ACOUSTIC RESIDUES

Number theory, the paradigm of pure mathematics, helps change the sound of small rooms and concert halls

By IVARS PETERSON

There's a new ingredient in the sound of The Oak Ridge Boys. On the rear wall of the control room in their recently completed recording studio in Hendersonville, Tenn., hangs a cluster of panels that looks like it belongs in a trendy art gallery or on the set for a "Star Trek" episode.

Sculpted from wooden strips separated by thin aluminum dividers, each panel consists of an array of wells of equal width but different depths. These panels, called reflection phase gratings, scatter sound waves. The result is a richer, livelier sound with an enhanced sense of space: The walls seem to disappear, some listeners comment, and a small room takes on the air of a great hall.

The secret lies in the varying depths of a panel's wells. With depths based on specific sequences of numbers rooted in the mathematics of number theory, these wells scatter a broad range of frequencies evenly over a wide angle. Developed by RPG Diffusor Systems, Inc., a small, young company in Largo, Md., such panels are starting to show up in modern broadcast booths, concert halls, churches and even nightclubs. The next step may be into private homes.

The scientist who pioneered the ideas now being applied and refined by RPG is Manfred R. Schroeder of the Drittes Physikalisches Institut at the University of Göttingen in West Germany. He is also associated with AT&T Bell Laboratories in Murray Hill, N.J. Recently, at an Acoustical Society of America meeting in Nashville, Tenn., Schroeder outlined some of the steps that led to his discovery. He titled his address: "The unreasonable effectiveness of number theory in acoustics."

About a decade ago, Schroeder and two collaborators undertook a major study of more than 20 famous European concert halls. One of their findings was that listeners like the sound of long, narrow halls better than that of wide halls. Perhaps the reason for this, Schroeder reasoned, is related to another finding — that listeners prefer to hear somewhat different signals at their two ears.

In a wide hall, the first strong sound to arrive at a listener's ears, after sound traveling directly from the stage washes over the listener, is the reflection from the ceiling. Such reflections produce very similar signals at the two ears. In narrow halls, the first reflections reach the listener from the left and right walls, and these two reflections are generally different.

AT&T Bell Labs

Manfred R. Schroeder

This may be why so many modern halls are acoustically unpopular, says Schroeder. Economic constraints dictate the construction of wide halls to accommodate more seats, and modern air conditioning systems allow lower ceilings. To improve such halls, sound must be redirected from the ceiling toward the walls.

A flat surface can't do the job. It reflects sound in only one direction, according to the same rules that govern light reflecting from a mirror. "The ceiling must have ups and downs," explains Schroeder. "How deep the notches should be comes from number theory." The result is an acoustic grating analogous to diffraction gratings used to scatter light.

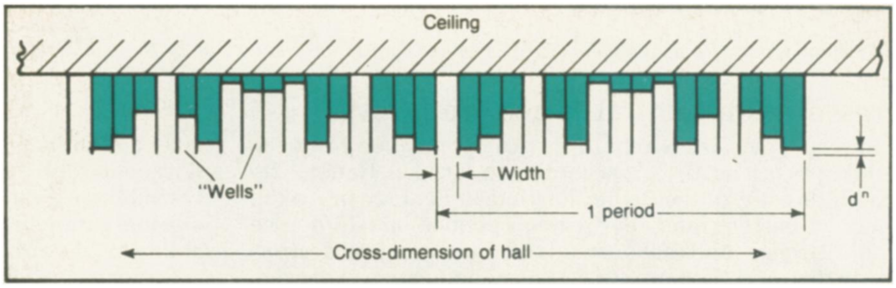
Although number theory suggests several answers, the most effective grating is based on "quadratic-residue" sequences. One example of such a sequence is based on the prime number 17. The first sequence member is the remainder (or residue) after the first number, 1, is squared and divided by 17. The answer is 1. Squaring all the numbers from 1 to 16, then dividing by 17 to find the residue, produces the sequence: 1, 4, 9, 16, 8, 2, 15, 13, 13, 15, 2, 8, 16, 9, 4, 1. For larger numbers, the pattern simply repeats.

Finding the depth of a given grating well involves multiplying the appropriate number in the sequence by the longest wavelength for which the grating is designed to scatter sound efficiently and then dividing by a factor that depends on the well's numerical position. Mathematical analysis shows that for such an arrangement, the spectrum of energies scattered into different directions is essentially flat.

"Why does number theory work so well?" asks Schroeder. The answer is in the way waves cancel or reinforce each other, depending on whether a crest of one wave meets a trough or a crest of another wave. For perfectly periodic waves, destructive interference occurs whenever one wave lags behind the other by half a wavelength or one-and-a-half wavelengths or two-and-a-half wavelengths and so on. In every case, the result is the same.

Hence, in wave interference, it's not the path difference that determines the resulting pattern but the residue after dividing by the wavelength. Similarly, in modular arithmetic what counts is not the numerical value itself but the remainder after division by the modulus (17 in the example).

Reflection phase gratings now provide architectural acoustics designers with a new way of spreading sound around both in space and in time. "You've got only three ingredients to conjure up every imaginable type of acoustic environment, namely, absorption, reflection and diffusion,"



Am Inst of Physics

This design for a reflection phase grating is based on a quadratic-residue sequence for the prime number 17. The pattern, shown in cross section, repeats itself every 17 "wells."

says Peter D'Antonio, RPG president. Sound-absorbing surfaces made of foam or fiberglass and sound-reflecting surfaces such as flat or curved panels are widely used. "Until [reflection phase gratings] came along," says D'Antonio, "there really were no commercial diffusive surfaces." It was like trying to type a paragraph without using, for instance, the letter "d," he says.

RPG now manufactures a variety of panels that can be clustered to create the right kind of sound patterns for control rooms, orchestra pits or choir lofts. Conventional ensemble reflectors, which can often be seen standing behind groups of performers, are usually just curved or flat surfaces that reflect sound in only a few directions. An appropriately placed cluster of gratings, however, distributes the sound energy much more evenly.

"It's amazing to see musicians' faces light up when they have an appropriate ensemble reflection," says D'Antonio. "They can hear their instruments. They get a sense of pitch again."

To make small rooms seem acoustically bigger, the company has also developed a scheme that involves putting absorbing material around and near speakers to reduce early reflections. This mimics the pleasing delay heard in a concert hall between the initial sound from a stage and the first reflections.

With reflection phase gratings, says D'Antonio, "three-dimensional sound coming from two speakers is essentially becoming a reality. It's only when you have sufficient diffusion from the [gratings] and when you take care to minimize those very early reflections that you can create a listening environment that allows you to hear these spatial textures."

With better digital recordings, electronic instruments, superior home playback systems and new forms of music, the demands for improved acoustic surroundings for recording and listening will likely increase. "Just like the performance of a symphony orchestra in a great hall," says Bob Todrank, president of Valley Audio in Nashville and designer of Acorn Sound Recorders for The Oak Ridge Boys, "time cues in recorded material give us the perception of musical size, space and depth. The preservation of these cues is vital to the overall enjoyment of any musical performance." ❑