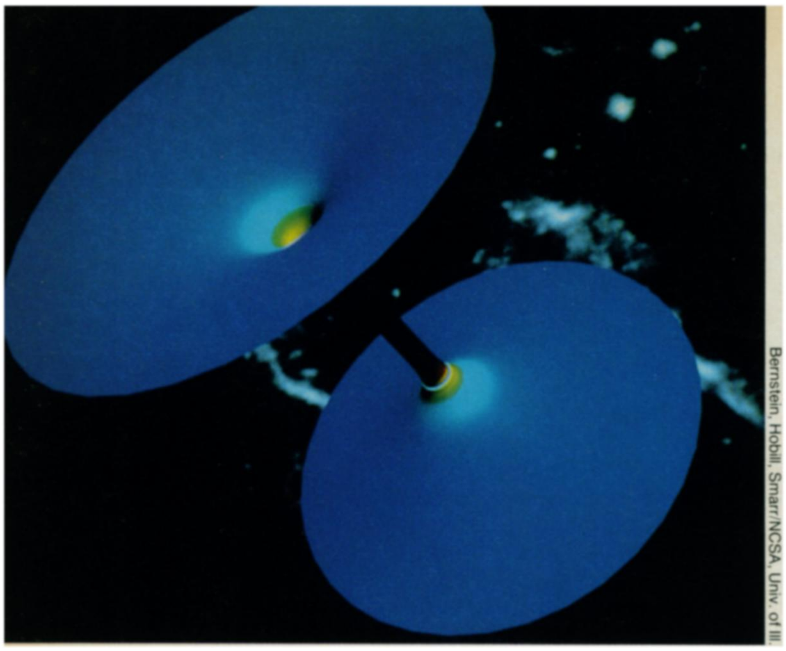


Bernstein, Hobill, Smarr/NCSA, Univ. of Ill.

A computer can simulate the physically impossible flight through the center of a black hole from one universe to another.

Relativity by the Numbers

Supercomputers help physicists picture collapsing stars and gravitational waves

By IVARS PETERSON

To the uninitiated, the mathematical equations used by theoretical physicists to convey their ideas can be baffling and intimidating. These compact notations pack a tremendous amount of information about the interactions between forces and particles. Even physicists often need help in teasing out what such equations mean—what predictions they make about the behavior of electrons, galaxies or cosmic strings.

Einstein's general theory of relativity is a particularly rich and elegant example. In his theory, Albert Einstein found a way to describe a physical force—gravity—in terms of a mathematical construct—geometry. According to his theory, gravity, time and three-dimensional space are fused into a single universal entity. Great masses such as stars warp the geometry of space and time. Cataclysmic events such as supernova explosions generate space-time ripples that propagate as gravitational waves.

Written in condensed form as $\mathbf{G} = 8\pi\mathbf{T}$, the 10 field equations expressing the general theory of relativity look simple enough. They state the relationship between **G**, the curvature of four-dimensional space-time, and **T**, a measure of the extent to which matter and energy distort this geometry. Yet despite their apparent simplicity, these equations describe gravity in a host of astrophysical systems, from exploding stars to collapsing galaxies.

"The difficulty lies in trying to get some kind of physical information out of the Einstein equations," says astrophysicist David W. Hobill of the University of Illinois at Urbana-Champaign. Traditional methods for solving the equations, like pencil-and-paper methods used for solving typical college calculus problems, work only for the most simple cases, which bear little resemblance to real stars and galaxies.

When Einstein published his theory, he explained only one phenomenon that Newton's laws were unable to account for: a slight shift in Mercury's orbit. He predicted two other effects, which were subsequently observed, namely the bending of starlight by the sun and the gravitational redshift corresponding to the amount of energy light loses as it fights the effects of gravity.

