Plastic Math

Growing plastic models of mathematical formulas

By IVARS PETERSON

The graphic images that Stewart Dickson creates on a computer screen are but pale shadows of the vivid, three-dimensional reality they represent. No amount of shading, color, animation or other forms of graphic prestidigitation brings such images truly to life.

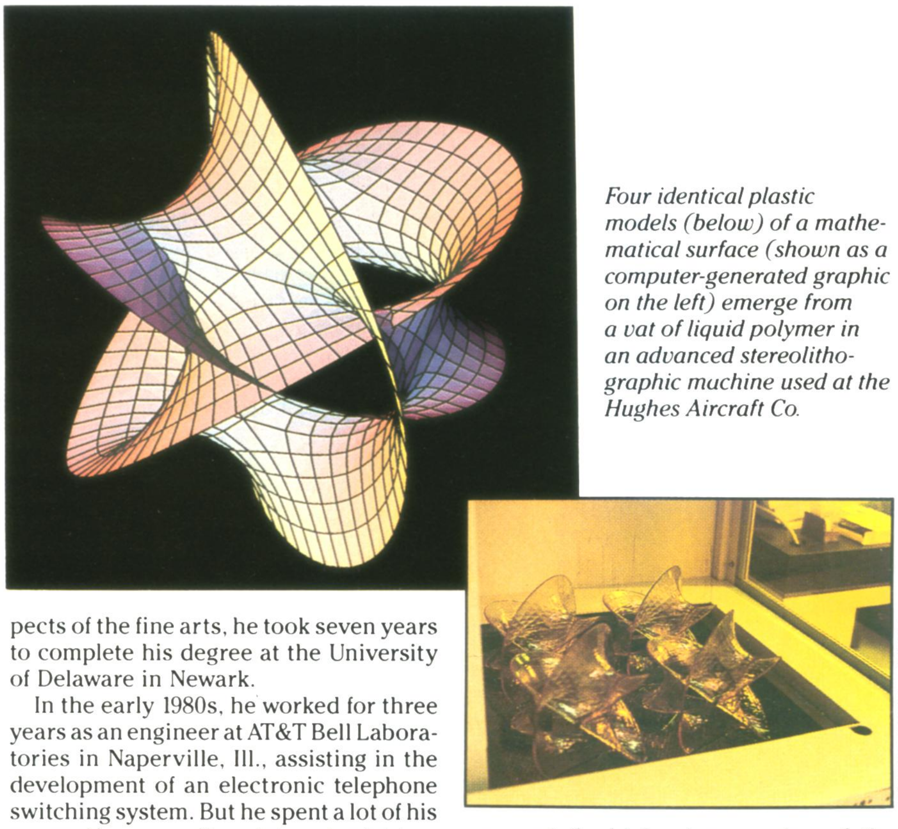
Dickson's frustration with the inherent flatness of two-dimensional displays spurred him to turn to sculpture and to new technologies for creating models of mathematical forms in all their three-dimensional splendor. "A picture of a three-dimensional object is an abstraction," he insists. "A sculpture is not."

A recently developed technology known as stereolithography has now provided Dickson with the tools he needs to realize his vision of sculpting mathematics. By linking the power of computer graphics to the direct formation of a solid, shaped object, the technique permits a user to design an object on a computer screen, then generate a three-dimensional, plastic model. Already used for creating prototypes of commercial products ranging from perfume bottles to automobile wheels, stereolithography also shows promise as a method of visualizing mathematical shapes and scientific data.

A physical model can help you understand a complicated mathematical surface better than a picture of it on a two-dimensional screen, says Clifford A. Pickover of the IBM Thomas J. Watson Research Center at Yorktown Heights, N.Y. "You can hold it at whatever angle you like; you can put your fingers into the nooks and crannies," he notes.

To demonstrate stereolithography's versatility, Dickson has produced an array of exotic shapes based on a variety of mathematical equations. He described his most recent efforts at the Physics Computing '91 conference, held during June in San Jose, Calif.

Currently a computer programmer specializing in graphics and animation at The Post Group, a post-production video company in Los Angeles, Dickson started out in electrical engineering. Constantly sidetracked by his interest in sculpture and other aspects of the fine arts, he took seven years to complete his degree at the University of Delaware in Newark.



