Shareware, Mathematics Style

A unique, supercomputer-centered project brings together a group of leading mathematicians and computer scientists to explore exotic geometries

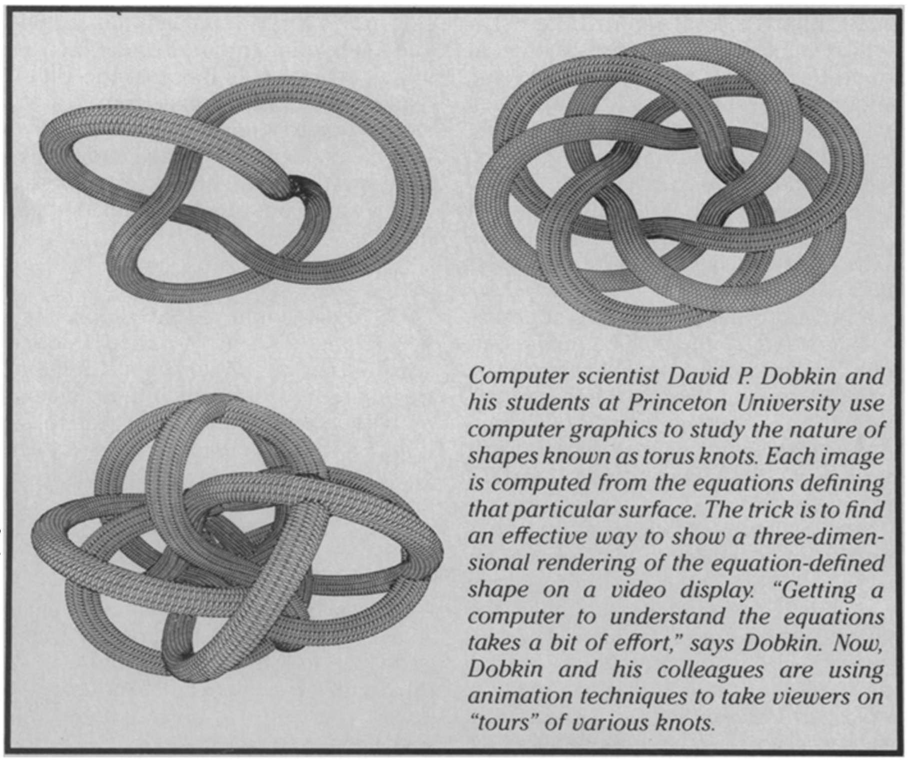
By IVARS PETERSON

The problem started out as a rough sketch hastily scribbled on a napkin during a lunchtime meeting several years ago. It concerned translating a particular set of equations into pictures. It was also the beginning of a fruitful collaboration at Princeton (N.J.) University between computer scientist David P. Dobkin, who was especially interested in computer graphics, and mathematician William P. Thurston, who was fascinated by the twists and turns of three-dimensional surfaces.

The napkin equations turned out to represent objects known as torus knots (see illustrations). "To go from the napkin to the pictures was much more than a day's work," says Dobkin. "I had to learn a whole lot about graphics, in addition to learning a whole lot about topology and mathematics."

Now the collaboration has been greatly expanded. Dobkin and Thurston are among 13 members of the recently established "Geometry Supercomputer Project." For the first time, an international group of prominent mathematicians and computer scientists, connected by a high-speed telecommunications network, will be able to work together on the same supercomputer to focus on solving some of the most challenging problems in geometry. Unlike other supercomputer projects, which are aimed at specific scientific or engineering applications, the geometry project represents a venture into pure, or basic, mathematics.

The use of computers in mathematics research is still relatively new. Individuals or small groups at places such as the National Center for Supercomputing Applications in Urbana, Ill., have already used sophisticated computer technology to visualize and study mathematical forms (SN: 10/24/87, p.264). But these efforts represent only a small portion of all mathematics research.



Computer scientist David P. Dobkin and his students at Princeton University use computer graphics to study the nature of shapes known as torus knots. Each image is computed from the equations defining that particular surface. The trick is to find an effective way to show a three-dimensional rendering of the equation-defined shape on a video display. "Getting a computer to understand the equations takes a bit of effort," says Dobkin. Now, Dobkin and his colleagues are using animation techniques to take viewers on "tours" of various knots.

