Forest Fires, Barnacles and Trickling Oil

Chance plays a key role in some simple mathematical models that suggest natural processes

By IVARS PETERSON

The spread of a raging forest fire is an awesome sight. Flames leap from tree to tree. The fire fans out in a blazing ring, consuming fresh timber at its outskirts and leaving a burnt residue within.

Mathematicians have their own, tamer version of this fiery ring. It's a simple mathematical procedure that on a computer screen seems to mimic the step-by-step growth of a forest fire. This particular procedure is one example of an "interacting particle system."

The study of interacting particle systems is a relatively new activity for mathematicians. It began in the late 1960s as a branch of probability theory. During the two decades since then, says Thomas M. Liggett of the University of California at Los Angeles, "this area has grown and developed rapidly, establishing unexpected connections with a number of other fields."

The first examples of interacting particle systems were suggested by research in statistical mechanics. Physicists wanted to understand how a collection of wandering or randomly scattered particles can suddenly organize itself, as happens when a liquid solidifies or a material is magnetized.

The mathematical models developed to simulate such phase transitions proved to be a rich source of inspiration. The idea was to look at what happens to particles scattered across a grid when each particle is allowed to interact with its neighbors according to certain rules. It became clear, says Liggett, that models with a very similar mathematical structure could also be useful for studying neural networks (SN: 8/1/87, p.76), tumor growth, ecological change and the spread of infections.

Moreover, the dynamic behavior of such models suggested intriguing mathematical questions. Mathematicians became interested in how certain models evolve over time and began searching for unusual types of behavior.

"Mostly, it's a mathematician's game of seeing what happens," says Richard T. Durrett of Cornell University in Ithaca, N.Y. The mathematician defines the rules, sets up the game board and lets the game play itself out.

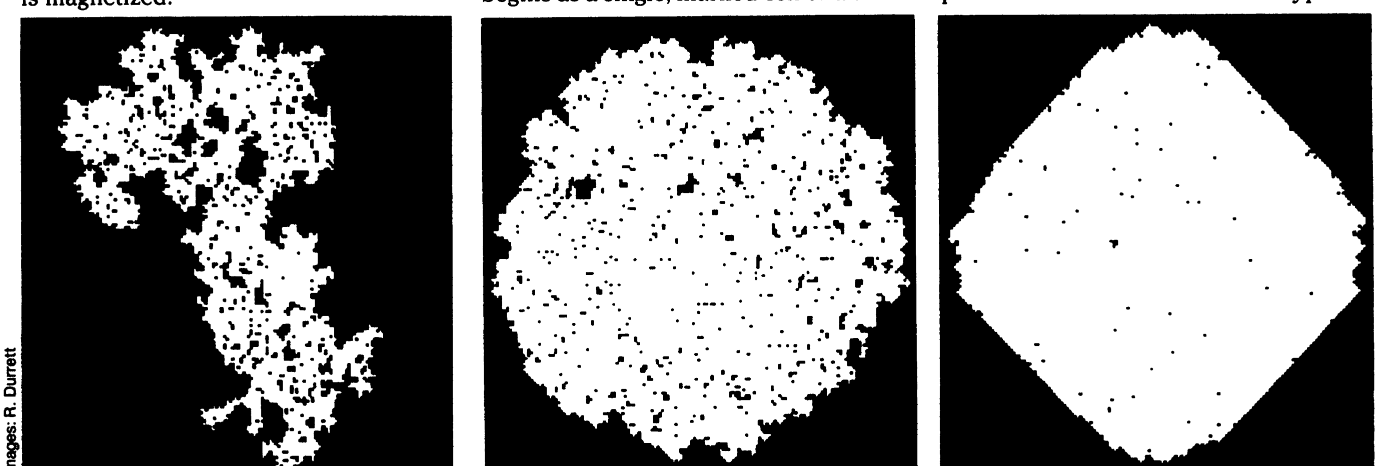
The rules for a mathematician's forest fire are simple. The playing field is a checkerboard grid, on which each cell represents a tree. The fire begins as a single, marked cell or a small cluster of cells at the grid's center. At each time step, a burning cell has a certain probability of spreading the fire to its four nearest neighbors, unless those neighbors have already been burnt.

The same rules also lead to a rough model for the spread of an infectious disease such as measles. In this case, each cell represents an individual who is healthy, ill or immune.

Of course, such a model isn't complicated enough to simulate a real measles epidemic, just as the forest-fire version doesn't take into account all the factors that influence a real forest fire. Nevertheless, the behavior of the model suggests some important features that these natural processes may possess. Eventually, researchers hope to develop more sophisticated variants that come closer to simulating the real thing.