The earliest solutions of Einstein's equations concerned simple situations—for example, the gravitational field surrounding a single, isolated, spherical mass. That led to estimates of how large and concentrated a mass had to be to cause space to curve significantly, and to the concept of a black hole, an object with such strong gravity even light can't escape. No one, using standard equation-solving techniques, could work out solutions for more complex cases, such as two massive stars spiraling in toward each other.

"You really have to go to computer methods to explore interesting astrophysical systems," Hobill says. "Computational methods are now becoming more and more popular, and it's becoming easier because of supercomputers."

It's remarkable how little we know about the solutions to the Einstein equations, says Larry L. Smarr, director of the National Center for Supercomputing Applications at Illinois. "With supercomputers, we are now capable of solving for and exploring the physics of much more complex solutions of the Einstein equations than ever before."

Last May, about 60 researchers gathered at Illinois to discuss the use of computers for solving Einstein's equations, a field now known as numerical relativity. The five-day workshop offered participants a forum for sharing recent research results, assimilating progress made in the last few years and discussing future directions for the field.

One of the main driving forces in numerical relativity is the shift of general relativity from a purely theoretical pursuit to a fledgling experimental or observational science. That transformation will probably begin in the early 1990s, when gravitational-wave observatories with new instruments about 1,000 times more sensitive than any now available will be ready to detect gravitational signals arriving from sources outside our galaxy. "They're going to see something," says L. Samuel Finn of Cornell University in Ithaca, N.Y. "But interpreting what's seen is going to require quite a bit of thought."

The first detected gravitational signals will likely come from distant objects more massive than the sun and moving at nearly the speed of light, or from the violent explosion of a massive star. By computing in advance what gravitational waves coming from various complex astrophysical events would look like, theorists should be able to tell observers what kind of signals to expect and how to interpret any signals received.

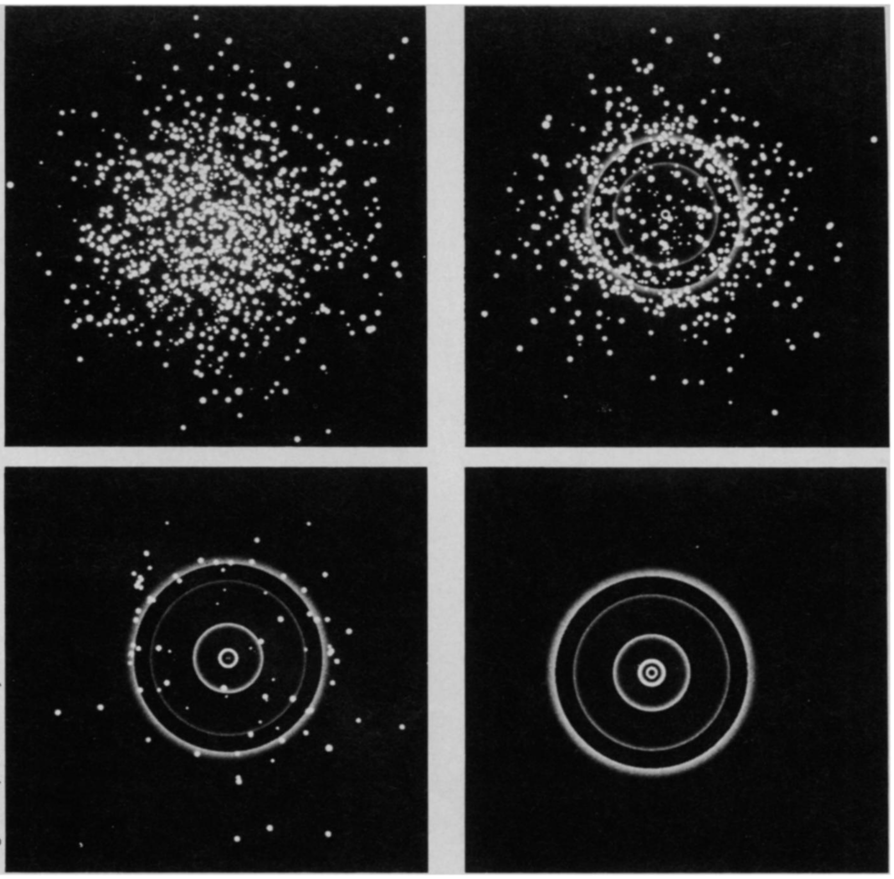
"We have to build up a catalog of gravitational radiation waveforms, so that we can go back and forth between what the observer sees and what the theoretician can calculate on his computer," Finn says. That kind of interaction would give theorists a way to check their computations and experimenters a better understanding of what they're observing.

