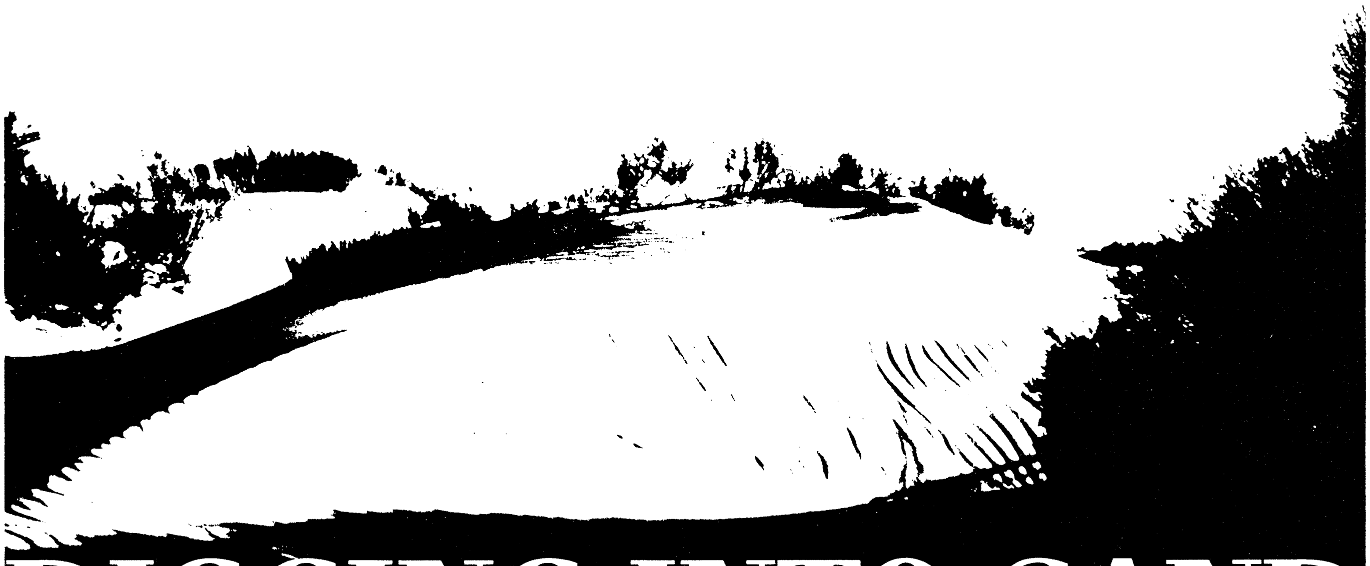


I. Peterson

DIGGING INTO SAND

Sandpile avalanches suggest new ways of looking at turbulence and other complicated phenomena

By IVARS PETERSON

Playing with sand can be instructive not only for children but also for physicists. Dumped onto a table, a bucket of dry sand settles into a cone-shaped mound. Whether the pile is small or large, its slope is always the same. Adding extra sand to an established pile triggers small avalanches — but by the time the avalanches die out, the pile's characteristic profile is back.

"We've played with sand so much that we think we have a good feeling for what happens in sand," says physicist Sidney R. Nagel of the University of Chicago. Indeed, physicists sometimes use the behavior of sandpiles as a metaphor for other phenomena in physics.

Yet sand's behavior is actually complicated and poorly understood, Nagel says. Dry sand can be heaped into a pile, which, like a solid, retains its shape. If the pile is disturbed or its slope becomes too steep, sand grains behave like a fluid and flow downhill. A closer look, however, reveals that the flow resembles a dense gas of heavy, colliding particles rather than a true liquid. Such multifaceted behavior compounds the difficulties of studying and understanding sand's behavior.

Recent interest in sandpiles has been fueled by speculation that the behavior of avalanches on sandpile surfaces may exemplify a new theoretical model for the dynamical behavior of systems made up of large numbers of interacting parts, whether sand grains, molecules or galaxies. First suggested in 1987 by Per Bak, Chao Tang and Kurt Wiesenfeld of the Brookhaven National Laboratory in Upton, N.Y., this theory of "self-organized criticality" has attracted considerable attention.

According to its proponents, the theory provides a fundamentally different way of viewing a wide range of phenomena, including certain types of noise in electronic devices, turbulence during fluid flow, the pattern of energy release during earthquakes, the ebb and flow of sunspot activity, the irregular flickering of radiation from distant quasars, the large-scale structure of the universe and the erratic fluctuations of stock market prices and other economic indicators. At the same time, although the theory has generated a great deal of excitement among scientists, recent attempts to apply it to real sandpiles and other systems have largely failed.