Four identical plastic models (below) of a mathematical surface (shown as a computer-generated graphic on the left) emerge from a vat of liquid polymer in an advanced stereolithographic machine used at the Hughes Aircraft Co.

Photos © Stewart Dickson

In the early 1980s, he worked for three years as an engineer at AT&T Bell Laboratories in Naperville, Ill., assisting in the development of an electronic telephone switching system. But he spent a lot of his spare time — often late at night — experimenting with sophisticated graphics devices and laser printers to produce a variety of images.

His outside activities weren't confined entirely to the visual arts. Dickson's interest in electronic music sparked a novel design for a stringed musical instrument, for which he received a patent in 1981.

By 1984, Dickson was deeply immersed in computer-generated imagery, working first for a design and film company in Illinois, then coming in 1988 to The Post Group, which provides special effects and other video production services.

"It was also the year that stereolithography first hit the press, and I got right on it," Dickson says.

That year marked the introduction by 3D Systems, Inc., in Valencia, Calif., of commercial equipment suitable for building solid plastic objects from computer-aided designs. The process involves scanning a computer-guided beam of ultraviolet laser light across the surface of a photosensitive liquid, thereby turning to solid any areas exposed to the beam.

The creation of a three-dimensional plastic model begins in a vat of a liquid polymer resin such as an acrylate. An overhead laser beam "draws" a layer, corresponding to a slice or cross section through the object, on the liquid's surface, just a fraction of a millimeter above a submerged, movable platform. This initial layer hardens and sticks to the platform, which then descends a fraction of a millimeter. The laser then draws a second layer on the fresh liquid surface, the exposed liquid hardens on top of the original layer, and the platform sinks another fraction of a millimeter. The process repeats itself until the object is complete.

Through his persistence, Dickson persuaded initially reluctant customer support engineers at 3D Systems to let him try generating several mathematical shapes using the new equipment.

The first object took nearly a week to build. "It was a solid object, and you had to scan its entire interior," Dickson says. "It was the first time this particular engineer had run into this [time] problem, but he succeeded in producing an object anyway."

Dickson's contribution lay in developing a method for converting a mathematical description of the surface of a three-dimensional object into terms that the

Trinoid (Jorge-Meeks):

$$x = \mathrm{Re}\int_{0}^{re^{i\theta}} \frac{1-\zeta^4}{(\zeta^3-1)^2}\, d\zeta\ ;$$

$$y = \mathrm{Re}\int_{0}^{re^{i\theta}} \frac{i\left[1+\zeta^4\right]}{(\zeta^3-1)^2}\, d\zeta\ ;$$

$$z = \mathrm{Re}\int_{0}^{re^{i\theta}} \frac{2\zeta^2}{(\zeta^3-1)^2}\, d\zeta\ ;$$

$$\overset{4}{\underset{0}{r}}\ ;\ \overset{2\pi}{\underset{0}{\theta}}$$

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(* The Global Enneper-Weierstrass representation of minimal surfaces *)
(* X := Re[Integrate[phi, {z, p0, p}]]; p = {u + I v} *)
Clear[phi]
Clear[X]
Clear[r]
Clear[th]
Needs["Graphics`ParametricPlot3D`"]
phi[f_, g_, eta_] := {(1 - g ^2)f, I (1 + g ^ 2) f, 2 f g} eta
X[z0_, f_, g_, eta_]:=
X[z0,  f, g, eta] = Re[Integrate[phi[f, g, eta], {z, z0, Z}]] /. Z->r Exp[I
Xeval = X[0, 1, z^2, (z^3 - 1) ^ -2]

ParametricPlot3D[Xeval, {th, 0, 2 Pi, Pi/30}, {r, 0, 4.0, 1 / 20.3},
        Boxed->False, RenderAll->False, LightSources ->
    {{{0.7071, 0, 0.7071}, RGBColor[0.9481, 0, 0]},
     {{0.5773, 0.5773, 0.5773}, RGBColor[0, 0.8888, 0]},
     {{0, 0.7071, 0.7071}, RGBColor[0, 0, 1]}},
        ViewPoint -> {0.7654, -0.9924, 1.5831}
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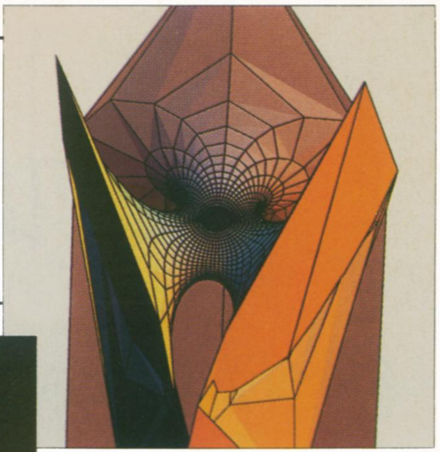