"Without numerical relativity, we would never be able to interpret the waveforms that we discover with the [gravitational] wave instruments," Smarr says.

"The big problem that we would like to solve in the next 10 years before the [gravitational] wave observatories come on line is the coalescence of two orbiting compact objects," says Charles R. Evans of the California Institute of Technology in Pasadena. Of special interest is the behavior of black holes. A pair of orbiting black holes, spiraling in toward each other until they merge, is likely to be a strong source of gravitational waves.

"While we don't have the actual calculations in hand yet, we can make estimates and we have every expectation that such a system is a very strong source of gravitational radiation," says Evans. "The main uncertainty is that we don't know how often these things occur. We have yet to identify a black hole-black hole binary somewhere in the galaxy." However, binary systems consisting of two neutron stars – compact, dense stars composed almost entirely of neutrons – or of a neutron star and a black hole have been identified, enhancing the chances of finding a system made up of a pair of black holes.

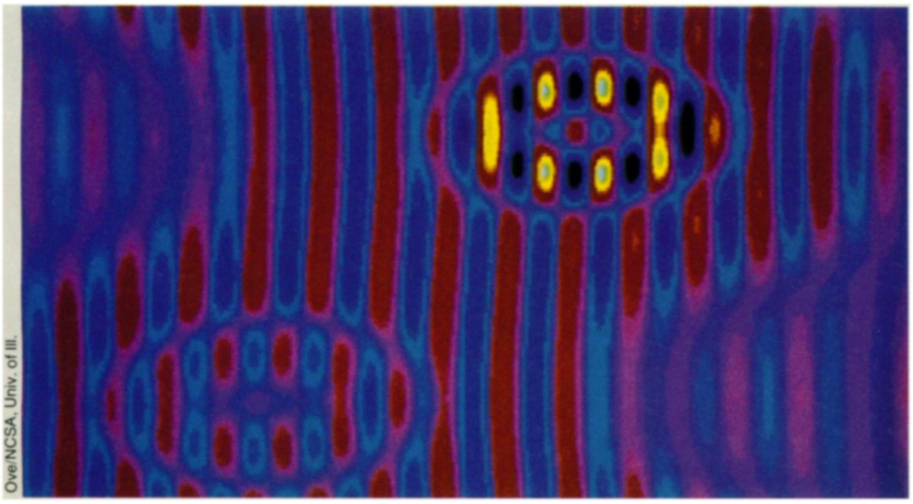
The fact that such a cosmic event must be simulated in three dimensions makes the computations particularly difficult. So far, most computations in numerical relativity have been done for one- or two-dimensional cases. Computers are not yet fast enough and have too little memory to handle a three-dimensional problem in sufficient detail.



Images: Shapiro, Teukolsky/Cornell

Astrophysicists Stuart Shapiro and Saul Teukolsky used a supercomputer to calculate the motion of thousands of stars, showing how a quasar might form from a collapsing star cluster. At first, the stars are moving at nearly the speed of light (top left). Under the influence of gravity, as described by Einstein's general theory of relativity, the cluster begins to collapse, and a black hole forms at its center (top right). Concentric circles (bottom left) show how the black hole traps light from the cluster's center. Eventually, the black hole consumes most of the mass in the cluster's central region (bottom right).

