Mathematics

The business of busy beavers

Start with a string of zeros and ones printed on a strip of tape and a device that reads, prints and erases symbols. The device scans the tape and, following a set of precise instructions, makes appropriate changes in the printed symbols. Given sufficient time, such a simple device — known as a Turing machine — can perform any computation that a modern digital computer, no matter how powerful, can do.

Of the many possible Turing machines, a few researchers have been looking for one dubbed the "five-state busy beaver." The idea is to find a set of five instructions for printing the largest possible number of ones on a tape originally filled only with zeros. Its mission accomplished, the machine would then automatically shut down.

Finding this creature turns out to be no simple matter. In 1985, amateur mathematician George Uhing discovered a five-state Turing machine that prints 1,915 ones as it goes through more than 2 million moves before finally stopping (SN: 2/9/85, p.89). Now, Heiner Marxen of the Technical University of Berlin has topped Uhing's mark by finding a five-state Turing machine that prints 4,098 ones on a tape, taking nearly 12 million steps.

Scientists don't know whether this particular machine is the long-sought five-state busy beaver. Other sets of five instructions might conceivably print out even more ones before coming to a stop. But Marxen's solution already requires so many moves that it would probably take an army of people, using the fastest available computers, many years to search for better machines. "It begins to make it somewhat improbable that we can solve the five-state problem," says Allen H. Brady of the University of Nevada at Reno, who confirmed Marxen's discovery. In contrast, computer scientists already know that no conceivable four-state Turing machine can print more than 13 ones before halting.

With so many moves involved in just a five-state machine, it's difficult to distinguish in general between Turing machines that stop and those that continue computing forever — a way of determining the truth of certain mathematical conjectures. Translated into mathematical terms, this means the number of steps required to solve particular mathematical problems can easily outstrip the ultimate problem-solving capacity of real computers.

Computing a prime champion

More than 2,000 years ago, the Greek geometer Euclid proved there is no largest prime number. But proving that a particular whole number is a prime — that is, divisible evenly only by itself and the number one — is a time-consuming task that limits the size of numbers that can be tested for primality. Last month, a team of six computer scientists at the Amdahl Corp.'s Key Computer Laboratories in Fremont, Calif., succeeded in showing that the number $391,581 \times 2^{216,193} - 1$ is a prime, setting a record for the largest known prime. The number has 65,087 digits — 37 digits more than the previous record holder.

The researchers sifted through 350,000 huge candidate numbers before settling on 7,000 behemoths for final testing. Using an advanced version of the Lucas-Lehmer primality test, they spent more than a year checking out the numbers whenever company computers were otherwise idle. By the time they had refined their testing algorithm, the group could test a candidate number in 33 minutes, using a program that took up only a small fraction of the computer's memory.

"The main benefit is the fine-tuning we did on the algorithm, primarily speeding up the multiplication of high-precision numbers," says team member Sergio Zarantonello. Weather forecasters and other researchers may find the improved multiplication techniques useful for speeding up their own computer models.

Science & Society

Big dividends from pollution cleanup

Annual U.S. expenditures on pollution abatement and controls rose from $18 billion in 1972 to almost $86 billion in 1988. But a new analysis of the economic impacts of that spending does "not lend support to the widely held belief that environmental programs generally hurt the economy by crippling industries and increasing unemployment," three economists report in the current AMBIO (Vol.18, No.5).

Businesses fund about 60 percent of U.S. pollution control and cleanup. About a quarter of their antipollution spending goes into capital investments for pollution abatement and control technologies, such as stack-gas scrubbers and clean coal technologies. And those investments appear to yield handsome returns, conclude Jonathan D. Jones of the Treasury Department and Roger H. Bezdek and Robert M. Wendling of Management Information Services, Inc., a Washington, D.C., consulting firm. Using computer models, they estimated how 1985 investments in particular pollution control technologies translated into sales for each of 80 individual industries. Then they calculated derivative effects of these investments throughout the economy — such as increased sales by suppliers of parts and raw materials and increased transportation costs to haul those materials.

The group's findings suggest that industry's $8.5 billion investment in controlling air, water and solid-waste pollution in 1985 translated into corporate sales of $19 billion, profits of $2.6 billion and 167,000 new jobs. But benefits were distributed quite variably. Per dollar spent, investments in air pollution controls yielded the greatest sales and profits, while solid-waste disposal generated the most new jobs. Though these estimates do not account for the job and profit losses that some individual companies undoubtedly experienced, they also do not account for the social benefits of a cleaner, healthier environment, Bezdek says.

Tracking seaside medical wastes

After hypodermic needles washed up on East Coast beaches last summer, 22 states enacted new infectious waste laws or regulations and Congress passed the Federal Medical Waste Tracking Act. But a waste industry group says those efforts probably won't do much to prevent "syringes on the sand." The reason: Most of this medical waste didn't come from hospitals or other medical facilities, the primary targets of regulatory efforts. A report issued last month by the National Solid Wastes Management Association says disposable syringes represented the bulk of the medical wastes found, and most of those probably came from diabetics or intravenous drug abusers.

Sewer system overflows and U.S. Navy vessels also contributed beach medical waste, says Leslie Legg, who prepared the report for the Washington, D.C.-based trade association.

"We feel that future regulations should address the proper management of medical waste from all sources, including households," Legg says. Such regulations could call for separate packaging of potentially infectious medical wastes from patients' homes for disposal by health care providers, she adds.

The volume of hospital waste getting special handling has increased since 1987, when the Centers for Disease Control in Atlanta upgraded its "universal precautions" and urged health care providers to treat as infectious any material contaminated with blood or body fluids. Robert Peters, manager of the association's Biomedical Waste Treatment Institute, says those precautions have increased the average amount of hospital waste managed as infectious from an estimated 3 to 7 percent to 30 percent. Some hospitals have apparently interpreted the regulations stringently, handling 70 to 90 percent of their debris as infectious, according to a 1988 Office of Technology Assessment report.