expelled through the pouch's single opening. All this is visible to any three-dimensional observer, who can readily peek inside these creatures to see their skeletons and internal organs. "What you see is what you get," says Dewdney.

On Arde, nails and saws are useless. Glue is the most common fastener, and plywood is a composite made up of hair-like threads. Houses are built underground, dug into the planet's edge so that they do not get in the way of inhabitants moving around. Yet Ardean life is still full of delays. Obstacles continually impede travel; inhabitants must wait, stay in line or climb over one another to get anywhere. But sailboats need only a mast to cross the planet's sea, and the use of balloons is rapidly expanding. Ardean writing —strings of dots and dashes—is slow and takes up a lot of room, so books are few, terse and rapidly recycled. A form of volleyball (played with an inflated circle) exists, but bowling is boring because nobody ever misses.

Looking at two-dimensional science and technology "throws light on our world," says Dewdney. "It explains why things [in three dimensions] are so hard." In particular, there is wide interest in the consequences of relativity, electromagnetism, gravitation and quantum theory in two dimensions because this simpler, special case may lead to a better way of looking at and understanding these phenomena in higher dimensions.

The appeal of *Flatland* and its successors, says Banchoff, is in their "challenge to the imagination." He adds, "My job as a pure mathematician is to help prepare people for the challenge of seeing and understanding higher dimensions." *Flatland* is a good place to start. □