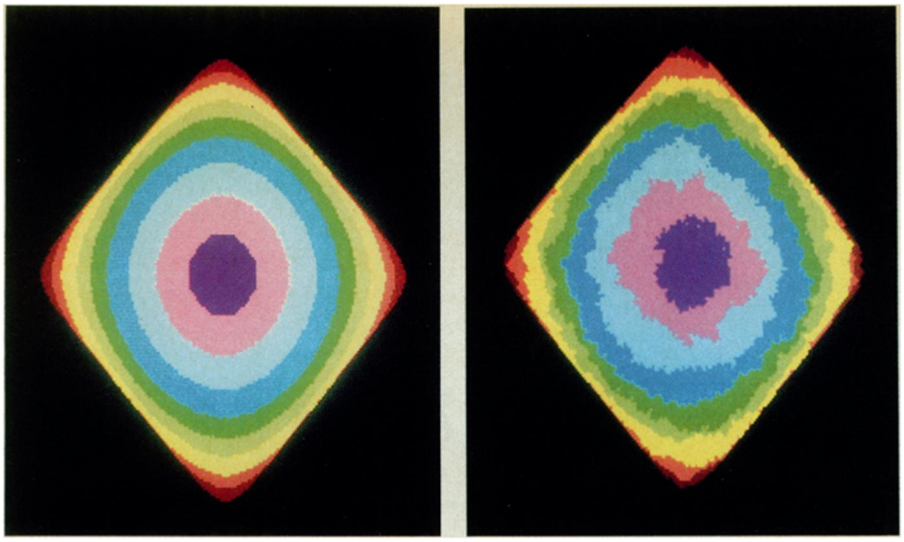
The simplest set of rules mathematically worth investigating makes the unrealistic assumption that the fire (or infection) in a given cell lasts for only one unit of time. At each step, the toss of a coin or a similar randomizing procedure decides whether a certain burning cell spreads its flames to each of its neighbors. Hence, at any given time, a computer screen would show three types of

Images: R. Durrett



p = 0.51 p = 0.60 p = 0.75

In these computer-generated illustrations, areas of unburnt forest are shown in black, and areas where trees are on fire or have already been burnt appear white. As the probability of transmission of the fire from one cell to a neighboring cell increases, the fire's boundary changes from a fractal-like pattern (left) to a rough circle (middle) to a square (right).



Richardson's model mimics the spread of a plant species or the propagation of immobile animals such as barnacles. Each cell has a certain probability of spreading into unoccupied neighboring cells. Because there is no provision for death, the pattern grows to cover the plane. This pair of illustrations superimposes the patterns generated by 10 different colonization probabilities, from p = 0.1 (center pattern) to p = 0.9 (outer pattern). The image on the right shows the results of one run for each of the probabilities, while the image on the left is an average of 10 runs.

cells: those that are burning, those already burnt up and those that have so far escaped unscathed. The burning cells usually form an irregular, broken ring that gradually expands as time goes on.

Mathematicians, particularly probabalists, are interested in how the process depends on the probability of transmission from one cell to another. One way to study this dependence is to assign a certain transmission probability to each affected cell and then see what the model does.

In general, the researchers find that when a cell has only four chances in 10 (a probability of 0.4) of passing on the fire to a given neighbor, then the fire eventually dies out. If it has six chances in 10 (a probability of 0.6), then the fire continues to spread. The critical probability that establishes the dividing line between these two types of behavior turns out to be 0.5. Other models often have different critical probabilities.

The shape of the ring also seems to depend on the transmission probability. At a probability of 0.5, the ring's outer edge looks like a fractal, having an incredibly convoluted boundary that when magnified, instead of looking smoother, appears to be equally complicated on every scale (SN: 5/2/87, p.283). Theorists have no idea why a fractal should appear at this critical value. By the time the probability reaches 0.6, the ring is smoothed out into a circle. At even higher probabilities, the ring takes on the shape of a square.

Another example of an interacting particle system, oriented percolation, resembles the downward trickle of a surface pool of oil through an underlying bed of sand or clay. Fingers of oil penetrate this layer if the ground is porous enough. The trick is to find the critical probability at which enough air spaces are present so that an open pathway exists. Because no one knows precisely what this critical probability is, theorists must resort to computer simulations to get a feel for what the process looks like.

The oriented percolation model can be pictured as a gigantic diamond-shaped grid of connected pipes. Each section of pipe in the grid has a valve, which may be either open or closed. At the top of the network is a reservoir of fluid. If all valves are open, fluid will flow down through the network of pipes. If all valves are closed, no flow occurs.

