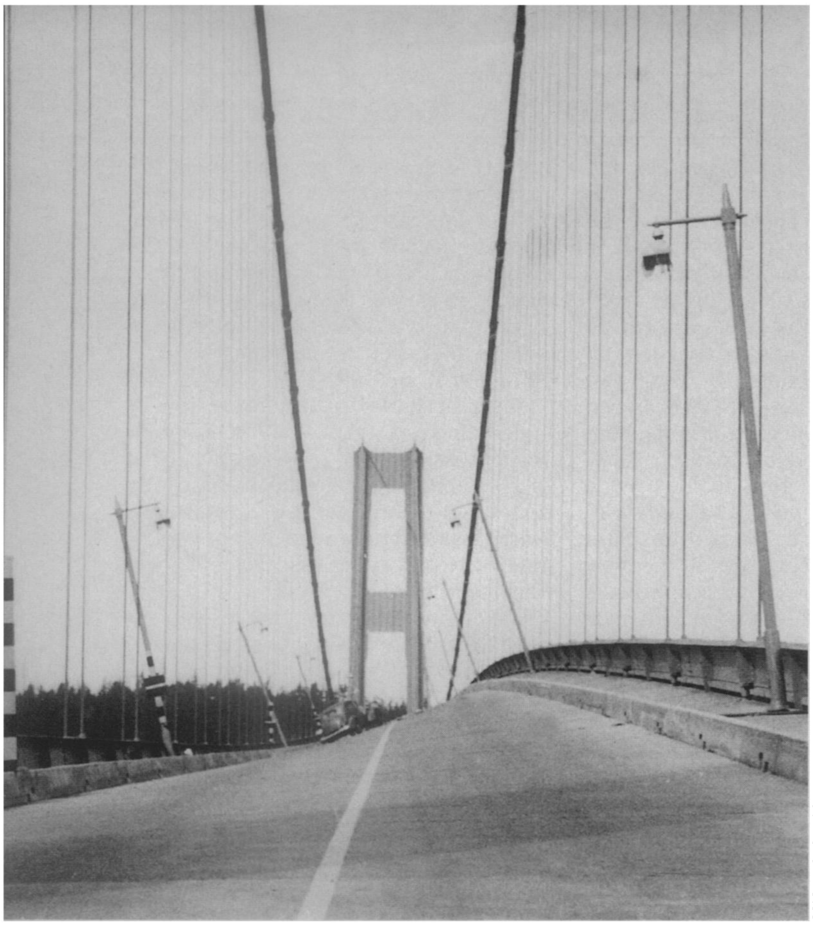


Rock and Roll Bridge

A new analysis challenges the common explanation for a famous collapse

By IVARS PETERSON

Startling scenes of rippling pavement, featured in a classic film that captured the 1940 destruction of the Tacoma Narrows suspension bridge in Washington state, rank among the most dramatic and widely known images in science and engineering. This old film, a staple of most elementary physics courses, has left an indelible impression on countless students over the years.

Many of those students also remember the standard explanation for the disaster. Both textbooks and instructors usually attribute the bridge's collapse to the phenomenon of resonance. Like a mass hanging from a spring, a suspension bridge oscillates at a natural frequency. In the case of the Tacoma Narrows bridge, so the explanation goes, the wind blowing past the bridge generated a train of vortices that produced a fluctuating force in tune with the bridge's natural frequency, steadily increasing the amplitude of its oscillations until the bridge finally failed.

"This explanation has enormous appeal in the mathematical and scientific community," observes applied mathematician P. Joseph McKenna of the University of Connecticut in Storrs. "It is plausible, remarkably easy to understand, and makes a nice example in a differential-equations class."

But the explanation is flawed, he says.

Resonance is actually a very precise phenomenon. Anyone who has seen sound waves shatter glass knows how closely the forcing frequency must match an object's natural frequency. It's hard to imagine that such precise, steady conditions existed during the powerful storm that hit the bridge, McKenna says.

Furthermore, the structure displayed a number of different types of oscillations. Initially, its roadway merely undulated vertically. Then the bridge abruptly switched its oscillation mode, and the roadway started to twist. It was this extreme twisting that actually led to the bridge's demise.

Indeed, even the 1941 report of the commission that investigated the disaster concludes: "It is very improbable that resonance with alternating vortices plays an important role in the oscillations of suspension bridges."

If simple resonance doesn't explain the Tacoma Narrows destruction, what does? Fascinated by that question, McKenna and Alan C. Lazer of the University of Miami in Coral Gables, Fla., have spent the last six years developing an alternative mathematical model that may help elucidate the catastrophic collapse.