Says Evans, "It's going to be a real race as to whether the numerical relativists can calculate and predict what the waveform will look like before the observers get their antenna on line to detect one."



Ove/NCSA, Univ. of Ill.

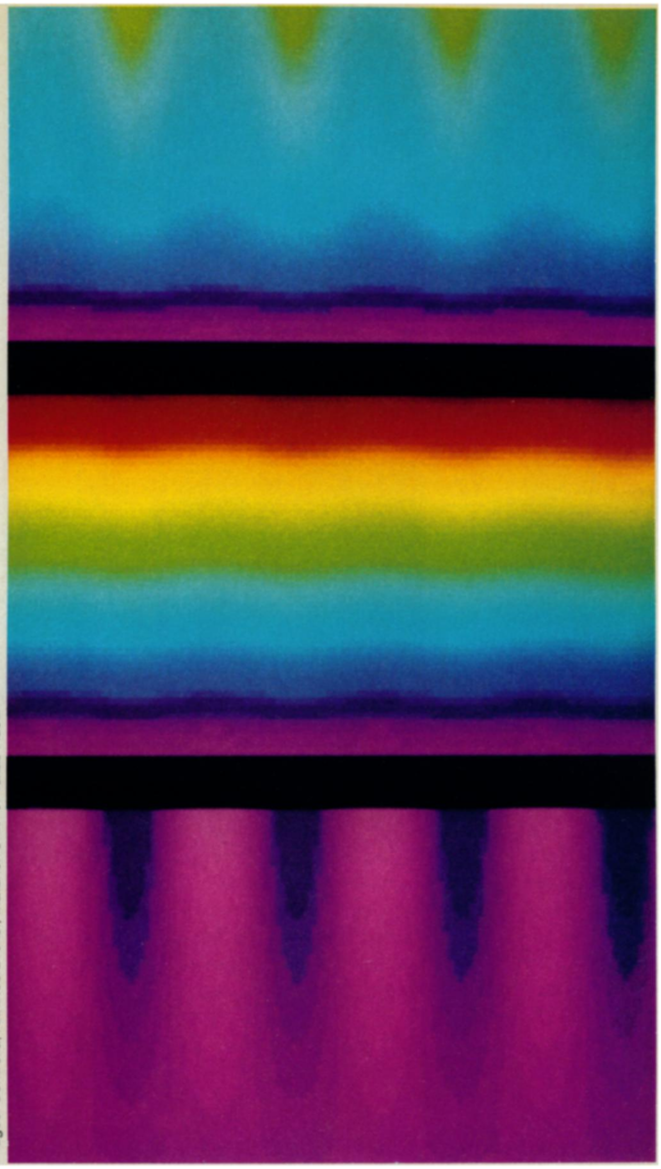
In Einstein's general theory of relativity, gravitation is a manifestation of the curvature of space and time. Roger Ove is investigating the dynamical behavior of solutions to Einstein's equations under a variety of special conditions. His computer-generated images illustrate how irregularities in such a gravitational field propagate in a universe that is curved like a doughnut, or torus. In this way, Ove can study the circumstances leading to the formation of gravitational waves, and he can follow the progress of these waves through space and time.

"There's going to be a period during which people learn all the tricks they need to do three dimensions," Finn says. For example, researchers need to work out ways of visualizing the reams of numbers produced in a three-dimensional simulation. "People don't have a good handle on the best way to represent three dimensions," says Finn. "That's one of the issues that is going to hold up productive work in this field. Until people figure out a way to look at their results and understand them, they're not going to know how to advance the science and communicate the results."

Observers also have a good chance of detecting gravitational waves from the collapse of a massive star into a neutron star, which results in a supernova explosion. "With gravitational radiation, you can see all the way inside a supernova," Finn says. "You know what is going on to the deepest levels. This gives you an entirely new window onto what's going on there."