What if some valves are open and others closed? Clearly, no fluid will flow until enough valves are open. By studying the results when the probability of a particular valve being open lies somewhere between 1 (all valves open) and 0 (all valves closed), mathematicians can determine the critical probability at which flow is first established.

Computer experiments show that when the open-valve probability is 0.55, the process dies out. This means that, on the average, when 55 out of 100 valves are open, fluid gets into the pipes but doesn't get very far before all paths for fluid flow are blocked. As the probability of a valve being open increases, the fluid penetrates deeper. Percolation — long-distance flow — is established when the probability of an open valve gets close to 0.645. However, a mathematical proof that this number must be the critical value has been elusive. The best that anyone has done is to prove that the lower limit on the critical probability is 0.6298 and the upper limit is 0.84.

A similar, two-dimensional model can be set up for the spread of a plant species or the propagation of a population of immobile animals such as barnacles or mussels. This particular model is named for British mathematician Daniel Richardson of the Polytechnic of the South Bank in London, who first suggested the model in a 1973 paper.

Partly because there is no provision for death, the model's pattern always grows to cover the entire plane, leaving only a few holes near its fringes while growth occurs. Theorists are interested in the times at which a growing shape exceeds a certain boundary, and how this time depends on the probability of an occupied cell sending an offspring (or root) to an adjacent, unoccupied space.

These examples illustrate just a few of the infinite number of possible models that could be investigated. Even in something as simple as the forest fire model, the number of possibilities is striking. The rules could be changed to involve a different number of neighbors. The burning stage could last for more than one unit of time. The grid itself could be hexagonal or triangular instead of square. The possibilities seem limited only by the researcher's imagination.

How do mathematicians decide which models are worth studying? Some are suggested by studies in other fields, such as physics and biology. Others show some type of novel behavior.

"It's definitely true that there is an insane number of models," says David Griffeath of the University of Wisconsin at Madison. "It's just that the overwhelming majority of them do absolutely nothing that's the least bit interesting. There's a very small set of them that end up doing something special, and those are the ones you try to find."

"What a mathematician looks at," says Liggett, "is the extent to which this new model is likely to show a different kind of behavior, which seems inherently interesting and which is not the same as some other model that has been studied, and the extent to which a new technique is required."

And there's another barrier. Many candidate models are too complicated to be analyzed mathematically. "One tries various rules," says Durrett, "until one finds a system that one can analyze." Adds Griffeath, "So few of these systems are amenable to rigorous analysis that when you find one that is, you milk it for all it's worth."

The emphasis on diversity and the difficulty mathematicians have in rigorously proving general statements about these models lead to what seems a curiously fragmented type of mathematics. "The unity of the subject does not come from general theorems," says Liggett. "It comes from the same techniques being used to analyze a variety of models. It really is a very complex business, and it's not in the nature of things to be able to get a simple statement that describes all models."

One topic that has produced some interesting results is the behavior of various interacting particle systems in different dimensions. Such models need not be restricted to planar cells or even to three-dimensional blocks. Although difficult to visualize, these models can be studied in any dimension.

