Linking doughnuts, soda straws and energy

Knowing how magnetic field lines in a neutron star are tangled or linked gives scientists a good estimate of the minimum energy such a system can have. That surprising result is one consequence of a recently proved mathematical theorem linking the topology, or geometric form, of lines representing the flow of an incompressible fluid (whether an ideal liquid or a magnetic field) and the energy embodied in that fluid flow.

"What's of interest is the relationship between the topology of the lines and the energy in the system," says mathematician Michael H. Freedman of the University of California, San Diego.

One way to picture the situation is to think of a solid torus, or doughnut shape, made up of stretched rubber bands, each running once around the doughnut's hole. The idealized rubber bands want to shrink as much as possible, but because the material is also incompressible, they can't make the doughnut shrink to nothing. However, the rubber bands can release their energy by forcing the doughnut's shape into that of a long, thin soda straw. Like the doughnut, the soda straw has a central hole, but because of the straw's much narrower configuration, each rubber band has a significantly shorter distance to stretch.

When one doughnut winds through the hole of a second doughnut, the situation changes dramatically. The soda-straw solution is no longer possible. If one doughnut were to spring into its soda-straw configuration, the change would stretch the other doughnut in a way that increases the energy of its rubber bands. In other words, the linking of the two doughnuts prevents the dissipation of energy. Similar arguments hold for more complicated links and knots.

In the case of a neutron star, the material within the star moves around so as to let magnetic field lines straighten out and separate as much as possible. But any linking and knotting of these lines gets in the way, limiting how much the system can "relax" and setting a lower bound on its energy. Similarly, a solid torus of spinning fluid may give up energy by elongating like a soda straw, but such a shift is prevented when several tori link.

Freedman, working with graduate student Zheng-Xu He, builds on a theorem proved by Soviet mathematical physicist V.I. Arnold, who showed that energy bounds exist for the special case when a quantity called the "linking number" can be computed for a given tangle of lines. Using ideas from knot theory, Freedman and He extended Arnold's theorem to cover a much broader range of knots and links.

"Geometric linking, independent of linking number, can be used to estimate a lower bound on the energy of an incompressible flow," Freedman says. "This suggests wider applicability of the topological lower-bound principle than was visible from Arnold's work."

The new results provide an intriguing hint of how to keep a hot, ionized gas, or plasma, from leaking out of a magnetic "bottle" — a problem faced in fusion-energy research. In this case, creating local "tangles" in a confining magnetic field may keep energy from dissipating below a certain level.

"Any real-world system is much more complicated than what we addressed," Freedman says. "But that's often the nature of mathematical physics. You don't try to build a model as complicated as the world. You try to build something you can analyze, and then you take it from there."

— I. Peterson

Reforming math education

The way mathematics is taught and learned in U.S. schools, from kindergarten to college, requires a major overhaul, says a National Academy of Sciences report. It notes that three out of four students leave school without mastering enough mathematics to "cope with either on-the-job demands for problem solving or college expectations for mathematical literacy."

Recognizing the failure of the "new math" reforms imposed several decades ago, the report emphasizes the need for a new approach that establishes and builds on "appropriate national expectations." It urges the adoption of a coordinated national strategy to be implemented voluntarily by local school systems. One key proposal calls for a shift in mathematics education at all levels away from pencil-and-paper exercises and rote memorization toward using calculators and computers and solving more realistic problems.

The report, called "Everybody Counts," is the first of several addressing current problems in mathematics education. Later this year, the National Council of Teachers of Mathematics will release the first detailed set of standards for teaching mathematics and the Mathematical Sciences Education Board will propose a framework for curriculum revision. □

Caterpillar disguise: You are what you eat

A biologist has found what he calls the "best worked-out" example of diet influencing an animal's development: a caterpillar that grows to look like either a flower or a twig, depending on what it eats after hatching. The caterpillar provides a good model for studying how environmental cues turn genes controlling development on and off, a subject about which little is understood, says discoverer Erick Greene of the University of California, Davis.


Greene

Caterpillar (arrow) and catkin.

Although several insects develop into different forms depending on external variables, this is the first known case in which diet dramatically influences an individual insect's appearance, Greene says. *Nemoria arizonaria* caterpillars that hatch in the spring feed on an oak's male flowers, called catkins, and soon begin to take on catkins' golden color and fuzzy appearance. They also develop rows of dots resembling catkin pollen sacks on their backs. In stark contrast, summer-born *N. arizonaria*— hatched after catkins have disappeared — are greenish-gray and look like oak twigs, Greene says.

To test how three environmental variables might influence the caterpillar's appearance, Greene subjected eight groups of recently hatched *N. arizonaria* to different temperature, day-length and diet regimens for 15 days. He found that only the diet, composed of either catkins or leaves, mattered. Then, using artificial diets, he discovered that plant chemicals known as tannins alone will prompt a caterpillar to develop a twig-like appearance. Greene still does not know which tannins are important or how tannin levels affect development.

Although catkin types survive better than twig types, the twig-like caterpillars probably maintain their evolutionary foothold because they allow the species to produce a second yearly brood, Greene suggests in the Feb. 3 SCIENCE. Because female *N. arizonaria* don't wait a full year to become moths and lay eggs, catkin mimics always produce twig-like young, whereas twig mimics always produce offspring destined to become catkin-like.

Greene discovered the charlatan catkin while studying insect-eating birds. The moment came when "one of the catkins started to walk away from me," he recalls. "I just about fell out of the tree when I found it."

— I. Wickelgren