The Troubled State of Calculus

A push to revitalize college calculus teaching has begun

Calculus: a cavernous lecture hall, 200 or so dozing and fidgeting students, a lecturer talking to a blackboard strewn with Greek symbols, a thick, heavy textbook with answers to even-numbered problems, a seemingly endless chain of formulas, theorems and proofs.

By IVARS PETERSON

More than half a million students take an introductory college calculus course in any given year, and the number is growing. A large proportion have no choice. Calculus is a barrier that must be hurdled on the way to a lucrative professional career in medicine or engineering. Even disciplines like history now sometimes require some college mathematics. But for many people who in the last few years have passed through such a course, the word "calculus" brings back painful memories.

In many universities about half of the students who take introductory calculus fail the course. A surprisingly large number must take the course several times to get through. At the same time, engineering and physical sciences professors complain that even the students who pass don't know very much about calculus and don't know how to use it.

"The teaching of calculus is a national disgrace," says Lynn A. Steen, president of the Mathematical Association of America, based in Washington, D.C., and a professor at St. Olaf College in Northfield, Minn. "Too often calculus is taught by inexperienced instructors to ill-prepared students in an environment with insufficient feedback," he says. "The result is a serious decline in the number of students pursuing advanced mathematics, and a majority of college graduates who have learned to hate mathematics."

Lost is a sense of what calculus means as a remarkable intellectual achievement. Largely hidden is the notion of calculus as one of the foundations of modern science. Obscured is its role as a useful tool for understanding concepts like statistical averages and how quickly things change — in situations ranging from fluid flow to stock market prices.

Now a small group of educators has started a movement to change what is taught in an introductory calculus course, to improve the way it is taught and to bring the teaching of calculus into the computer age. Earlier this year, 25 faculty members, administrators, scientists and others representing diverse interests met at Tulane University in New Orleans to see what could be done.

One big surprise was a general agreement that there is room for change. When participants came to the meeting, says mathematician Peter L. Renz of Bard College in Annandale-on-Hudson, N.Y., although they recognized the problem, "we all believed that there was nothing we could do about calculus." Yet despite this pessimism, many of the participants brought worthwhile suggestions.

"I [originally] had the feeling that engineers were worried but that the mathematicians [who taught the courses] were not," says Mac E. Van Valkenburg, engineering dean at the University of Illinois at Urbana-Champaign. "It was pleasant for me to find that they were not only concerned but also willing to do something about it."

What emerged was a call for "a leaner, livelier, more contemporary course, more sharply focused on calculus's central ideas and on its role as the language of science," says Paul Zorn of St. Olaf College. "Significant change is possible, desirable and necessary."

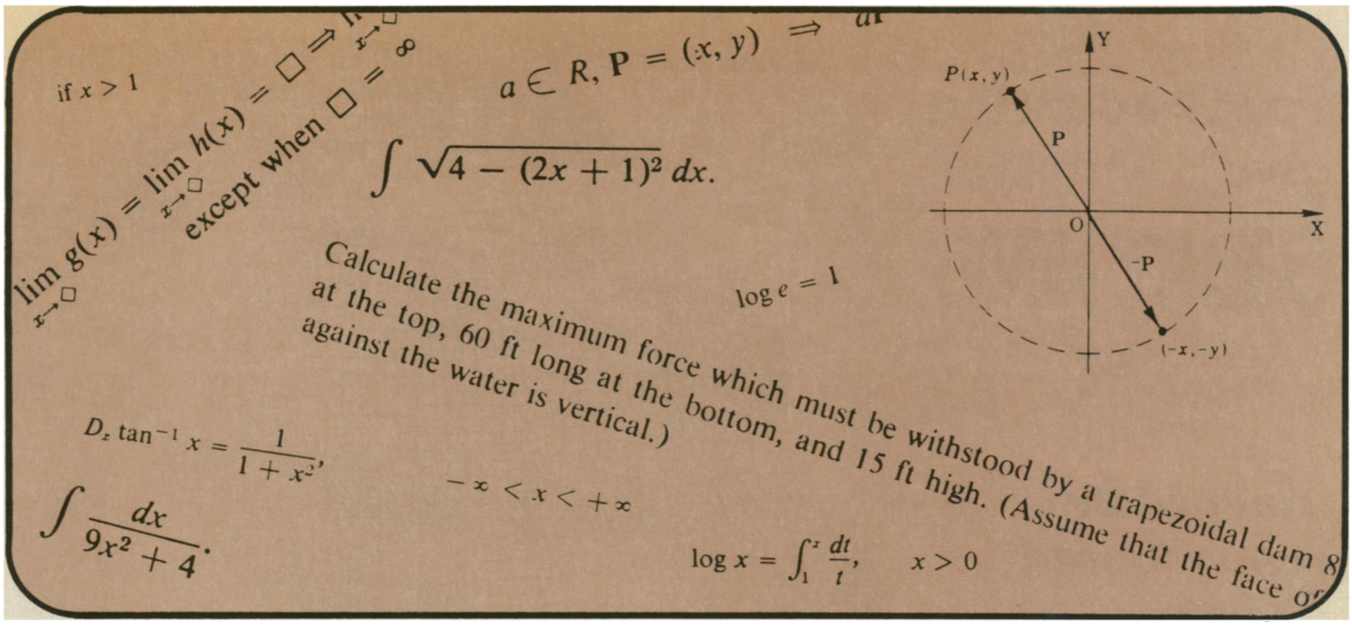
A key question is the role of hand-held calculators and computers. Unlike other disciplines, mathematics courses rarely involve the use of such machines. "It's as if the computer didn't exist," says Van Valkenburg. "It's time to make a change to use the technology that's now available."