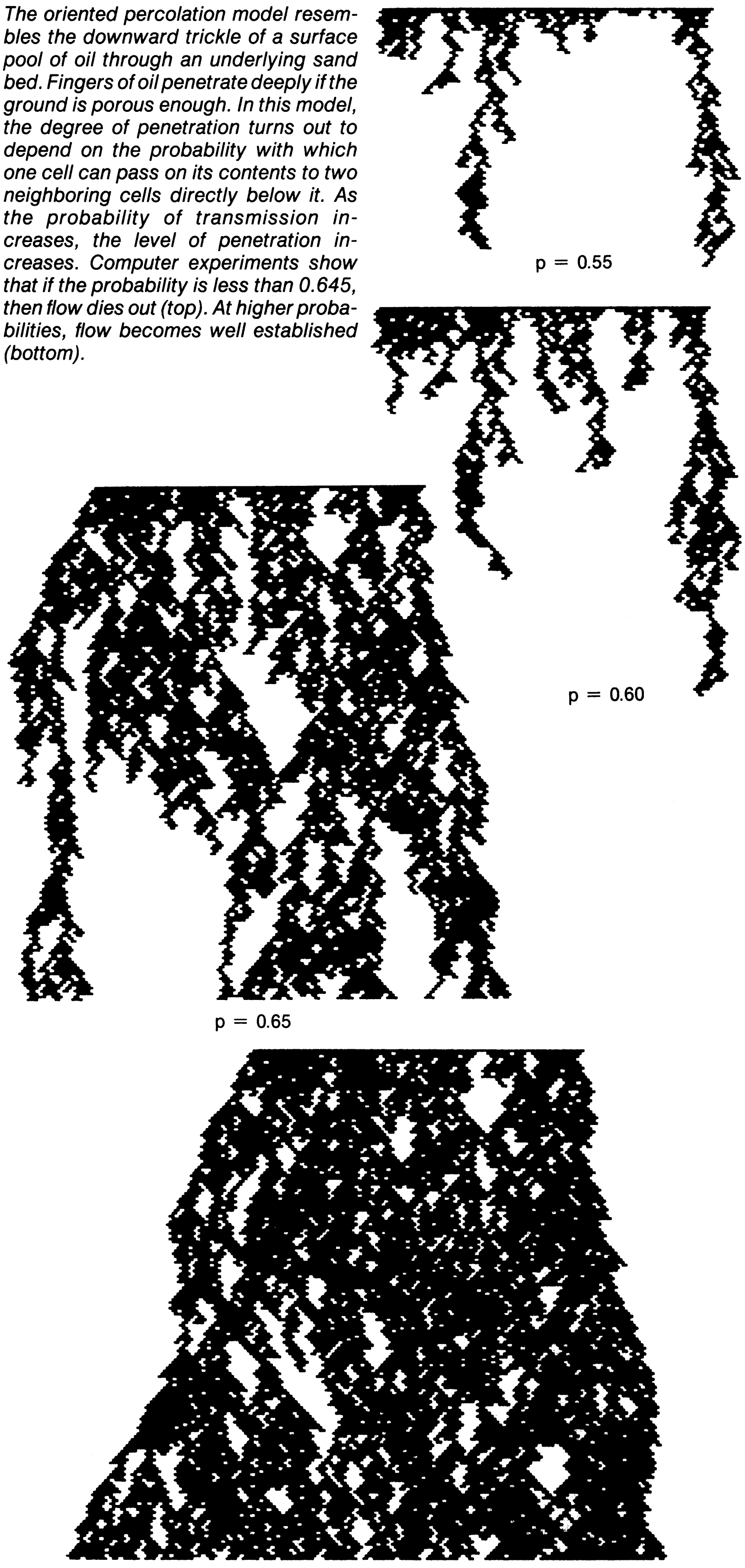
"All dimensions have been studied a lot," says Liggett. "One of the interesting things that has come out is that one does observe different behavior at different dimensions. The critical dimension, which means that at lower dimensions one thing happens and for bigger dimensions something else happens, is different for different models."

In fact, the behavior of most models seems to get simpler when the dimension gets larger. Because higher dimensions have so much room, the interactions between particles get weaker. Particles tend to be much farther apart. Frequently, higher-dimensional versions of various processes behave almost as if there were no interactions. They begin to look a lot like processes traditionally studied in probability theory — those, such as random walks, made up of independent steps.

Mathematicians working on interacting particle systems are divided in their opinions on the utility of computer simulations. "My feeling is that [computer simulation] isn't that useful," says Liggett. "It's fun. You produce pretty pictures. It certainly demonstrates the concepts of theorems to people who aren't mathematically that sophisticated. But in terms of actually pointing toward mathematical results, there are a few examples, but I don't think it's a major factor."

Others disagree. "I think it's undeniable that learning to use the power of computer simulation is something that will be more and more important in the future," says Griffeath. "For the first time, we're getting to the point where the technology is powerful enough that these simulations can give accurate answers to complex problems."

With the help of computer simulations, Griffeath is studying the clustering behavior of certain systems that seem to mimic the patterns seen in oscillating chemical reactions (SN: 11/22/86, p.329). He's examining systems that start off with a fine-grained structure and, because of a



The oriented percolation model resembles the downward trickle of a surface pool of oil through an underlying sand bed. Fingers of oil penetrate deeply if the ground is porous enough. In this model, the degree of penetration turns out to depend on the probability with which one cell can pass on its contents to two neighboring cells directly below it. As the probability of transmission increases, the level of penetration increases. Computer experiments show that if the probability is less than 0.645, then flow dies out (top). At higher probabilities, flow becomes well established (bottom).

capacity for self-organization built into their rules, develop into a pattern of bigger and bigger globs.

Durrett is interested in formulating techniques for obtaining useful quantitative information about a model's critical values and survival properties. He hopes to achieve this by using a judicious blend of rigorous proof and computer simulation. "When that's done," he says, "you'll find that these models will have a wider application." So far, most models are not complicated enough to be taken seriously as stand-ins for natural processes.