Illustrations: Dobkin/Princeton Univ.

Furthermore, even mathematicians interested in using computers have difficulty obtaining the equipment necessary to convert their ideas into images. Few university mathematics departments have the technical staff needed for writing computer programs, operating computer facilities and developing appropriate graphics techniques.

Those mathematicians with the patience and interest to write software often find it hard to exchange programs with their colleagues because different computer systems are often incompatible. It sometimes takes much more time and effort than it's worth to polish a program—especially one that is evolving rapidly—so that it runs on different computer systems and can be used readily by other researchers.

"There's no really good vehicle for exchanging the sort of work that's done on computers," says Thurston. Traditional methods of information exchange such as journal articles and seminar presentations "don't have the immediacy of working on the same computer," he says.

The Geometry Supercomputer Project represents a systematic attempt to make it easier for mathematicians to gain access to the equipment and expertise needed to use large-scale computation cooperatively and effectively. "Joined together as a group," says project organizer Albert Marden of the University of Minnesota in Minneapolis, "we would be able to share results and techniques and to hire some very good people who would work for everybody in the group."

When the University of Minnesota established a supercomputer institute, with access to a Cray-2 supercomputer, Marden saw his opportunity. "People were running around excited about the supercomputer, without being quite sure of how scientifically to take advantage of it," he says. Marden already knew that Thurston and several other mathematicians were keenly interested in computation but lacked the necessary resources.

"I put two and two together," he says. "But I never realized how complicated it would be." It took two years to gather the group, organize the project and arrange for funding. The group obtained a three-year, $1.5 million grant from the National Science Foundation (NSF), and the University of Minnesota offered to contribute computer time, office space and other services.

Although project members have diverse backgrounds, the Geometry Supercomputer Project builds on earlier, small-scale collaborations like the one between Thurston and Dobkin. "There are strong ties," says Marden. "There's a good reason why every single person is on the project."

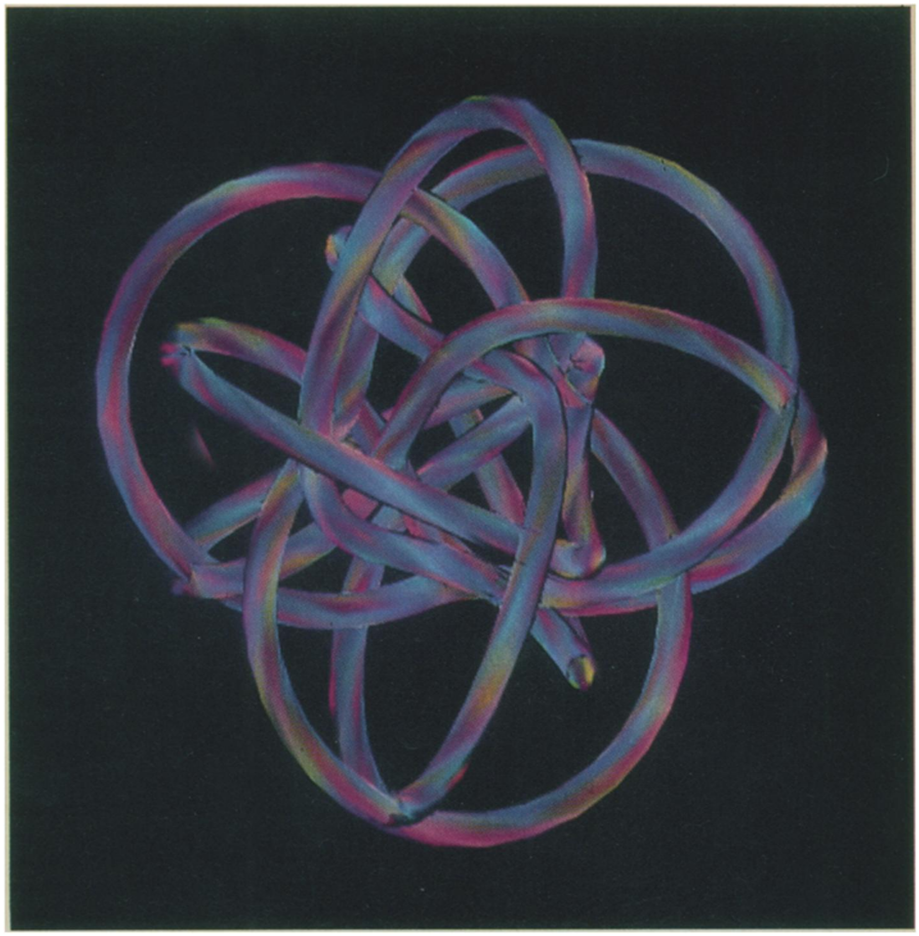
"We're all doing geometrical computations of one type or another," says Thurston, who is perhaps the central figure in the group. "It's something that not too many mathematicians have done in a serious way."

