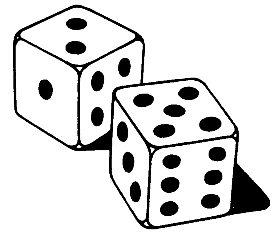
ROLLING ROCKS AND TUMBLING DICE



Studies of tumbling dice may suggest new ways of looking at shifting sand dunes and avalanches

By IVARS PETERSON

A quick flick of the wrist sends a pair of dice bouncing, rolling and skidding down a green felt surface. Another toss sends a second set tumbling across the table. But in subtle ways, the latter set — carefully weighted so that certain faces are more likely to land upward — seems to move a little more erratically than the first pair.

Illustrations: Werner/Caltech

Studying the movement of individual sand grains may lead to a better understanding of how sand dunes form and shift.

A loaded die tends to bounce around more than a "perfect" die, says graduate student Bradley T. Werner of Caltech in Pasadena. "It appears to move less smoothly," he says. For the last year or so, Werner has been using computer simulations to study the way loaded and unloaded dice roll.

Werner's research is part of a larger effort to see if the general behavior of granular systems such as rock slides, sand dunes, planetary rings and snow avalanches can be derived from the motions and interactions of the individual particles that make up a given system.

The problem seems immense. A sand dune, for instance, contains trillions of grains. Furthermore, these windblown particles are not randomly placed but highly organized. If it were possible to slice open a sand dune, one would see very fine laminations that consist of alternating layers of larger and smaller grains. Even more curious is the "yell" that certain sand dunes give out when they are kicked.

"There's a whole collection of peculiar things going on," says physicist Peter Haff, who supervises Werner's work. "You really wonder . . . how the grains are actually rubbing and bouncing off one another."

One way to study such a complex system theoretically is to begin with a simpler case. But even the motion of a single particle pulled by gravity down a slope is far from trivial to analyze, especially when the particle is allowed to have an arbitrary shape. Friction and energy losses further complicate the picture.

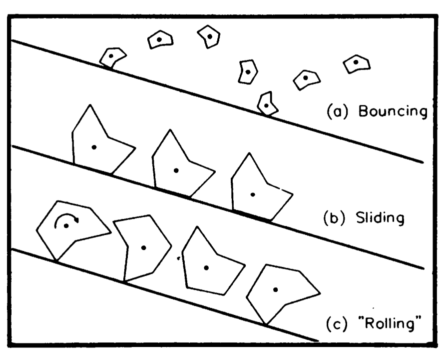
Werner began his study by looking at variously shaped, two-dimensional particles moving down a smooth slope. Initially, he was interested in seeing if sand grains of different shapes could be separated by letting the sand flow down a slope.

Werner's computer simulations showed that these particles, at various times, apparently exhibit three types of motion: sliding, bouncing and rolling. With enough friction, most bouncing particles eventually end up rolling. Sliding particles skid to a stop. Shape doesn't seem to have a great effect.

A field test in a lonely Mojave Desert canyon confirmed that "real" rocks behave similarly. Massive boulders tumbling down a mountainside may bounce at first, but this motion quickly decays into a roll. "We didn't take any measurements," says Werner, "but we didn't have to roll too many rocks down the slope to see what was happening."

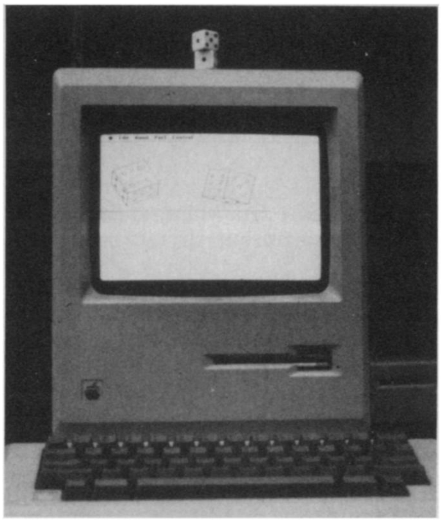
These results were good enough to encourage Werner to go one step farther with his computer simulations. This time, he chose a cube as his sample particle. A fellow graduate student, who happened to be interested in gambling, suggested that he also look into weighted cubes. This led Werner into a study of loaded dice in action.