One of Durrett's current projects concerns the effect of using a mixture of rules on a single grid. In other words, the rule that applies for one cell may not be the same as the rule that applies for a neighbor. In ecological terms, that situation could correspond to an environment in which, say, desert and forest patches occur side by side.

"Many of these problems can be explained in a few minutes to a person with no prior experience in the area," says Durrett. "That's what I find is one of the charms of the area. It's not something that you have to spend three years in graduate school to appreciate. And it's easy to play around with these things on a computer." □

Books

Books is an editorial service for readers' information. To order any book listed or any U.S. book in print please remit retail price, plus $1.00 handling charge for each book, to **Science News Books,** 1719 N Street, NW, Washington, DC 20036. All books sent postpaid. Domestic orders only.

Ada: A Life and a Legacy — Dorothy Stein. In this biography of Augusta Ada Byron, Countess of Lovelace, Stein strips away the myths surrounding Lady Lovelace's reputation as the inventor of the science of computer programming and reveals a life more fascinating than previous accounts have indicated. Lady Lovelace's life is told, often in her own words from letters, memoirs and little-known documents, to disclose a turbulent and complex 19th-century woman. Originally published in hardback in 1985. MIT Pr, 1987, 321 p., illus., paper, $9.95.

Einstein's Legacy: The Unity of Space and Time — Julian Schwinger. A Nobelist recounts Einstein's quest for the reconciliation of Newton's theories of space and time and Maxwell's theories of electricity, magnetism and light, which led to the unification of matter and energy and of space and time in Einstein's special and general theories of relativity. The scientific developments are blended here with accounts of the human side of the story. Describes past experimental tests of Einstein's theories as well as current space-age experiments intended to further confirm or perhaps challenge Einstein's theories. Sci Am Bks(W H Freeman), 1987, 250 p., color/b&w illus., $32.95.

A Field Guide to Hawks of North America — William S. Clark. Field identification of hawks and other diurnal raptors, including eagles, falcons and kites, is difficult, according to the author, partly because they are wary and difficult to approach and partly because they exhibit a variety of plumage and alter their shape with different flight modes. This well-illustrated guide presents the latest in field marks and behavior characteristics by which the 33 native and six accidental North American hawks may be more easily identified. HM, Peterson Field Guide Series, 1987, 198 p., color/b&w illus. by Brian K. Wheeler, paper, $13.95.

A Field Guide to Mushrooms of North America — Kent H. McKnight and Vera B. McKnight. This beautifully illustrated guide includes most of the common edible and poisonous species found in North America, along with many others that are of interest. Focuses primarily on identifying characteristics that can be observed with the specimen in hand rather than under a microscope. HM, Peterson Field Guide Series, 1987, 429 p., color/b&w illus., paper, $13.95.

In Search of the Double Helix: Quantum Physics and Life — John Gribbin. A readable account of evolution as it unfolds from Darwin to the modeling of DNA as a double helix. Describes how physics and chemistry have contributed to the emergence of molecular biology. Goes on to bring the story up to the present as scientists continue to search for the origins of life. Originally published in hardback by McGraw-Hill in 1985. Bantam, 1987, 369 p., illus., paper, $8.95.

The Media Lab: Inventing the Future at MIT — Stewart Brand. The Media Laboratory at MIT, a unique interdisciplinary center, is based on its director's conviction that the whole gamut of communications media — television, books, telephones, recordings, newspapers, magazines and film — is being transformed by computers. The lab is concerned with electronic communication technologies and, according to the preface, how humans connect, how they are connecting faster and wider with new technology and how they might connect better. Much of the book is a tour of some of the lab's research, and the rest is concerned with "the media lab of the world," discussing how the new communications technologies will affect the world. Viking Pr, 1987, 285 p., color/b&w illus., $20.

Prehistoric Britain — Timothy Darvill. Archaeological evidence is used here to examine the development of human societies in Britain from earliest times to the Roman conquest. This well-illustrated book emphasizes six themes: subsistence, technology, ritual, trade, society and population. Yale U Pr, 1987, 223 p., illus., $25.

The Psychology of Happiness — Michael Argyle. Relationships, work and leisure, the author found, are major sources of human happiness. Here, he presents research findings in this field and discusses these sources, along with the effects of wealth, social class, sex, age and health on happiness. Methods for enhancing happiness are described. Methuen Inc, 1987, 256 p., charts & graphs, $47.50, paper, $14.95.