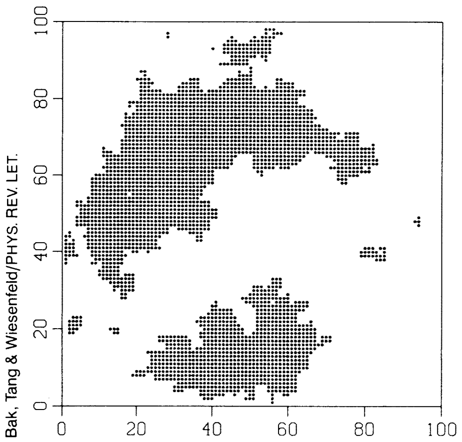
Bak and his colleagues contend that a complex dynamical system can naturally evolve into what they call a self-organized critical state, which is far from equilibrium and barely stable. Such a precariously balanced system always operates on the edge of collapse, yet responds resiliently to external stresses by returning in a characteristic way to its initial "critical" state.

One way to picture such a system is to imagine building a sandpile on a table. "I can do it by slowly adding grains of sand," Bak says. The pile starts out flat and gradually gets steeper and steeper. Now and then, avalanches occur, and as the pile grows, the avalanches become bigger.

Eventually, the sandpile reaches a critical state and the system regulates itself. Randomly adding sand grains to a pile triggers avalanches that bring the slope back to a particular angle known as the angle of repose. Even if thrown off balance by the sudden arrival of a load of sand, a sandpile readily recovers its angle of repose, always returning to its critical state. The amount of sand that accumulates balances the amount carried away by avalanches.

"It gets stuck at this state," Bak says. "You cannot build it further, and it will not collapse further."

Bak's theory predicts that when a sandpile reaches its angle of repose, adding more sand grains or slightly tilting the pile's base will generate avalanches of all sizes, limited only by the pile's size. Furthermore, the avalanches will occur at widely varying intervals. "If you were sitting at some place on a sandpile and measuring what was going on as a function of time and space, you would find



In this simulation of sandpile avalanches, as seen from above, each dark area represents the shifts triggered by the sliding of a single grain, which in turn forces other grains to slide.

features on all time scales and all length scales," Bak says.

To test this idea, he and his colleagues constructed a simple computer model designed to capture some of the crucial features of sandpile behavior. In their "theorist's view of a sandpile," each square of a two-dimensional grid has a certain numerical value. When a "sand grain" lands on a square, the square's value goes up by 1. If that value happens to exceed a predetermined critical value, then sand starts to flow and the value goes down by 1. That shift affects the square's nearest neighbor, which may then also exceed the critical value, and so on. The result is a chain reaction that continues until the system returns to a stable state.

"You can also drive the system by starting with a very steep sandpile and letting it collapse," Bak says. "This model is incredibly simple. Nevertheless, it has all the complexity of more usual, standard critical phenomena."

Computer simulations show that adding single grains to a sandpile in its critical state produces avalanches of many different sizes. "Sometimes nothing more happens," Bak says. "Another time, one unit of sand slides and that triggers two neighbors to slide, so we get a three-unit avalanche. Sometimes we see very large avalanches."

It's also possible to monitor how much sand is flowing at a given time. That flow is very irregular. "It goes up and down, sometimes almost dies out, hits a peak, then eventually dies out completely," Bak says.

Merely by changing the language, the researchers can use the same computer model for earthquakes. "You can think of this as being not a model of sand but a model of the Earth's crust and colliding tectonic plates," Bak says. In that case, the critical slope would correspond to a critical pressure. Thus, an earthquake is a chain reaction triggered by a local instability that propagates like a domino effect through a geologic fault system.

The model also predicts that small changes in the system can have widespread effects, a phenomenon that bodes ill for those who hope eventually to predict earthquakes. "The phenomenon is extremely sensitive to details far away from where you started," Bak says. "If this indeed represents the physics of earthquakes, [quakes] are unpredictable in the deepest possible way."

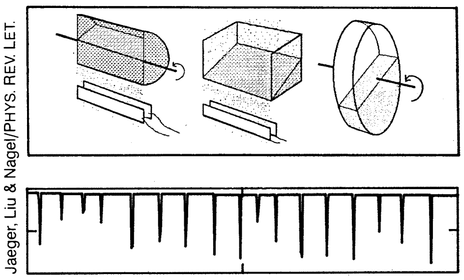
Bak's analogy between sandpile avalanches and self-organized criticality has prompted several researchers to take a closer look at the behavior of real sandpiles. "It's a nice idea, and that's what we set out to test," Nagel says.

Nagel and his colleagues induced avalanches in several different ways. In one setup, they dropped sand grains at random onto a sandpile in a rigid box with one open side. Sand tumbling down the slope and out of the box would fall into a narrow gap between two electrically charged parallel pieces of metal, allowing the researchers to monitor sand flow. In another setup, they observed sand's behavior in a slowly rotating cylinder half filled with sand.

