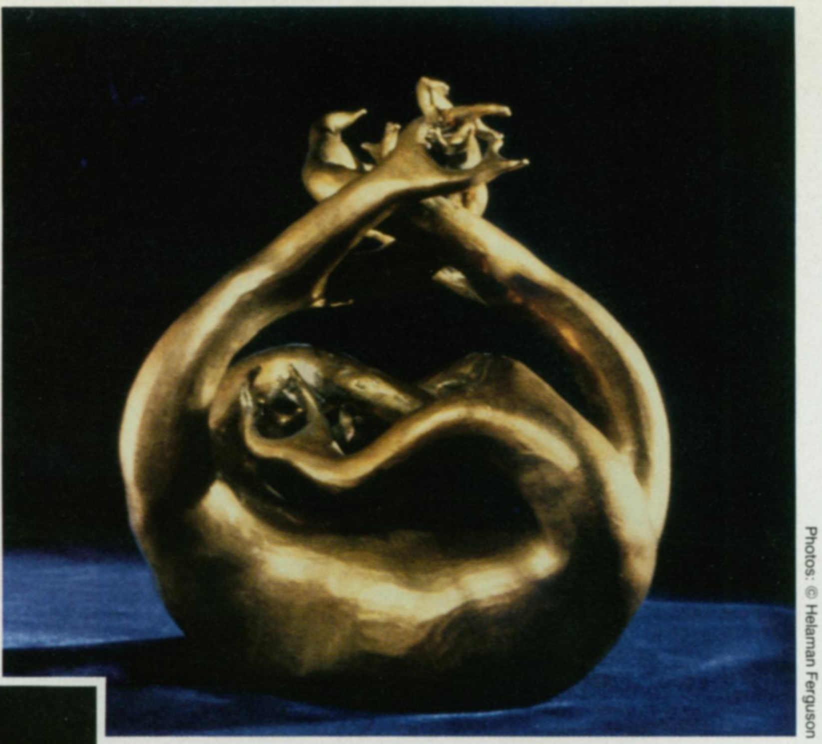
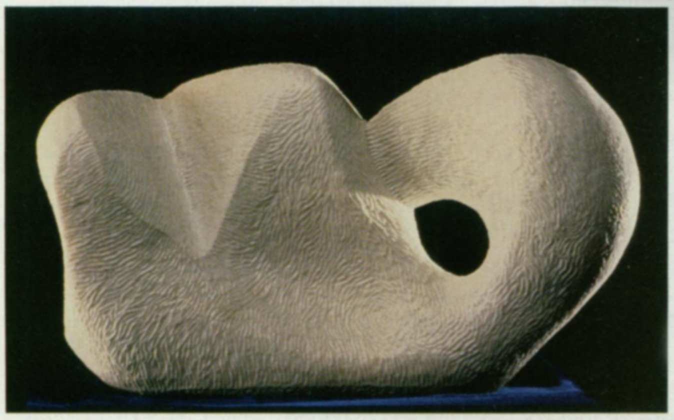
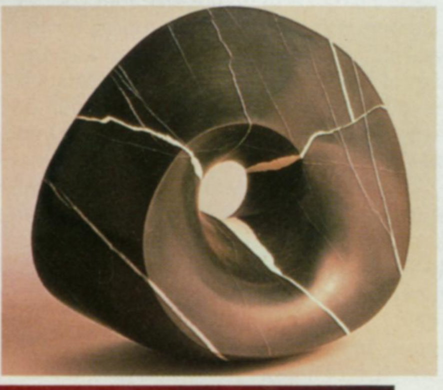
Equations in Stone





A mathematician turns to sculpture to convey the beauty of mathematics

By IVARS PETERSON



Helaman Ferguson's sculptures exhibit a variety of mathematical forms. From top to bottom: "Wild Sphere," "Klein Bottle with Cross-Cap and Vector Field," "(3,1) Torus" and "(1,5) Umbilical Torus with Vector Field."

A wild sphere flings its bronzed arms into the sky. A twisted ring pulls itself free from a slab of white Carrara marble. A glistening, silky-smooth knot writhes across the ground. A metallic eye blinks in harsh sunlight.

Each of these creations, realized in stone and bronze by mathematician and sculptor Helaman R.P. Ferguson, represents a concrete expression of a mathematical theorem. Each is an attempt to embody — in a form that anyone can see and touch — the special sense of beauty that mathematicians experience in their explorations of abstract realms.

"Aesthetic pleasures are a major motivation [in mathematical research], but we usually don't bring that out," Ferguson says. "I am interested in the adventure of affirming pure mathematical thought in unpredictable physical form."

Compelled to communicate mathematical beauty in tangible terms, Ferguson took an extended leave of absence nearly two years ago from Brigham Young University in Provo, Utah, where he taught mathematics. He now spends as much time as he can immersed in his art at his new home and studio near Laurel, Md. Last month, he exhibited 21 of his sculptures in Columbus, Ohio, at a meeting celebrating the 75th anniversary of the Mathematical Association of America.