"What distinguishes suspension bridges, we claim, is their fundamental nonlinearity," Lazer and McKenna state in a paper to appear in a forthcoming SIAM Review.

Linear differential equations, such as those typically used by engineers to model the behavior of structures such as bridges, embody the idea that a small force leads to a small effect and a large force leads to a large effect. Nonlinear differential equations, such as those studied by Lazer and McKenna, have more complicated solutions. Often, a small force can lead to either a small effect or a large effect. And exactly what happens in a given situation may be quite unpredictable.

Lazer and McKenna say their new theory provides key insights into why suspension bridges oscillate the way they do. It applies not only to the Tacoma Narrows bridge and San Francisco's Golden Gate bridge—which may be prone to large-scale, potentially destructive oscillations during earthquakes — but also to large, flexible structures, such as space stations, giant space-based robot arms and certain types of ships. The theory even suggests ways of constructing extremely light, flexible bridges that won't oscillate wildly.

Suspension bridges have a long history of large-scale oscillations and catastrophic failure under high and even moderate winds. The earliest recorded problem involved a 260-foot-long footbridge constructed in 1817 across the River Tweed in Scotland. A gale destroyed that bridge six months after its completion.

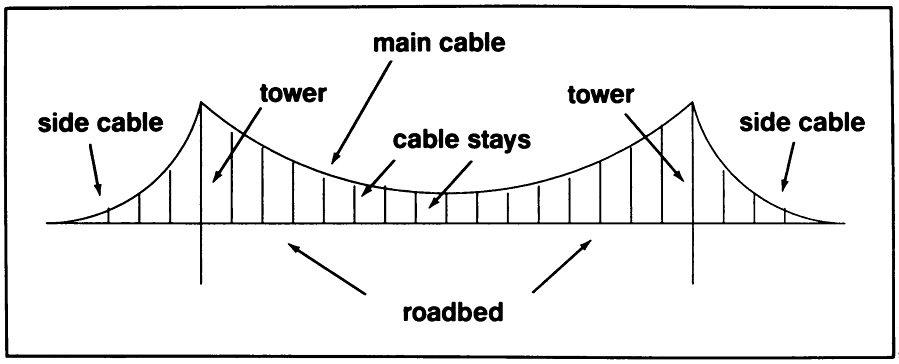
Photo above: On Nov. 7, 1940, gusting winds induced twisting motions in the Tacoma Narrows suspension bridge, causing its collapse only four months after the bridge had been opened to traffic.

In 1854, winds completely destroyed the roadway of a 1,010-foot suspension bridge across the Ohio River at Wheeling, W. Va. An eyewitness wrote that the structure lunged like a ship in a storm, finally crashing into the waters below.

There are a number of other examples. In some cases, the bridges didn't actually shake themselves to pieces, but the oscillations grew large enough that a traveler crossing the bridge would get seasick.

What distinguished the design of the Tacoma Narrows bridge from its predecessors was the extreme flexibility of its narrow, thin, two-lane roadbed. Unfortunately, this graceful, streamlined design gave the bridge a pronounced tendency to oscillate vertically under widely varying wind conditions. Even before its completion, several workers had felt seasick as a result of its motion. Later, thrill-seeking motorists would come just for the novelty of driving over "Galloping Gertie's" undulating surface.

Engineers tried to correct the problem but failed. Then, early in the morning of Nov. 7, 1940, with a stiff breeze blowing at roughly 40 miles per hour, the undulations became more serious. Officials closed the bridge at 10 a.m. — just before it began twisting itself to pieces.



McKenna

The main ingredients for a one-dimensional model of a suspension bridge.

Lazer and McKenna say a complete mathematical explanation for the Tacoma Narrows disaster must isolate the factors that make suspension bridges prone to large-scale oscillations; show how a bridge could go into large oscillations as the result of a single gust and at other times remain motionless even in high winds; and demonstrate how large vertical oscillations could rapidly change to a twisting motion.

One significant clue lies in the behavior of the vertical strands of wire, or stays, connecting the roadbed to a bridge's main cable, McKenna says. Normally, those stays would remain in tension under a bridge's weight. Civil engineers usually assume that the stays always remain in tension, in effect acting as rigid rods. That allows engineers to use relatively simple, linear differential equations to model the bridge's behavior.

However, when a bridge starts to oscillate, those stays begin alternately loosening and tightening. That produces a nonlinear effect, changing the nature of the forces acting on the bridge. When the stays are loose, they exert no force, and only gravity acts on the roadbed. When the stays are tight, they pull on the bridge, countering the effect of gravity.

