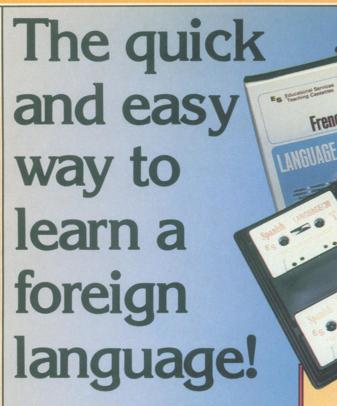


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# The Constrained Curve

The geometric path traced by a robot arm is independent of time. Now a mathematician at the General Motors Research Laboratories has devised a simple, innovative way to relate the path to time so that the machine can track the path and meet specific performance objectives without exceeding its physical operating limits.

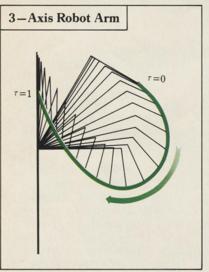


Figure 1: Schematic diagram of a 3-axis robot tracing a path in 3-space.

Figure 2: Results for Figure 1 path. I: Plot of the change of variables,  $t=h(\tau)$ . II, III, and IV: Normalized velocity, torque, and rate of change of acceleration for the waist, shoulder, and elbow (for any variable, a value of  $\pm 1$  indicates operation at a limit).

at repeating a well defined motion with a high degree of accuracy. A robot with a welding tool, a paint sprayer, or a grasping device at its tip can weld in the right spot, spray a precise pattern, or locate a part in a given place time after time.

This untiring precision makes robots valuable in a quality-oriented manufacturing process such as the assembly of an automobile. That's why General Motors has installed so many robotic manipulators in its plants, and why GM is intent on developing technology and software to use these machines to their best advantage.

When a robot is to apply sealant to a windshield opening, or move a part from one point to another, its tip is positioned at points along a fixed geometric path, always maintaining the orientation needed to perform the task.

Mathematically, tip position along the path can be described as a function of a one-dimensional position parameter  $\tau$  that ranges from 0 to 1 as the path evolves from beginning to end. Actually, for a robot having three joints, Figure 1 for example, tip position is determined by a set of three functions of  $\tau$ , one for each joint of the arm. Each separate joint function relates a specific angle of rotation,  $\theta$ , about that joint axis to a given value of  $\tau$ .

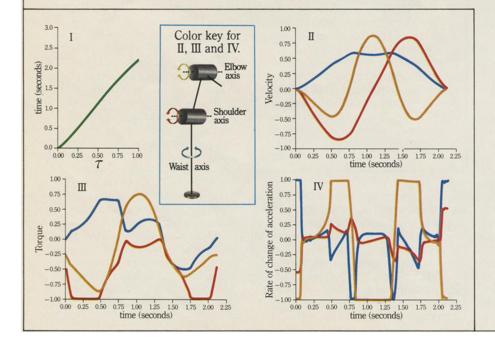
To get the robot to perform a task, however, its computer controller must associate each point on the path with some value of time—in effect telling the robot to be in position A at a certain time, position B at another time, and so on, throughout the path.

Establishing an appropriate correspondence between time and the path position parameter is an important prerequisite to actually controlling the robot to follow the path.

Dr. Samuel Marin, a mathematician at General Motors Research Laboratories, has devised an effective and efficient means of computing the required correspondence. His work addresses productivity concerns. Dr. Marin's objective is to make cycle time (the time it takes the robot to trace the path from beginning to end) as small as possible, yet to respect at all times the physical operating limits of the robot.

Dr. Marin noted that by seeking a correspondence that gives time explicitly in terms of the path position parameter,  $t=h(\tau)$ , the problem's character changes. It appears not so closely associated with control theory, where the problem has also been studied, but more like a problem of nonlinear optimization.

Setting  $g(\tau)=h'(\tau)$ , the derivative of h with respect to  $\tau$ , allowed Dr. Marin to pose the minimum time prob-



lem in the following way: minimize  $\int_0^1 g(\tau) \, d\tau$ , subject to some constraints dictated by the physical operating limits of the robot mechanism. These limits on the robot—limits on velocity, acceleration or torque, and on rate of change of acceleration (Fig. 2)—can all be formulated as differential inequality constraints and are all expressible in terms of the unknown function  $g(\tau)$ , as:  $g(\tau) \geq G(\tau, g, g', g'')$   $\tau \in [0,1]$ .

If the problem could be discretized, making it in some sense finite, it could be put on a computer and solved numerically. So Dr. Marin replaced the unknown function with a piecewise cubic approximation.

This allows the search for the unknown function to be confined to a class of functions that are completely characterized by a finite number of coefficients in a B-spline series.

He similarly discretized the constraints, replacing the infinite set of constraints with a finite dimensional subset that could be dealt with numerically.

He completed the formulation of the discrete problem by incorporating a grid-refinement strategy. Now the problem's dimension could be gradually increased to better approximate the continuous case.

What resulted was a classic nonlinear optimization problem, a finite dimensional problem in which it remained only to find the coefficients of the B-splines while satisfying the constraints.

A monotonicity property of this problem coupled with properties of the approximation method suggests that the simple technique of cyclic coordinate descent might best provide a solution. "While not so effective in other applications, a cyclic coordinate descent-based algorithm appears to be exactly what is needed in this class of problems," notes Dr. Marin. "With modifications introduced to ensure that the iterates are strictly feasible, this method has consistently and rapidly solved the problem."

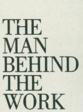
Working closely with mathematicians at Rensselaer Polytechnic Institute, Dr. Marin is confirming this method's utility. In comparisons so far with several widely used, general-purpose optimization codes, the special method consistently shows itself to be superior.

"My work in path parametrization is just part of the story here at GM," emphasizes Dr. Marin. "Many aspects of this problem's formulation are rooted in deeper concerns about how robots can be made to move faster and more accurately. These concerns originated in the work of Dr. Robert Goor, my colleague in the Mathematics Department, and have motivated several significant advances in robot control and trajectory planning.

"Until all the pieces are put together in a production system, it's difficult to gauge the full value of this work. However it will help reduce our manufacturing costs and will enhance our product quality."

## **General Motors**







Dr. Sam Marin is a Senior Staff Research Scientist in the Mathematics Department of the General Motors Research Laboratories. He is also the Manager of the Department's Mathematical Analysis and Computation Section.

Dr. Marin received his undergraduate degree in mathematics from St. Vincent College in Latrobe, Pennsylvania, and holds both an M.S. and a Ph. D. in that discipline from Carnegie-Mellon University. Between graduate degrees, Sam was an officer in the U.S. Navy, teaching mathematics at the Naval Nuclear Power School.

Since joining General Motors in 1978, Dr. Marin has pursued interests in numerical analysis and approximation. He has published research relating these areas to a variety of applications, including robotics, geometric curve design, and acoustics.

Sam is a member of the Society for Industrial and Applied Mathematics. He lives in Rochester Hills, Michigan, with his wife and two children.