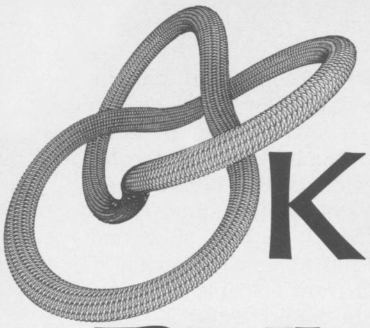

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KNOT PHYSICS

Newly discovered links between quantum physics and knot theory may tie together a wealth of mathematical ideas

By IVARS PETERSON

As a graduate student in the late 1960s, James I. Lepowsky wanted to study both physics and mathematics. But he found little interaction between the two disciplines. Since their unfortunate divorce at the turn of the century, physics and mathematics have largely set off on separate paths, developing their own languages and specialties. Lepowsky, presently at Rutgers University in New Brunswick, N.J., chose the path of mathematics.

In the last few months, however, many mathematicians, including Lepowsky, have rediscovered physics. They are plunging into the intricacies of quantum field theory, seeking to understand the special mathematical methods physicists have developed for solving theoretical problems related to the nature of matter and the structure of the universe.

Behind the mathematicians' sudden fascination lies a new approach to quantum field theory developed by physicist Edward Witten of the Institute for Advanced Study in Princeton, N.J. His approach appears to tie together a wide range of mathematical ideas and may lead to a deeper insight into string theory, which attempts to provide a unified picture of gravity and quantum mechanics. Witten's "grand scheme" was the main topic of a recent, month-long special program at the Mathematical Sciences Research Institute in Berkeley, Calif.

"It's all fantastically exciting," Lepowsky says. "It's remarkable how many superficially different branches of mathematics are becoming understood as different aspects of the same thing."

The present excitement traces back to 1984, when Vaughan F. R. Jones of the University of California, Berkeley, formulated a new way of distinguishing knots. He produced a system of mathematical expressions called polynomials that can be used as labels for knots as pictured in a two-dimensional diagram. Different knots presumably have different polynomial labels (SN: 5/21/88, p.328).

Meanwhile, mathematicians at Oxford University in England were exploring connections between the shapes of surfaces and certain complicated equations used by physicists to describe the behavior of fundamental particles. Those connections led to the discovery that four-dimensional space has unexpectedly bizarre properties. In the course of this exploration, Michael F. Atiyah noticed a strong resemblance between Jones' knot polynomials and some aspects of quantum field theory. He suggested the two were somehow related.

Witten, one of the chief proponents of string theory, took Atiyah's provocative suggestion seriously. In string theory, the world is no longer made up of elementary particles but of tiny strings that wriggle about. Different particles, whether electrons, quarks or neutrinos, correspond to strings vibrating in particular ways. This idea of a vibrating loop of string sitting in physical space resembles the mathematical notion of a knot as a twisted circle embedded in three-dimensional space.

Moreover, just as Einstein based his theory of gravity on geometric principles (the curvature of space), Witten wished to find a similar geometric foundation for string theory. String theory at its finest should be a new branch of geometry, Witten says.

Witten's "topological quantum field theories" hint at what such a geometrical foundation may look like. He starts with an appropriate mathematical description of an abstract geometrical space. To that description he applies techniques that physicists often use to introduce quantum rules into a system. Because Witten's theory has no physical content, it doesn't directly say anything about particles and forces in the real world. But the results do provide information about the space for which the calculations are done.

"At this point, it's still not clear what the impact will be on physics," says physicist Gary Horowitz of the University of California, Santa Barbara. "The main potential application is to our understanding of quantum gravity and string theory. We need to understand string theory and topological field theories better to see exactly what the relationship is, but there is already a glimpse of a connection."

Mathematics, however, has already felt the impact of Witten's ideas. Because Witten's method can be applied to any twisted or deformed three-dimensional space containing a knot, his theory supplies a way of defining or identifying a knot in three dimensions. In addition, it provides for the first time a picture of what geometric information is encoded in a Jones polynomial. In the past, knot theorists could calculate a Jones polynomial for a given two-dimensional representation of a knot but had no idea how that information was related to the knot's three-dimensional form.

"Witten has done what I was trying to do," Jones says. "He has given a three-dimensional definition in terms of quantum field theory, which was what I suspected existed. But I didn't know enough quantum field theory to get it right."

Furthermore, mathematicians now see the possibility of a network of links joining quantum field theory to a variety of mathematical ideas, from knots and the geometry of curved spaces to algebra and group theory. Lepowsky, for one, is working on reconstructing knot theory from an algebraic viewpoint. "The underlying structures are so similar one should look for a deeper understanding of that connection," he says.

"Everything seems to be connected to everything else," says Robion C. Kirby of the University of California, Berkeley. "The subject is in a great state of flux, and nobody quite understands what's happening."

"The physicists are teaching us something that we didn't see in pure mathematics," adds John W. Morgan of Columbia University in New York City. "It's having a lot of influence on very active areas in mathematics." □