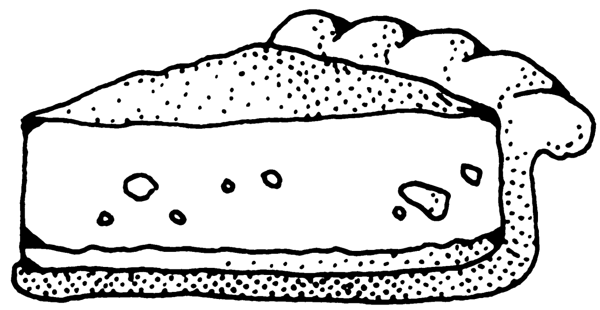
Millions of Digits of Pi



'Digit hunters' have now pursued the value of pi to more than 29 million decimal places

By IVARS PETERSON

For most people, knowing the value of pi (π) to two decimal places is enough. In some practical applications, this ratio of a circle's circumference to its diameter may need to be known correct to 10 or 15 decimal places, but rarely more.

Then what's to be done with 29 million digits of pi? This recently computed, record-breaking number would fill every issue of SCIENCE NEWS for the next four years.

Mathematicians know that pi is an irrational, transcendental number. It cannot be written out fully as a decimal. The number never ends.

Nevertheless, pi can be expressed, for example, as the sum of an infinitely long series: $\pi = 4/1 - 4/3 + 4/5 - 4/7 + 4/9 - \ldots$. Using such a series, anyone, in theory, can calculate pi to as many decimal places as desired.

Over the centuries, many amateur and professional mathematicians have taken up the challenge, performing the tedious calculations involved and looking for series that reach the answer faster. For some, it becomes an obsession.

In the last century, an obscure British mathematician, William Shanks, spent 20 years calculating the value of pi, laboriously multiplying by hand numbers hundreds of digits long. Eventually, he reached 707 decimal places and published the result. But a century later, a new calculation showed that Shanks had made an error in the 528th digit.

The coming of high-speed computers with large memories has now added a little glamor to this centuries-old digit hunt. Last month, a newly installed Cray-2 supercomputer found the value of pi to 29,360,000 decimal places. The previous verified record had been a Japanese effort that reached about 10 million digits (SN: 1/5/85, p. 8).

"We wanted to do something flashy," says David H. Bailey, a computer consultant at the NASA Ames Research Center at Moffett Field, Calif. Bailey wrote the pi-calculating computer programs in his spare time. "We wanted something that would show off the power of the Cray-2," he says, and something that would thoroughly test the computer's hardware and software.

"If even one error occurs in the computation," Bailey says, "then the result will almost certainly be completely in error after an initial correct section." On the other hand, if the result of the computation, even to 100,000 decimal places, is correct, then the computer has done billions of operations without error, he says. Both manufacturers and purchasers of new computer equipment often compute the expansion of pi to certify system reliability.

Applying an algorithm recently discovered by mathematicians Jonathan M. Borwein and Peter B. Borwein of Dalhousie University in Halifax, Nova Scotia, Bailey's program took up 28 hours of processing time and 138 million words of main memory on the Cray-2 to yield 29,360,128 digits of pi. This computation required 12 trillion arithmetic operations.

To check the result, Bailey ran a program implementing an older, slower, somewhat different algorithm, also developed by the Borweins. Only the last 17 digits were different. "Thus it can be safely assumed," says Bailey, "that at least 29,360,000 digits of the final result are correct." He has submitted his results and an outline of his method to the journal MATHEMATICS OF COMPUTATION.

The result is very credible, says Peter Borwein. "It's virtually impossible that a bug would give the same answer from two algorithms as different as these."

One reason for interest in the decimal expression of pi is that most mathematicians suspect that the sequence of digits is random. On the average, each number from 0 to 9 should appear one-tenth of the time, they say. However, this conjecture has never been proven.

Very little is actually known about the mathematical properties of pi, says Stan Wagon of Smith College in Northampton, Mass. "It is not even known that all digits appear infinitely often," he writes in THE MATHEMATICAL INTELLIGENCER (Vol. 7, No. 3). Perhaps π = 3.1415926 ..... 01001000100001000001 .... This is just one of many mysteries that encircle pi.

Thus, there is continuing interest in performing statistical analyses on the decimal expansion of pi to search for possible patterns. "I did a quick analysis of the frequencies of digits," says Bailey. He didn't find anything statistically abnormal. Bailey plans to do more detailed analyses when he finds time. A Japanese statistical analysis of the first 10 million digits also showed no unusual deviations from expected behavior.

Another interesting mathematical question involves the newest algorithms used to compute the decimal expansion of pi. "They converge quickly," says Borwein. "It's hard to think of any other process that can compete with them." This means that significantly faster algorithms are unlikely to be found.

"In some ways, a more interesting and probably harder question," he says, "is to determine what the theoretical lower limit is on how fast you can calculate this." Proving that the algorithms can't be sped up beyond a certain point, he says, "would give us a lot of information [about pi] that we don't currently have."

Meanwhile, the digit hunt continues. With a one-line change in his program, Bailey says he can easily reach 60 million digits using a little more than twice as much computer time. Furthermore, new software allowing him to use all four of the Cray-2's central processors instead of just one would also help.

The limiting factors really are how much energy a programmer is willing to put into the hunt and how long somebody can get access to a supercomputer, says Borwein. "If it were important enough, without any technological breakthroughs, one could probably do a billion digits," he says. "After that, who knows?"

In Japan, Yasumasa Kanada of the University of Tokyo and his colleagues are planning a new assault that may take them to 33 million digits and then to 100 million. But at Symbolics, Inc., a small company in Palo Alto, Calif., Bill Gosper, who was one of the original computer "hackers" at the Massachusetts Institute of Technology, is following an independent course. Using a mathematical technique involving "continued fraction expansions" and just a Symbolics minicomputer, he has managed to reach 17 million digits.

For the digit hunters, the pursuit continues. □