"Mathematics is both an art form and a science," Ferguson writes in the August-September AMERICAN MATHEMATICAL MONTHLY. "I believe it is feasible to communicate mathematics along aesthetic channels to a general audience." His article describes in considerable detail how he created two of his best-known mathematical sculptures.

Umbilic torus

At first glance, Helaman Ferguson's "Umbilic Torus NC" looks like an artifact excavated from an ancient Maya, or possibly Chinese, ruin. The sculpture's burnished edges and inscribed surface seem to carry a cryptic message from bygone days.

This textured, twisted bronze ring, which stands about 27 inches high, actually represents a unique blend of the old and new. Its basic form emerged from mathematical formulas involving specific arrays of numbers, or matrices, associated with particular kinds of mathematical expressions known as "homogeneous cubic polynomials in two variables." Computer graphics allowed Ferguson first to create a variety of images and then to select a pleasing visual representation of these formulas. A specially programmed milling machine produced the intricately inscribed surface, and an ancient bronze casting technique rendered the final object.

The sculpture displays a number of intriguing mathematical properties. The mathematical term "umbilic" refers to the particular way in which this torus is twisted. Moreover, like a Möbius band, this shape has a single edge. Tracing the ring's edge carries your finger three times around the ring before returning it to its starting point. The ring's cross section corresponds to a curve called a hypocycloid, which is the path followed by a point on the circumference of a small circle that in turn is rolling inside the circumference of a larger circle.

The surface relief pattern approximates a fractal curve, which winds in and out to cover the whole surface. In fractal curves, large-scale patterns repeat themselves on ever-smaller scales so that a magnified image of a portion of such a curve looks just like the overall curve.

Ferguson is not the first to exploit the link between mathematics and art. Aesthetic considerations have long played a role in the development of mathematics, and mathematical ideas continue to inspire the work of both artists and artisans.

Mathematicians have a distinctive sense of beauty. They strive not just to construct irrefutable proofs but to present their ideas and results in a clear and compelling fashion, dictated more by a sense of aesthetics than by the needs of logic. And they are concerned not merely with finding and proving theorems, but with arranging and assembling the theorems into an elegant, coherent structure.

In the words of mathematician and philosopher Bertrand Russell, "Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the trappings of paintings or music, yet sublimely pure, and capable of stern perfection such as only the greatest art can show."

This sense of mathematical beauty remains foreign to most nonmathematicians. Without a well-trained eye, it's difficult to appreciate the bare-bones beauty of a theorem or an equation.

Artists have helped bring some of that beauty out of its cerebral closet. For example, the intriguing drawings of Dutch graphic artist M.C. Escher skillfully convey the illusion of infinitely repeating forms and the strange properties of hyperbolic planes. Other artists have dabbled in representations of four-dimensional forms. And a growing number now use the computer as a tool for creating art.

Ferguson's interest in art emerged early in his life. He learned to work with stone as a young apprentice to a stonemason, studied painting as an undergraduate and completed several graduate courses in sculpture.

Along the way, he also earned a doctorate in mathematics, later teaching the subject at Brigham Young University for more than 17 years and publishing research papers on a number of mathematical topics. To earn a living and support his artistic efforts, Ferguson currently designs new computer-based numerical recipes, or algorithms, for operating machinery and for visualizing scientific data. He's particularly interested in developing methods for "sculpting" data into forms that are meaningful and useful to researchers trying to cope with the overwhelming amounts of information that a supercomputer can produce.

But Ferguson's real passion is sculpture. He sees his work as a dialogue between abstract thoughts and physical materials. Mathematical ideas guide his hands, which in turn respond to the surprising patterns and flaws that lurk at the heart of a stone slab or emerge from a fiery brew of molten metal.

Because mathematical theorems are eternal, or "time invariant," he prefers to work with materials that survive on geological time scales. His favorite medium is stone, and the huge chunks of imported Carrara marble resting heavily on his front lawn testify to its allure.

For some sculptures, he uses traditional techniques and equipment similar to those Michelangelo employed more than four centuries ago. Ferguson's basic technique is called direct carving.

"I begin with raw material such as stone and systematically destroy the parts I don't want to see or feel," he says. "It is a subtraction process." His tools range from hand-held, chisel-tipped pneumatic hammers and special saws to various grinding wheels and polishing disks.

"With a stone, you really don't know what's in there until you get into it, particularly if it has a lot of grain or texture or different colors," Ferguson says. "These are all accidental things that you can't do anything about but which you can make use of in the process of creating a work of art. Part of the direct-carving-in-stone adventure is to make appropriate use of what you find."

At the same time, Ferguson has a hand

in the new world of computer-aided manufacture. He is testing an experimental, computer-guided system that allows him to audition his chosen mathematical form on a computer screen before committing it to stone, and then to make changes in the design as he carves away the stone and responds to its particular character.