"One of the most appealing aspects of this kind of research . . . is the simplicity of the experimental setup," says Dutch physicist Heinrich M. Jaeger of the Technische Natuurkunde in Delft, who collaborated with Nagel. "This makes it possible to study problems at one of the frontiers of current research with a decidedly low-tech approach."

The researchers discovered that nothing happens until the sandpile slope reaches an angle a few degrees higher than the angle of repose. Then the whole slope suddenly shifts. "The surprising thing is that the avalanches are almost always system-spanning," Jaeger says. "They are enormously large avalanches." Moreover, they don't happen at just any time but at particular times depending on the angle and rotation speed of the apparatus.

In the traditional picture of sandpile avalanches, the angle of repose represents a balance between friction and gravitational force. Avalanches start when the gravitational force on an exposed sand grain is larger than the amount of friction between sand grains. The new results show that for a sand grain on a surface to start moving, it must be able to roll or slide over its neighbors just below. Therefore, sand starts moving when the pile's slope is higher than the angle of repose.

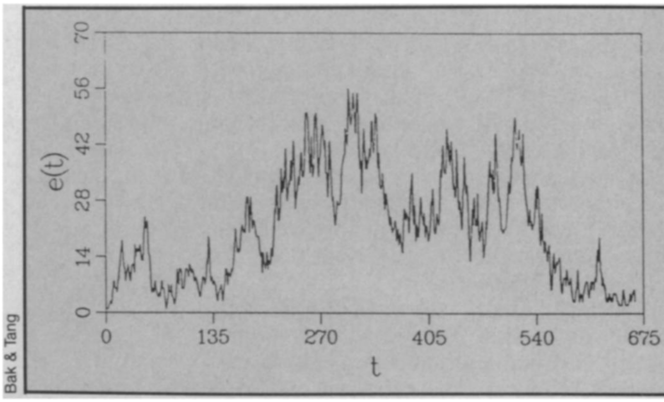


Schematic diagram (top) shows three possible configurations for observing the nature of sandpile avalanches. Tracking how much sand falls in a rotating drum shows that avalanches occur at fairly regular intervals (bottom).

"A sandpile doesn't collapse until you reach a threshold angle, at which point the whole thing collapses and you get a global landslide, which brings you back to the angle of repose," Nagel says. Sandpiles behave in ways that are not as simple or as obvious as many scientists had assumed, he notes. Such information has important implications for studies of flowing granular material (SN: 5/13/89, p.293).

"We find that the simple, intuitive picture of [the angle of repose] as the critical angle of the system has to be modified and that the direct analogy between the dynamics of sandpiles and physical systems exhibiting a critical point . . . is not well founded," Nagel and his collaborators conclude in the Jan. 2 PHYSICAL REVIEW LETTERS. "Perhaps other systems can be found which would show such critical behavior."

Nagel's negative results haven't slowed the theoretical exploration of self-organized criticality. "The



One computer simulation, based on the idea of self-organized criticality, produces fluctuations in energy released resembling those observed for a real earthquake and its aftershocks.

sand picture is only a vivid example," Bak emphasizes. "The concept of self-organized criticality is obviously much more general."

Bak and other researchers have developed a number of simple computer models to test the applicability of this concept to a variety of complex, interactive dynamical systems. "Of course, one can't make realistic models of meteorology, of geology, of turbulence, and so on," Bak says. "That would be way too complicated. Our philosophy is to create some simple toy models that capture some of the extensive physics of what is going on in these phenomena."

The idea is to look for "universal" mathematical quantities – numbers describing features that appear consistently in a wide variety of different models. Such universal quantities may reflect fundamental properties that apply not only to simple model systems but also to real situations.

One of Bak's models suggests how a fire might spread through a forest if an arsonist randomly set trees ablaze. A computer works with a grid in which each square is marked with a 0 (no tree), 1 (a tree) or 2 (a burning tree). At each step, burning trees die; neighbors of burning trees catch fire on the subsequent turn and then die. Fires propagate, but fresh trees appear randomly in unoccupied squares.

Starting with a random pattern of fires scattered over the entire grid, the system evolves to a critical state in which fires develop a string-like structure. That leaves trees in clusters of many different sizes.