The nonlinear differential equations that correspond to such an asymmetric situation have much less predictable solutions. "Linear theory says that if you stay away from resonance, then in order to create a large motion, you need a large push," McKenna says. Nonlinear theory says that for a wide range of initial conditions, a given push can produce either small or large oscillations.

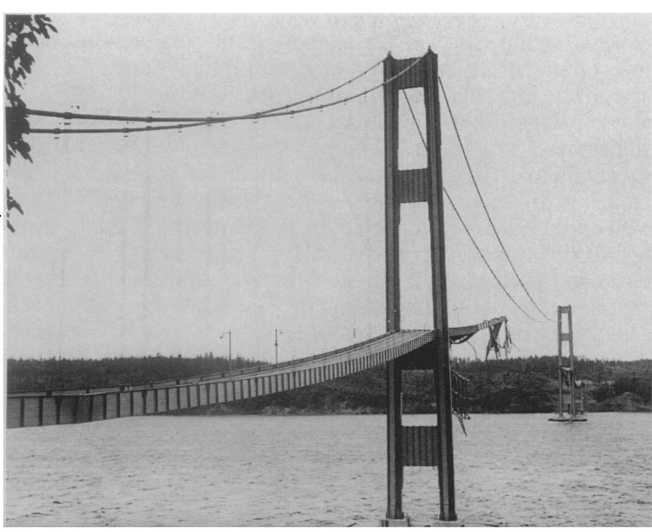
"You can think of it as being between two peaks on a mountaintop," McKenna explains. "You might slide down one side and end up in a small-amplitude solution, but you might equally well slide down the other side and end up on a large-amplitude solution."

According to nonlinear theory, a suspension bridge can respond to a whole range of forcing frequencies. "It's the opposite of saying that the [forcing] frequency has to match exactly the [bridge's] natural oscillation," McKenna says.

He and Lazer also find that their nonlinear equations yield mathematical solutions corresponding to waves traveling up and down a bridge's roadbed. On several occasions, the Golden Gate bridge has exhibited traveling waves that start at one end and ripple along the pavement to the other end. In one incident on a windy day in 1938, the bridge's chief engineer reported observing a cluster of ripples traveling down the roadway — a wave-like motion similar to the cracking of a whip.

Suspension bridges built or remodeled after 1940, including the Golden Gate, are unlikely to suffer the same stormy fate as the Tacoma Narrows bridge. Civil engineers responded to the Tacoma disaster by stiffening existing bridges and building new bridges heavy and rigid enough to resist wind-induced motion. In fact, the structure replacing the original Tacoma Narrows bridge has a much heavier, four-lane roadway. Because such bridges naturally flex very little, a linear analysis suffices. Only when flexibility becomes an issue and a bridge moves so much that its stays start loosening does the nonlinear theory come into play.

"So long as you build a big, heavy, rigid bridge, it's not going to get into the range where the cables are slackening without an incredible input of energy," McKenna



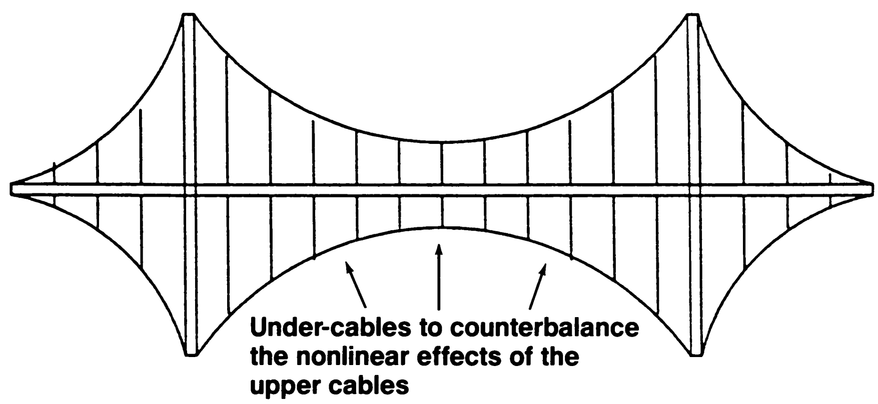
Photos: Tacoma Public Library

A heavier, stiffer bridge (right) replaced the original Tacoma Narrows bridge (left).

says. "If you don't get into the large oscillations, then you will never see any of this nonlinear behavior."

There is, however, one possible energy source that could send such a bridge into large-scale oscillations. "An earthquake is precisely the sort of energy source that will put you into the nonlinear mode," McKenna says.

Last fall, the Golden Gate bridge went into large-scale oscillations during the magnitude 7.1 Loma Prieta earthquake. The bridge oscillated for about a minute, roughly four times longer than the earthquake itself lasted. One witness noted that the stays connecting the roadbed to the main cables were alternately slackening and tightening "like strands of spaghetti" – a sign that the bridge was in a nonlinear state. Fortunately, the bridge didn't start twisting, perhaps because the earthquake waves hit it head-on rather than obliquely, McKenna says.



