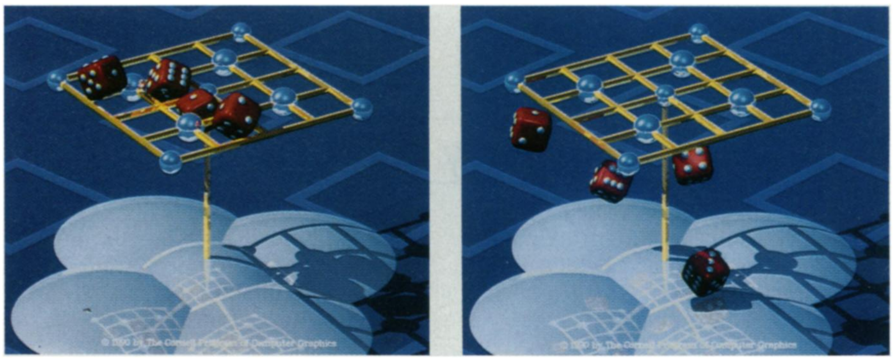
Recipes for Artificial Realities

Intriguing images emerge from a blend of mathematics, physics and computer graphics

By IVARS PETERSON



Two frames from Baraff's simulation of dice rattling through a grid.

Start with a blank computer screen. Throw in an equation or two. Fold in the relevant physical laws. Leaven with a pinch of intuition. Then watch the ensuing electronic images unfold.

Recipes that blend ingredients from mathematics, physics and computer graphics have turned computers into powerful tools for transforming information into pictures. Guided by carefully crafted instructions, computers can animate abstract concepts, display familiar objects and create new worlds beyond the realm of human experience — sometimes with startling results.

The technology for creating graphic images is rapidly changing the way researchers use computers in science and mathematics by providing increasingly sophisticated techniques for visualizing data and simulating physical reality. Conversely, new ideas emerging from mathematical and scientific research enrich the world of computer graphics.

To create the illusion of motion in a cartoon, an artist typically draws a sequence of images, changing the positions of objects frame by frame. But computer scientist David Baraff, a graduate student at Cornell University in Ithaca, N.Y., takes a different approach. In his visual simulations of dice rattling through a grid, bowling balls scattering pins, a child's ball-tipped jack clanking down stairs, and tennis balls and pencils tumbling around a room, he lets physics do the work.

"In computer graphics, people have gotten interested in doing things that are based on physical models," Baraff says. "At the most basic level, objects shouldn't go through one another. They should be solid. That's where I started."

Baraff's computer program uses Newton's laws of motion to define the trajectories of moving objects and relies on special, newly developed mathematical methods for calculating the forces between objects that touch or collide. "I don't specify the motions of the objects," Baraff says. "I place the objects, and the motion is created from the rules built into the simulator."

In his simulations, the computer establishes at each instant which objects in a scene have bumped against one another, and then computes the contact force between them. That analysis determines how the objects may bounce, roll or slide past each other. Baraff developed an efficient method, or algorithm, for accurately computing the forces between objects, even when their surfaces are curved rather than flat and regardless of the contact angle between them.

"Computing the forces between the objects — those forces that stop them from going through one another — actually turns out to be a fairly hard problem," Baraff says.

To speed up the generation of a sequence of images conveying movement, Baraff's algorithm uses information about the positions of objects at a single point in time to compute rapidly where the objects would be at the next instant. In other words, the computer doesn't need to create each new image from scratch. Because very little actually changes from one image to the next, information gleaned from the preceding scene hastens the generation of the next image in the sequence.

Baraff's main interest involves studying the factors that seem to limit the efficiency and speed of computer algorithms for detecting collisions between objects and for computing contact forces. His novel techniques for simulating motion realistically may prove useful in the classroom, allowing students to probe the role of friction and other factors in physical systems more complicated than those traditionally described in textbooks or investigated in the laboratory.

"You can control various things," Baraff says. "You can specify how slippery or sticky things are. You can certainly specify the distribution and masses of objects. You can also display the motion from any angle."

Baraff describes his method in the August COMPUTER GRAPHICS, which contains the proceedings of SIGGRAPH '90, a conference held in Dallas last summer.