"The uncertainty in this case is how

strong the signal will be," says Evans. Einstein's theory predicts that a collapsing star emits gravitational waves only if it is also rotating as it collapses. "There's not a whole lot of evidence available at the moment to tell us how rapidly the cores of these stars rotate," he says.



Images: Centrella, Anninos/Drexel; Matzner/Univ. of Texas-Austin

As a way of understanding the complex dynamical behavior shown by solutions of Einstein's equations, Joan Centrella and her colleagues have been studying the simplified problem of what happens over time to a standing or traveling gravitational wave in a one-dimensional vacuum. By using different methods for approximately solving the equations involved, the researchers can check the validity of their numerical techniques and search for evidence of unusual types of behavior. The three computer-generated images shown, each summarizing the consequences of changing a particular parameter in the given equations, demonstrate that three different methods for solving the equations give somewhat different solutions.

Researchers are also studying the fundamental mathematics underlying Einstein's equations. "They want to see what the properties of the equations are," says Hobill. "If you start off with certain conditions, will other conditions arise?"

What makes the equations particularly intriguing is that they are strongly nonlinear. Linear equations express a direct proportionality: As the value of one variable increases, the value of another variable increases correspondingly. In contrast, nonlinear equations express a more complicated relationship, and the results of changes in the value of one variable can show up in unexpected ways in the values of other variables. In Einstein's theory, gravitational waves, which appear as space-time ripples, themselves have energy and mass and consequently alter the curvature of space-time. That type of nonlinearity — a kind of feedback in which the gravitational wave feeds on itself — leads to some curious properties.

For example, normal waves pass right through each other undisturbed. Gravitational waves of sufficiently high amplitude slow themselves down when they meet. If the combination has enough mass, or energy, concentrated in a small volume, the waves may collapse on themselves to form a black hole.

Roger Ove of Illinois has studied what happens when only gravitational waves populate a universe shaped like the four-dimensional analog of a doughnut, or torus. In his calculations, there are no particles, just interacting gravitational waves propagating according to Einstein's equations.

Ove is interested in how waves propagate in such a setting, how small changes in geometry affect their motion and properties, and what unusual characteristics these waves may develop. "While this has very little physical relevance to the real world, Ove is using numerical relativity to explore the behavior of the Einstein equations," Hobill says.

Many questions arise. For instance, can two interacting waves form into a soliton — a packet with a definite shape traveling along as a single entity? Do gravitational waves steepen to form into shock waves just as acoustic waves do when, say, a jet plane breaks the sound barrier?

"We don't know if that can happen in general relativity," Smarr says. "Until we understand how the Einstein equations work — how strangely space and time can be warped — we don't know what we're likely to find when we look."

Says Finn, "These are important questions, particularly when you think back to the early universe." Gravitational shock waves, for example, would be more likely to form at a time when mass is more concentrated and the universe is just beginning to expand.

Numerical relativity is still a young field, barely out of the pioneering phase. Researchers still spend more time worrying about the details of their computer programs, or codes, than about the physics of general relativity. "Even two years ago, it was a real test just to see if you could do these simulations and have the codes hang together throughout a computer run without crashing," Evans says. "We're just getting past that stage."

Though small, the numerical relativity community is growing. Researchers are starting to redo early calculations, using improved, more accurate techniques to look for subtle effects swamped in the past by numerical errors. Confidence in the use of computational methods is also increasing as different groups of researchers attack the same problem using different techniques, and find they get similar results.

But this is only the start. "We have a stage to go yet," Smarr says. "To have a science, we must be able to get accurate and reproducible solutions to the Einstein equations. We must then be able to share these solutions with our colleagues, who may not be builders of numerical relativity codes, and they must be able to take a solution we generated and do further work based on what we've done."

"A lot is changing in general relativity," says Joan M. Centrella of Drexel University in Philadelphia. "From a sort of impenetrable curiosity that practically nobody ever studied because nobody could understand it, it's really becoming a science. Using computers, we can get solutions; we can understand these equations." ☐