McKenna

A bridge equipped with cables above and below the roadbed may be less prone to large-scale oscillations.

Lazer and McKenna emphasize that they don't yet have a complete, realistic mathematical model of suspension-bridge behavior. For example, their current model doesn't take into account the natural frequencies of the towers supporting the main cables or the main cables themselves, which also vibrate to some degree. And they haven't satisfactorily answered the question of where the twisting comes from, although their model provides some hints.

"However, if [our] simple model exhibits unexpected, complex oscillatory behavior, then one can reasonably expect a more accurate model to do so," Lazer and McKenna say.

According to their simple model, gusts of wind initially act as a random buffeting force on a suspension bridge, causing the towers and main cable to go into a high-frequency periodic motion like that of a randomly struck guitar string. That motion initiates low-frequency, vertical oscillations that ripple the roadbed.

The sudden transition from vertical oscillations to a twisting mode is more difficult to explain. Using computer simulations, McKenna has shown that a rod suspended from cables that behave nonlinearly can unexpectedly start twisting. Analyses of more realistic models should bear this out, he says.

"This, we feel, is the likely explanation of the destruction of the Tacoma Narrows bridge," Lazer and McKenna conclude. "An impact, due either to an unusually strong gust of wind, or to a minor structural failure, provided sufficient energy to send the bridge from one-dimensional to torsional [modes of oscillation]." The resulting twisting destroyed the bridge.

The nonlinear theory also suggests an intriguing new design for lightweight, inexpensive suspension bridges. In conventional suspension bridges, nothing keeps a stay from slackening during an oscillation. One possible answer is the installation of two sets of stays and cables: one set from which the roadway hangs and a second set that pulls down on the roadway from below. That modification would make the forces acting on the bridge more symmetric and less nonlinear.

"The mathematics predicts that this should work," McKenna says. Such a bridge would be less likely to oscillate wildly.

A suspension bridge built in 1850 across the Niagara River gorge provides a historical precedent for the efficacy of tie-down cables. Originally, the bridge was stabilized by a set of cables running from the roadway to the sides of the gorge. The structure survived without incident until the spring of 1864, when engineers temporarily removed the cables to keep them from snaring chunks of ice from the breakup of an unusually heavy ice jam. A heavy wind destroyed the bridge before the cables could be restored.

Nonlinear effects similar to those influencing suspension bridges also arise in flexible ships, especially when they're lightly loaded and ride high in the water. In a storm, waves can lift much of the ship out of the water. At that point, its behavior becomes nonlinear, and subsequent wave action can cause the entire ship to oscillate.

"Most ships probably won't go into this mode because they're very large and extremely rigid," McKenna says.

Nevertheless, nonlinear theory may explain a number of marine disasters, including the famous case of the *Edmund Fitzgerald,* which mysteriously sank in a storm on Lake Superior in November 1975. As a Great Lakes carrier, the vessel had considerably more flexibility than an oceangoing freighter. One explanation for the tragedy holds that the *Edmund Fitzgerald* dove into a "wall of water" and never recovered. Lazer and McKenna speculate instead that the ship sank after it went into a large-scale flexing motion.

"This would also account for one of the most puzzling aspects of the case, namely why the ship was broken not at its midpoint but at two points approximately 80 feet from the midpoint," Lazer and McKenna say. That's precisely what would happen if the vessel oscillated in one of the modes that appears as a solution of the simple nonlinear equation they use to model ship behavior.

The extremely light, large, flexible structures now on the drawing board for space applications pose even more serious problems. "If we can't even do it for a suspension bridge yet, [the designers] are going to have a great deal of difficulty predicting the nonlinear oscillations they're going to get in these structures," McKenna says.

Explorations of nonlinear theory require extensive use of computers. "What's happening now is that we can look at a very simple mechanics problem, like a rod suspended at both ends by nonlinear elastic cables, and we can search for interesting behavior on the computer," McKenna says. "When we find it, we can make predictions and prove theorems that would never have occurred to us before."

Step by step, that process leads to more realistic models of suspension bridges and other structures. "We start with some abstract results [theorems]," McKenna and Lazer write in their paper. "These results in turn are confirmed by the numerical calculations, which suggest both new theorems and new applications of engineering interest. Finally, these results make suggestions about how large structures should be constructed, or how ships should be handled at sea."

"Five years from now, I would like to have a complete model of a suspension bridge," McKenna says. "But that's going to be a big job." □