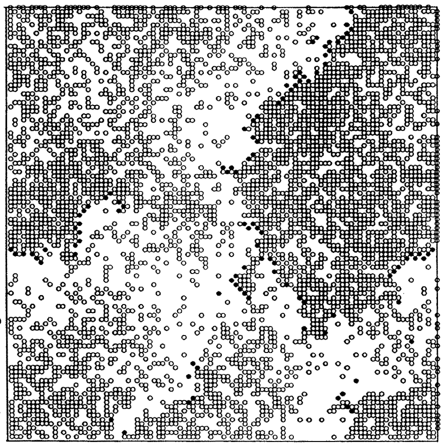
This can be thought of as a model for turbulence, Bak says. Just as tree growth adds fuel to keep fires burning in characteristic patterns, energy uniformly pumped into a fluid by stirring or heating turns a uniform flow into a swirling pattern of vortices that have no typical size.

Researchers at the Santa Fe (N.M.) Institute are studying the possibility of applying the concept of self-organized criticality to economic systems. Traditionally, economists tend to describe their systems in terms of simple equations relating a few quantities such as interest rates and employment levels. They usually study the effects of small deviations from an equilibrium situation.

Applying the concept of self-organized criticality to an economic system adds a new dimension. In such a state, a small perturbation can create either a small effect or a large one. There's no limit on how long the effect might last or how far it could extend through the system. These fluctuations are much stronger than those possible in an equilibrium model.

Simple computer models of economic systems, in which grid sites represent decisions made by individuals and simple rules define how information about those decisions spreads, actually do evolve to a critical state. "That's a totally different way of viewing an economic system," Bak says. "Indeed, if you plot economic indicators like the Dow-Jones [stock market] index, you have fluctuations on all temporal and spatial scales."

However, such models also demonstrate that self-organized critical systems are intrinsically unpredictable. In order to predict the fluctuations of economic indices, one must have complete information about the total system, and that is impossible to achieve. Even the tiniest disturbance can sometimes have an enormous impact, spreading through the whole system. This makes economic forecasting very difficult if not impossible.



Bak, Chen & Tang

In this snapshot of a computer-simulated forest fire, open circles represent live trees and filled circles represent burning trees. Living trees appear in units of various sizes separated by fractal-like strings of fires.

Two central questions about the theory of self-organized criticality remain unresolved. It isn't clear yet whether models showing this kind of behavior are universal, in the sense that a wide range of models produce the same results. For instance, Leo P. Kadanoff and his colleagues at the University of Chicago have shown that different simple computer models of avalanches don't necessarily have the same, universal properties. Nor is it clear whether the concept has any use for describing and explaining the behavior of real systems.

Bak argues that his concept of self-organized criticality provides a possible explanation for "1/f" noise, which has been detected in electronic devices and fits the kind of irregular fluctuations also seen in the luminosity of certain stars and in river flows measured over long periods of time (SN: 3/22/80, p.187). This type of noise, although irregular, is not random. "If the light from a quasar is measured over a long period, you see features that last for a very long time and other features that last for a very short time," Bak says. Taken together, such oscillations have no typical time scale.

Bak suggests that self-organized criticality is the common underlying mechanism accounting for phenomena that show a fractal structure in either time or space. A fractal is a self-similar geometric object in which the same pattern is repeated on ever-smaller scales. Magnifying a fractal by any amount reveals a miniature version of the larger form. Such structures have no characteristic length or time scale.

Self-organized criticality "is a new idea, and it has really struck a resonance," says Michael F. Shlesinger of the Office of Naval Research in Arlington, Va. "Many people are working on it, but I don't think it goes far enough toward giving real insight into problems."

Adds Michael B. Weissman of the University of Illinois at Urbana-Champaign, "I don't see any mathematical errors or anything obviously wrong with the theory as a construct that might describe something. But there's a fairly fundamental reason why that theory is extremely unlikely to be applicable to 1/f noise in electronic devices." Such devices, he says, don't show the fractal structures that Bak's theory suggests ought to give rise to 1/f noise.

"Often, a theoretical construct or even purely abstract math sits around for a while before somebody stumbles across something to which it actually applies," Weissman says. "That may be what will happen here."

Meanwhile, other researchers are still playing with sand (SN: 5/13/89, p.293). For example, gently shaking a sandpile induces a sort of behavior resembling relaxation – the recovery of materials after a stress has been applied (SN: 3/11/89, p.157). In the presence of vibrations, the decay of a sandpile's slope shows the same extremely slow time dependence also observed in glasses and in the decay of parcels of trapped magnetic field in many superconductors.

By carefully adjusting the intensity of the vibrations, researchers can also observe the transition from solid-like to liquid-like behavior in sand and other granular materials. In one experiment, researchers at the Université Pierre et Marie Curie in Paris demonstrated that vibrating a partially filled box of glass beads can induce convection patterns resembling those in a liquid.

"Sandpiles have properties that may be of generic importance," Nagel says. "If you could understand the behavior [of sandpiles], it might help you understand other physical systems, some of which are harder to get at because the key elements of their behavior occur at the microscopic level." □