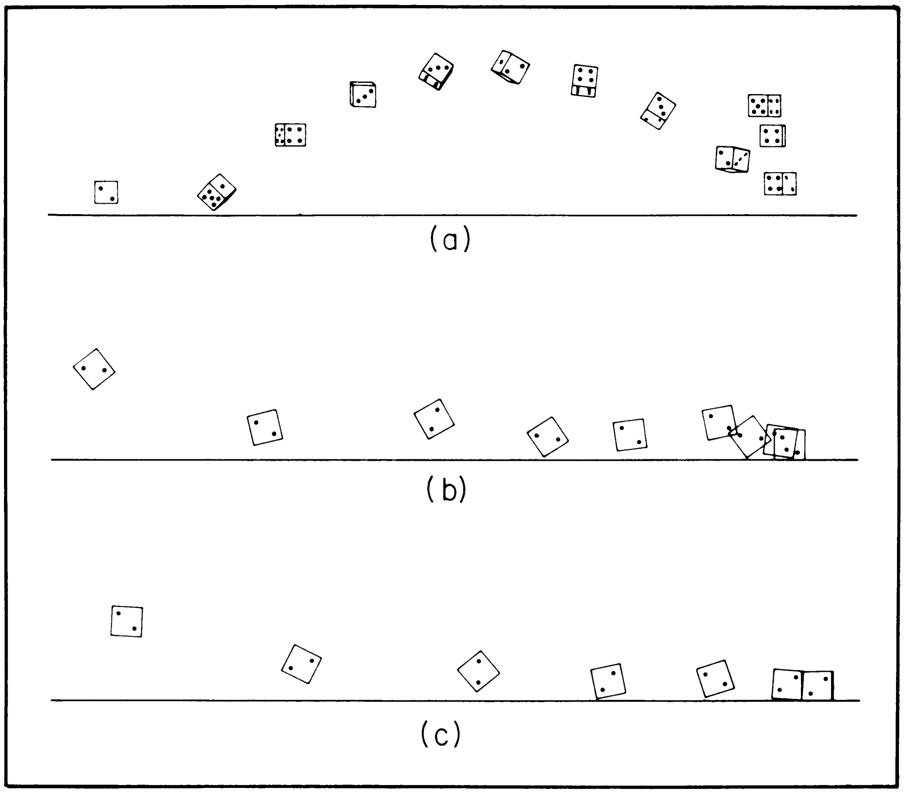
His observations, however, took place not at Las Vegas casinos but on the screen of his Macintosh personal computer. Werner wrote a computer program to simulate a cube's motion, carefully applying the appropriate physical laws of motion. His simulations embodied all of the important characteristics of a real, moving die: a nearly rigid particle that loses some energy on each bounce and is slowed by friction.



A particle moving down a slope can show three different types of motion.

Werner's aim was to see if he could predict the probability of a particular side landing face up, depending on how the die was weighted. In his initial simulations, he followed the behavior of a





Simulations run on a personal computer (above) allowed a Caltech researcher to study how loaded and unloaded dice tumble (right). Row (a) shows a fair die tumbling on all six sides; row (b), a loaded die constrained to tumble on just four sides; row (c), a fair die constrained to tumble on four sides.

square rather than the entire cube. The die rotated in such a way that only four of the faces could turn up.

"I picture the tumbling die as progressing from face to face and continually losing energy until it is finally captured, with one of the faces resting on the plane," says Werner.

For a loaded die, Werner defined a center of mass along a diagonal, halfway between the middle of an edge and the cube's center. This created two "low" faces close to the center of mass and two "high" faces farther away. His model ignored the remaining two faces.

"The computer simulations were very important for getting some ideas about how to go about calculating the interactions between a moving particle and a plane," says Werner.

In his simulations, Werner discovered that a loaded die tends to move a little more erratically than an unloaded die. Whether the differences can be seen in real dice isn't clear. "When I look at these [computer simulation] pictures, I'm looking at snapshots," says Werner. "I can sit there and stare at them for a good deal of time. But if you roll a die, it takes place so fast that to simply spot it with your eye is unlikely unless the die is heavily loaded."

Werner also noticed that the frequency with which the "low" faces of a loaded die ended up downward – a desirable outcome from the viewpoint of an unscrupulous gambler – depended on how much energy the die lost on each bounce. A soft, puttylike die loses energy and settles into position much more quickly than a hard, bouncy one. Thus, a soft, "inelastic" die is much less likely to come up favorably.

"A totally inelastic die would simply land on whatever face it first hits," says Werner. "If it's more elastic – loses less energy on each bounce – then it has more time to search out and find the preferred states."

Werner's studies aren't completed yet. "I expect the Macintosh to be chugging away at odd hours in the coming months," he says. One improvement would be to extend his simulation to the entire cube in three dimensions rather than just a weighted square.

"In addition," Werner writes in the March issue of Caltech's ENGINEERING & SCIENCE, "it would be of interest to investigate other methods of altering dice, including shortening some of the faces and altering the collision characteristics of one or more of the faces."

To check his simulations, Werner also plans some experiments with loaded dice. "Perhaps a few trips to Las Vegas will be necessary to investigate fully all aspects of this complex problem," he says. However, he says he has no plans to use his new knowledge to enhance his "meager earnings as a graduate student."

The dice study was a lot of fun, says Haff. "When [a student] comes up with an idea like that, I think it's really important to let him do it," he says. "You never can tell where anything's going to lead."

"I've learned a lot about how three-dimensional particles behave," says Werner. His work may be an early step toward a better understanding of granular flow, whether in a massive avalanche or in an emptying bin of corn flakes. "If you can't understand one particle," says Werner, "then you haven't a chance of understanding all those particles in a sand dune."

"In the last four or five years," says Haff, "there's been quite a burst of interest in the whole problem." Haff and his group are particularly interested in geological phenomena. There are a number of examples that are not understood at all, he says.

A mass of rocks in a landslide, for instance, after roaring down a mountainside, can sometimes "flow" along level ground for long distances. This system seems to behave more like water than separate rocks.

"The dry material has somehow fluidized itself," says Haff, "and behaved like a fluid of fairly low viscosity long after you would think the thing would have come to rest."

Computer simulations are playing an increasingly important role in these studies. The researchers begin with a simple analytical model – a set of equations that tries to reproduce the essential elements of the system's behavior. This gives the researchers a general impression of what's going on. Then computer models are used to add complications, such as the actual shape of particles.

"You add those things one at a time to see what difference they make," says Haff, "and to see what kind of role they might play. It's a good way to get a handle on things that are difficult for us to imagine."

In the case of dice, it also provides some insight into the ancient art of cheating. □