This unique sculpting system, developed by researchers at the National Institute of Standards and Technology in Gaithersburg, Md., first senses the stone's location and orientation. Then, supplied with the equations defining the mathematical figure Ferguson wishes to sculpt, the computer displays point by point how much stone the sculptor must yet cut away to attain the desired form.

Because the system is interactive, Ferguson can modify the design at any time, and the computer quickly incorporates the new information and changes its instructions accordingly. "When carving stone, things occur that you can't predict," he says. "That's part of the excitement in creating something out of a physical material."

This kind of technology may prove useful in industrial applications for machining intricate parts or using exotic materials. "The major difference between my sculpture and industrial parts is that superficially an aesthetic object tends to be more complex and interesting than a functional part," Ferguson says. Thus, solving the difficult problems of creating an aesthetic artifact helps solve the comparatively easier problems of automating the manufacture of a machine part.

Ferguson's dusty basement studio is filled with the results of his dialogues between materials and mathematical theorems — sculpted geometrical forms that go by exotic names: Klein bottles, trefoil knots, cross-caps, horned spheres and tori. These figures play important roles in topology — the branch of mathematics dealing with the fundamental characteristics of geometric shapes that remain unchanged even when the shape itself is stretched, twisted or otherwise distorted, so long as the transformation doesn't involve tearing or breaking.

In topology, for example, the surface of a sphere and the surface of a bowl belong to the same category of geometrical objects because it's possible to deform a sphere's surface into a bowl's surface without tearing it apart. In contrast, the surface of a doughnut, or torus, belongs to a different category because there's no way to turn a sphere's surface into a doughnut's surface without performing surgery of some kind.

Ferguson's sculptures often involve what mathematicians call non-orientable surfaces. The simplest example of such a surface is a Möbius band. This remarkable surface, constructed by gluing together the two ends of a long strip of paper after giving one end a half twist, has only one side and one edge. Both the cross-cap and the Klein bottle offer somewhat more complicated examples of non-orientable surfaces.

"Most of my work deals with very fundamental [mathematical] ideas," Ferguson says. That often involves questions of category and equality: When are two things really the same (or equal)? When do they belong in different categories? What do we mean when we use symbols to define something?

But the sculptures transcend mere models of precisely defined mathematical objects. Ferguson's work has a warmth and softness that belies its cold, inorganic origins. The freely flowing lines, subdued details and references to organic forms characteristic of his sculptures bring in a human element, imparting a familiar, tactile quality to esoteric mathematical ideas.

"We could just make a model . . . , as exactly as physically possible," Ferguson writes. "When we got done, we would have just that — a model. Nothing more, something less. A sculpture can have . . . nuance, timbre, mystery, warmth, history, numerous levels of meaning, and many references other than what its humble origins would hope or explain."

Photographs fail to capture many of the elements crucial to appreciating Ferguson's pieces. The textures and contours of his sculptures invite viewers to touch their surfaces and trace their designs. And only by walking around a piece to see it from different vantage points can a viewer sample its full spectrum of patterns. Each new look brings startling revelations.

Ferguson's sculptures are starting to appear in public. His "Torus with Cross-Cap and Vector Field" — a 550-pound, gracefully textured and smoothly contoured sculpture in pure white marble — stands at the headquarters of the American Mathematical Society in Providence, R.I. A few other pieces grace university campuses. A number of corporations, private collectors and a few of Ferguson's colleagues have also obtained examples of his work.

Ferguson's sculptures allow non-mathematicians to experience some of the pleasure that mathematicians take in their work and to share in the wonder they feel. "We see our [mathematical] beauty in the mind," says mathematician James W. Cannon of Brigham Young University. "It is wonderful to have someone show the rest of the world how beautiful it is."

"Mathematics has tended to be largely unknown, partly because few mathematicians seem willing to expend the effort to communicate with less prepared souls than themselves," Ferguson says. "I have seen people change their feelings about mathematics dramatically while viewing one of my sculptures."

He adds, "I have seen their feelings about art and its possibilities change, too." □

Whaledream

Imagine a sphere with two arms reaching out to each other. Then imagine that those two arms have a pair of fingers, and that each of these fingers has a pair of smaller fingers, and so on. Now intertwine the fingers so that one arm appears inextricably linked to the other. The result is a "wild" figure known in mathematical circles as the Alexander horned sphere, named for mathematician J.W. Alexander, who in the 1920s first visualized this strange, topological form.

Helaman Ferguson has spent a great deal of time and effort trying to tame this fearsome beast — creating several versions of the Alexander horned sphere in both marble and bronze. "Whaledream II," carved out of white Carrara marble, is one of his more soothing realizations. This large sculpture, weighing about 550 pounds and standing about 15 inches high on a 30-inch base, represents the first 2¾ stages in the construction of the horned sphere, extending only to the first set of fingers, which appear clasped in an ethereal handshake.