Mathematics

Closing in on Fermat's last theorem

One of the most famous unsolved problems in mathematics started out as a note scrawled in the margin of a book. Now known as Fermat's last theorem, the problem was first proposed by 17th-century French mathematician Pierre de Fermat. Although he had a wonderful proof for the theorem, Fermat wrote in the book's margin, he didn't have enough room to write it out. After Fermat's death, scholars could find no trace of the proof in any of his papers, and ever since, mathematicians have struggled in vain to solve the problem. Some recent mathematical discoveries, however, have tied Fermat's theorem more closely to modern mathematics, suggesting a possible avenue to a proof.

Fermat's conjecture (as it should properly be called until a proof is found) is related to a statement by the Greek mathematician Diophantus, who observed that there are whole numbers, x, y and z, that satisfy the equation $x^2 + y^2 = z^2$. For example, $3^2 + 4^2 = 5^2$. In fact, this equation has an infinite number of whole-number solutions. Fermat proposed that there are no solutions to the equation $x^n + y^n = z^n$, when n is greater than 2. In other words, when $n = 3$, no set of whole numbers satisfies the equation $x^3 + y^3 = z^3$, and so on.

A century after Fermat's death, Leonhard Euler proved that Fermat's conjecture is true for $n = 3$. Later mathematicians found proofs for other special cases, and a computer search performed a decade ago showed that Fermat's last theorem was true for all exponents less than 125,000. In 1983, Gerd Faltings, now at Princeton (N.J.) University, showed — as one consequence of his proof of the Mordell conjecture (SN: 7/23/83, p.58) — that if there are any solutions to Fermat's equations, then there are only a finite number of solutions for each value of n. But until recently, a proof for the general case has remained elusive.

The current effort to tame Fermat's conjecture started with the work of Gerhard Frey of the University of the Saarlands in Saarbrucken, West Germany. Frey, who happened to be looking at equations for elliptic curves, written generally in the form $y^2 = x^3 + ax^2 + bx + c$, where a, b and c are constants, found a way to express Fermat's last theorem as a conjecture about elliptic curves. That put Fermat's problem into an area of mathematics where mathematicians have developed a wide range of tools and techniques for solving problems.

Frey wrote down the elliptic curve that would result if Fermat's conjecture were to be false. That elliptic curve turns out to have peculiar properties, and mathematicians examining the curve's equation had the feeling that the curve could not exist. If the curve's impossibility could be proven, then Fermat's conjecture would have to be true. Frey started on this task, but he failed to fill all the gaps in his attempted proof that the curve couldn't exist.

Then Jean-Pierre Serre of the College of France in Paris suggested that one of his own conjectures, if proven, would help patch up Frey's effort. Earlier this year, Kenneth A. Ribet of the University of California at Berkeley worked out the necessary proof of Serre's conjecture for a large class of situations. Ribet ended up showing that Fermat's theorem is true if certain elliptic curves arise from "cusp" forms.

Now Fermat's last theorem is tied to a central question in number theory, known as the structural conjecture. That conjecture states that all elliptic curves arise from cusp forms. Most number theorists, for a variety of convincing reasons, believe the structural conjecture to be true, although it has not been proven. Even without a proof, this conjecture plays a major role in number theory. In fact, a proof that the conjecture is false would come as a shock to the mathematics community. Fermat's last theorem finally seems to rest on reasonably firm ground.

Physics

Perseus flasher: Satellite glints

About a year ago, a group of Canadian astronomers reported strange flashes of light coming from the direction of the constellation Perseus (SN: 8/23/86, p.117). In a period of about two years approximately three dozen such bursts of light had been recorded. There are objects out somewhere in the cosmos that flash from time to time in other kinds of radiation, X-rays and gamma rays particularly. If any of the light flashes should happen to come from the same sources as the gammas and X-rays, the light flashes might contribute a good deal to understanding the astrophysics of the apparently extremely energetic processes that produce those bursts.

However, one possible source that the observers could not rule out was satellite glints, momentary reflections of sunlight by rotating artificial satellites. Two reports now attribute the observed Perseus flashes to such glints.

In a paper in the June 1 ASTROPHYSICAL JOURNAL LETTERS Paul D. Maley of the Rockwell Shuttle Operations Co. in Houston concentrates on what he calls "the main physical evidence," a flash that occurred March 19, 1985, and was photographed from Schomberg, Ontario. According to Maley, the trajectory of the Soviet satellite Cosmos 1400 put it in the place where that flash was seen at the right time. Furthermore, a study of the optical properties of that satellite indicates that it is capable of making such glints. Maley suggests that "Earth satellites are a likely source of many isolated nonmeteoric flashes seen by ground-based observers."

A group of 11 observers, including Maley and Bradley E. Schaefer of the NASA Goddard Space Flight Center in Greenbelt, Md., with members of the Santa Barbara (Calif.) Astronomy Group, the American Meteor Society and the Amateur Telescope Makers of Boston, comes to the same conclusion in a paper to appear in the Sept. 1 ASTROPHYSICAL JOURNAL. Similar flashes occur all over in the sky, and observers have attributed them to satellites, aircraft and even high-flying fireflies.

It has been suggested that the Perseus flasher is an astrophysical object that flashes about once every 12 hours. In this latest work, however, there were no flashes observed in 3,287.9 hours of observation of the spot in Perseus. Furthermore, a statistical analysis of the orbits and optical properties of a class of Soviet satellites called Molniyas indicates that they can account for the Perseus flashes.

However, not all light flashes from the sky have a near-earth origin. In his paper Maley mentions three that seem to have come from a gamma-ray burst source in the supernova remnant N49. Analysis shows that these do not coincide with the known satellite population.

Galactic center affects solar system

Since ancient times, astronomers have wondered why the planets of the solar system are located where they are. One of the famous empirical facts about this configuration is the Titius-Bode law, observed by Johann Titius in 1766 and published by Johann Elert Bode in 1772. It says that the radii of the orbits of the planets from the asteroid belt out (except Neptune) form a geometric progression, each radius being just about twice the radius of the orbit just inside it.

Jaume Llibre and Conchita Piñol of the University of Barcelona in Spain say they can explain the Titius-Bode law by gravity if they take into account the sun's motion around the center of the Milky Way galaxy. This sets up a four-body problem involving the gravitational interaction among the galactic center, the sun, a given planet and the planet just inward of the given planet. Presenting their calculations in the May ASTRONOMICAL JOURNAL, the researchers say that no previous attempt to explain the law has taken into account the center of the galaxy.