One of Thurston's major interests is compiling a comprehensive catalog of surfaces known as three-dimensional manifolds (SN: 7/17/82, p.42). These manifolds can take on a bewildering array of complex shapes, and the complete classification of these forms has stymied many a mathematician in the past. Just about every member of the group, while pursuing his own interests, is likely to contribute in some way to Thurston's classification effort.

Fractal geometry is another important element in the project (SN: 3/21/87, p.184). The idea of patterns that repeat themselves on ever smaller scales – patterns within patterns within patterns – was first proposed by project participant Benoit B. Mandelbrot of Yale University. He coined the word "fractal" to describe the self-similarity he observed.

Several project members have explored the effects of repeatedly evaluating a mathematical expression, such as $z^2 - 1$, for various values of z. The idea is to start by substituting a certain number into the expression, finding the answer, then plugging the answer back into the same equation, and so on, to see where the sequence of answers leads. This process of iteration has led to colorful, intricate portraits of equations, many of which show fractal patterns (SN: 2/28/87, p.137; 9/19/87, p.184).

Group members such as computer scientist Robert E. Tarjan of Princeton are interested in algorithms – the recipes used to achieve computational goals. From his work on sorting methods (SN: 9/15/84, p.170), Tarjan has found connections with the kind of geometric problems that Thurston is tangling with. And there's a great deal not yet known about which algorithms work best for solving particular geometrical problems.



An example of a torus knot.

David Mumford of Harvard University is searching for algorithms that mimic the pathways followed by nerve signals governing visual memory in humans. "We want to know," he says, "which of the ways that can be used to describe mathematically similar shapes would be most useful in simulating rapid and precise memory and recognition."

Although many features of the collaboration are still uncertain, project participants have already started to purchase new equipment, to discuss ideas and to develop software for joint use. However, says Thurston, "the communications system is not as good as we're hoping it will be one day."

Originally, the researchers had wanted to use a high-speed, satellite-based communications system, but they had to settle for a data network called NSFNET, which presently transmits information at 56,000 bits per second. This transmission rate is too low for sending pictures, which typically require millions of bits of data each. A few images could tie up communications lines for hours. NSF has plans to raise the network's transmission speed to 1.5 million bits per second later this year.

A good communications system, says project member James W. Cannon of Brigham Young University in Provo, Utah, means that "you can collaborate with someone in Virginia, New Jersey or England on a day-to-day basis, the way in the past you collaborated with someone in your own department." It reduces the sense of isolation sometimes felt by individual mathematicians at locations far from major research centers.

"It's exciting to be able to work in a common environment – sharing a computer facility – with people scattered all over the world," says Thurston.

How well the Geometry Supercomputer Project will work out is hard to predict. The project involves strong personalities from diverse backgrounds working together on difficult problems. "This is really a pilot project," says Dobkin. "Communities like this build up in computer science all the time and seem to survive happily. There's nothing to be lost by trying."

"We would like to think this is not a private club," says Marden. Initially, the project could serve a useful purpose simply by focusing the attention of mathematicians on the role of large-scale computation in mathematics. "Then we could enlarge the group," he says, "as resources permit."

"I hope," says Mandelbrot, "this project establishes for good among mathematicians the realization that the computer is an extraordinarily useful tool for exploring geometrical problems and making conjectures, and for communicating intuitions to other people." □