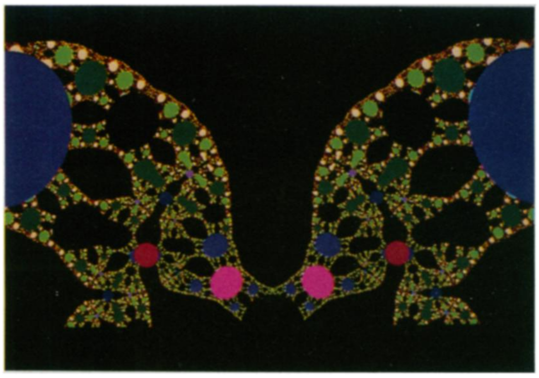
It's easy to imagine filling ordinary, three-dimensional space with a cubic scaffolding. One sees row upon row of cubes, each with six square sides. In such a lattice, the beams making up the structure always meet at right angles or lie parallel to each other. Like the squares on a sheet of graph paper, this cubic framework serves as a convenient background against which to depict, locate, measure and analyze mathematical objects.

But mathematicians sometimes find it more convenient and useful to study mathematical objects in a curved, or hyperbolic, space, where the rules of geometry differ from those we normally encounter. For example, the sum of the angles within a triangle is less than 180° in hyperbolic space, whereas the sum is exactly 180° in ordinary, Euclidean space.

Hence, a scaffolding that fills hyperbolic space, and serves as a suitable reference grid for that space, can't be cubic. Instead, one must try to imagine the effect of filling the space with a latticework of dodecahedra, each having 12 pentagonal faces (see cover).

When viewed in the Euclidean space of a computer screen, this structure appears strangely distorted. In particular, the framework beams look curved, because parallel lines in hyperbolic space don't

Pickover's mathematically derived, graphic "sculptures" correspond to equations that play important roles in modeling physical phenomena.



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remain the same distance apart as they do in Euclidean space.

Depicting such a hyperbolic framework accurately is no simple matter. Because the rules of hyperbolic geometry differ from the geometric rules that underlie most conventional computer-graphic techniques, graphics expert Charles Gunn had to develop special techniques for creating his images of structures in hyperbolic space.

"Algorithms for ordinary computer graphics make certain assumptions about angles that aren't true for this particular [way of representing] hyperbolic space," says Gunn, who works on the Geometry Supercomputer Project at the University of Minnesota at Minneapolis-St. Paul.

Visualization of hyperbolic geometry plays an important role in an innovative attempt by mathematician William P. Thurston of Princeton (N.J.) University to classify three-dimensional surfaces, or three-manifolds. These difficult-to-imagine shapes — the surfaces of four-dimensional objects — can take on a bewildering array of complicated forms, and their classification has stymied many a mathematician.

Thurston hopes that studying the details of how such manifolds "fit" into hyperbolic space will make it possible to prove that all conceivable three-manifolds of a certain type fit into a strictly limited number of categories. Moreover, studies of manifolds in general provide insight into the nature of the equations used to describe and model physical phenomena.

Pictures displayed in hyperbolic space also make it easier to communicate tricky mathematical concepts to colleagues, students and others, Gunn adds.

Computer scientist Clifford A. Pickover of the IBM Thomas J. Watson Research Center in Yorktown Heights, N.Y., is one of the more prolific creators of computer graphic images, using an astonishing range of mathematical and computational tools to create what he calls "mathematically derived sculptures." Pickover argues that computer graphics can help both artists and scientists extend their imaginations and break through the constraints imposed by physical systems.

Many of his most striking images emerge from chaos theory, a new discipline at the intersection of mathematics and physics. "Chaos theory is an exciting, growing field which usually involves the study of a range of phenomena exhibiting a sensitive dependence on initial conditions," Pickover writes in the September/October COMPUTERS IN PHYSICS. "Although chaos often seems totally . . . unpredictable, it . . . obeys strict mathematical rules derived from equations that can be formulated and studied."

Chaos researchers investigate the behavior of differential equations, which describe the way a certain quantity changes over time or varies across space. For example, a particular set of differential equations based on Newton's laws of motion models the movement of the planets orbiting the sun. Using differential equations expressed in an appropriate form, computers can calculate step by step the future behavior, or trajectory, of any physical system described by such an equation.

Pickover employs a variety of graphic techniques to picture what happens to the trajectories for different starting points, especially when the equations lead to irregular behavior. For each image, the computer generates sequences of overlapping spheres to trace out the twisted curves defined by the given mathematical formula. Pickover then graphically "dresses up" his creations, often giving them a wet, shiny look.

Out of these ingredients, he can create a veritable zoo of bizarre creatures. One set of formulas, for example, produces graphic depictions of seashell-like forms. Others generate gaping, toothless mouths, plump, glistening worms and butterfly wings speckled with brilliant color. □