'Ptarmigans Wheeling Over the Gorse'

At the recent Third Loyola Conference on Quantum Theory and Gravitation, held at Loyola University in New Orleans, David Hestenes of Arizona State University in Tempe urged all physicists working toward a unified theory of physics to use the same mathematical language. Such a theory would eventually explain everything in physics in a connected way, and Hestenes thinks those working toward it should talk the same language.

As anyone who has ever done a translation will know, the verbal languages of humanity are not exactly congruent to one another. The meanings of words and the images they evoke seldom coincide exactly from language to language. Disparities of grammar and syntax compound the difficulty. Many years ago in a course on French stylistics I was subject to a textbook that contained snippets from famous English authors, which we were supposed to translate into French. One of those passages had "ptarmigans wheeling over the gorse." The translating dictionary gives *lagopède* for "ptarmigan." Assuming it's the same species — and remember, for example, that the crustaceans designated by *homard, langouste* and *langoustine* in French do not divide up the same way as those called "lobster" and "crayfish" in English — that's a start. "Gorse" grows in both countries; it is broom plant, *plante de genèt,* in French. The real kicker is how to express in French the images called up by the verb "wheel." For sophomores that was a serious problem, and I don't remember how any of us solved it, but that's not the point.

Surely such things don't happen with mathematics, which is designed to be logical, denotational and nonaffectional. But, *au contraire,* they do. A famous historical example comes from the early days of quantum mechanics. Erwin Schroedinger had devised a way of representing quantum mechanical processes by wave equations, known as wave mechanics. Werner Heisenberg developed a formulation using matrices, arrays of numbers that represent groups of related algebraic equations. Physicists were disturbed by these two radically different ways of representing the same thing — surely one of them had to be wrong. According to the story, confusion persisted until the great Göttingen mathematician David Hilbert showed that the two formulations were mathematically equivalent. But that didn't stop them from looking different.

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Part of the problem is that physicists often do not study mathematics systematically. They tend instead to learn the mathematics they need to do the physics — or, as Isaac Newton did, to invent it. When they try to formulate a new piece of physics, they usually reach for something familiar even if it is not always the most apt choice. Mathematicians often complain about the cavalier attitude of physicists to mathematics.

"How to design a language for mathematical physics?" Hestenes asks. "How to express its geometric content?" Physics has always been close to geometry. In classical physics, objects can have characteristics such as velocity or acceleration for which a direction as well as an amount must be specified to have a complete description. Other properties — for example, temperature or pressure — can vary from point to point in a given region of space.

Modern physics makes the relationship with geometry even more intimate. In classical physics, geometry defines the playing field on which physical processes work themselves out; in modern physics, geometry becomes part of the game. After Einstein had made time into a geometrical dimension, he proceeded to make gravitational forces identical with a geometric quality, the curvature of space. Kaluza's and Klein's attempt of 60 years ago (SN: 7/7/84, p. 12) to relate electromagnetic phenomena to a proposed fifth dimension has recently been revived in an altered form that allows a large number of extra dimensions to relate to a variety of physical properties.

Even before the resurrection of Kaluza and Klein many of the "internal" properties of subatomic particles, the properties that go together to define one kind of particle or another, had close connections to geometry. The schemes most widely used to make some sense and order of the way these properties vary as one kind of particle changes into another kind made use of schemes, the so-called Lie groups, that were devised to make sense and order out of the possible rotations of geometric figures, triangles and hexagons. (Mathematicians spend their time playing with triangles and hexagons; physicists spend their time playing with lambda hyperons and sigma hyperons.)

In view of this intimacy between physics and geometry, Hestenes suggests using Clifford algebra. This is not the place for a systematic definition of how Clifford algebra works — although Hestenes gave one; such a disquisition might well elicit the response of the schoolboy who began his book report by saying: "This book told me more about penguins than I really wanted to know."

Hestenes recommends Clifford algebra because it is an algebra of directions. In the ordinary algebra learned in high school, people combine known and unknown numbers in various ways, by addition, subtraction, multiplication, etc., to solve problems. Clifford algebra does the same sort of manipulation with directions. Therefore, says Hestenes, it is well qualified to handle the many directed quantities in physics in a very natural way, including the complicated ones called spinors, which other mathematical languages that have been used cannot handle.

Can physicists handle Clifford algebra? Will they want to? —*Dietrick E. Thomsen*