For the price of a calculus textbook, any student can buy a scientific calculator that numerically solves equations and evaluates definite integrals — two important operations in calculus. "This means that you can do things that don't have to come out as whole numbers," says Thomas W. Tucker of Colgate University in Hamilton, N.Y. "There's some very nice mathematics involved in things you can't do by hand."

"We won't be teaching how to use a calculator," says mathematician Ronald G. Douglas, dean of the physical sciences school at the State University of New York at Stony Brook. Douglas organized the Tulane meeting and is one of the leaders in promoting calculus reform. Rather, calculators would be used to illustrate calculus itself, he says.

"The first thing that one can do on that basis is to eliminate an awful lot of the routine problems," says Douglas. The ideas are still important, and instructors may need some of these techniques to illustrate what is going on, he says, but drilling students in something that any calculator or computer can now do becomes much less important.

The conference participants agreed that the routine use of calculators and computers would help shift the focus of calculus back to its fundamental ideas and away from students mechanically plugging numbers into formulas to get "nice" answers. Until now, says Douglas, "all we've been teaching people in some



sense has been a kind of pattern recognition."

Over the years, calculus courses have also picked up a lot of baggage, leading to weighty textbooks crammed with material that really isn't essential. "We were putting in so much stuff that people were forgetting the point," says Renz. It isn't difficult to cut much of the excess, specialized material.

Another thrust of the calculus reform effort is toward reinforcing the important role of approximations. Traditionally, introductory calculus courses have dealt mainly with algebraic functions: exact mathematical expressions involving square roots, exponentials and trigonometric and other functions. "You simplify and try to come up with some nice, clean formula for whatever you're doing," says Tucker. "Calculus has a tendency to encourage those sorts of answers."

The truth of the matter is that the functions that one gets in real life are not well-defined algebraic expressions, says Tucker. Instead, they appear as waveforms on an oscilloscope screen, as numbers in a table or as bar charts in a report or a newspaper.

Tucker asks: How do you deal with functions when they come to you pictorially, for example, rather than algebraically? "We're hoping that we can teach students how to treat these kinds of functions," he says. In this case, calculator computations would be very helpful.

Adds Douglas, "Applications would be woven into the very fabric of the course."

Overall, conference participants suggested enough syllabus changes to require a new generation of textbooks. "My reaction is that the [new] syllabus would be very difficult to teach from a present textbook," says Tucker.

However, curriculum changes alone won't solve the problem. The teaching environment must also be altered. "One of the most difficult things about calculus – teaching it or learning it – is that calculus builds all the way through," says Douglas. In order to understand the second week, you have to understand the first week, and so on. That means checking that students do their homework week by week. Classes should be small enough to allow this, says Douglas. "Too many financially squeezed universities have tried to do calculus on the cheap."

At the same time, "calculus is the course mathematicians love to hate," as one conference participant put it. Although just about every mathematician at one time or another has taught first-year calculus, at many colleges and universities this task generally falls to inexperienced lecturers and graduate students, some of whom are simultaneously learning to speak English.

"The conference demonstrated that we could obtain agreement on how calculus should be changed and how it should be taught," says Douglas. "But it's only the first step. The next step, if one is going to try to translate this agreement into actual calculus courses, is going to be even more formidable." That, he says, would involve starting a textbook-writing project and setting up pilot programs at about a dozen universities to test the ideas.

With so many vested interests, adds Douglas, "the idea of proposing anything radical or even much different from what exists now is extremely difficult." But Douglas and his group are pushing ahead, hoping to have a new calculus program in at least a few universities by 1990.

Says Van Valkenburg, "We wish it could be sooner."

"There's enormous inertia," says Tucker. He notes that there is no guiding committee or group that oversees college mathematics curricula. In the United States, "nobody dictates a college calculus syllabus for the whole nation," he says.

At some universities, special calculus committees guard against and quash any radical innovations that individual faculty members may want to introduce. The business of these bureaucratic committees is to maintain a comfortable status quo. The economics of textbook publishing, which leads to encyclopedic volumes that try to satisfy everyone, itself encourages a do-nothing attitude.

One place where a change may come sooner is in high school calculus. Tucker, who is on the committee setting the syllabus for the advanced placement program in calculus, plans to take many of the conference recommendations back to the committee. "Because there's a uniform, national final exam every year, the syllabus is tightly controlled," says Tucker. "If those recommendations are incorporated in the advanced placement syllabus in this country, then all the courses will have to change. You'd see a dramatic change almost immediately." However, he notes, many high schools may be reluctant to take the lead and would rather wait until something happens at the college level.

Meanwhile, "it doesn't take much imagination to realize what [current calculus teaching] is doing to the engineering and scientific base in this country," says Steen. "All students who are planning careers in science and engineering have to go through calculus, and if they get shut out of it either by bad teaching or by out-of-date curricula, it's not doing them or this nation any good." □