Photos © Stewart Dickson

The minimal surface known as a "trinoid" starts as a set of mathematical equations (far left), which are then converted into a Mathematica program (top middle) for generating a graphic image (above). Stereolithography transforms that image into a plastic model (left).

stereolithographic equipment can understand. Because mathematical surfaces have zero thickness, one key step involves manipulating the data to "thicken" a surface so that it can exist as a physical object. Another involves testing whether everything in the object is actually connected. Otherwise the object would fall apart.

"It's the typical computer graphics tradeoff between accuracy and practicality," Dickson says.

Nowadays, Dickson uses Mathematica, a software system for doing mathematics by computer, to design and specify his mathematical shapes and to create two-dimensional graphic images of the surfaces. A second computer program converts this output into the steps needed for building a three-dimensional plastic model.

Stereolithography isn't for everyone — at least, not yet. Depending on its size, each of Dickson's convoluted mathematical objects takes from 24 to 36 hours to generate using the most advanced machine available. At this rate, he estimates that it costs a minimum of $1,000 to produce a model 8 inches high.

Moreover, the choice of colors and types of plastics available to create three-dimensional models remains somewhat limited. And the equipment needed for producing the models is both expensive and bulky.

Researchers at 3D Systems and elsewhere hope to improve the process so that future stereolithographic machines can make larger parts faster and more accurately. The future may see the development of speedier, more compact machines that create objects with finer detail.

At the same time, stereolithography isn't the only technology now available for building three-dimensional models out of a sequence of layers. In laminated object manufacturing, a high-power laser cuts the outlines of an object's cross sections out of sheets of plastic, metal or paper. These layers are then glued or welded together.

Ballistics particle manufacturing constructs a layered object using a high-speed jet of tiny plastic beads that melt readily and bond together on impact. Laser sintering uses a high-power laser beam to fuse powdered plastic into a continuous solid, with fresh powder spread out atop the completed layers as the shape builds up.

Each scheme has its own commercial and creative advantages and disadvantages, Dickson says, but they all offer interesting possibilities for sculpting mathematical shapes.

Dickson's artworks — his "abstractions made concrete" — are starting to appear in public. Six of his finished pieces currently highlight an exhibition of electronic art in Finland. And about two dozen of Dickson's three-dimensional mathematical models are emerging one by one from a stereolithography machine on display at this summer's Heureka National Research Exhibition in Zürich, Switzerland.

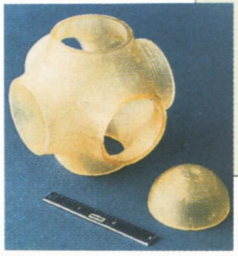
At the same time, Dickson is exploring various ways of expanding his repertoire of mathematical shapes, looking at different types of equations and pondering complicated forms that few have ever attempted to visualize or model.

"A physical entity promotes a more profound understanding than a flat image on a display screen," Dickson asserts.

Using stereolithography, one gets close approximations of objects that can be described mathematically but can't actually exist in the ordinary physical world, he says. In essence, stereolithography allows one to put the results of an abstract human process into physical form.

"Dickson's work speaks to the future of [scientific] visualization and the future of art," Pickover says. "You can hypothesize that in the future, rather than looking at a computer screen to visualize a scientific model, you'll be able to hit a button and walk away with a physical model. It's interesting to speculate what that capability would do for science." ☐

Dickson has created a scheme for constructing complicated lattices from a set of fundamental pieces. For example, he can build up a smoothed cubic lattice (shown on the right as a computer graphic) by joining together octahedral units (one unit shown below as a plastic model). Such lattices represent the extension of geodesic dome construction to nonspherical geometries.



Hughes Aircraft Co.

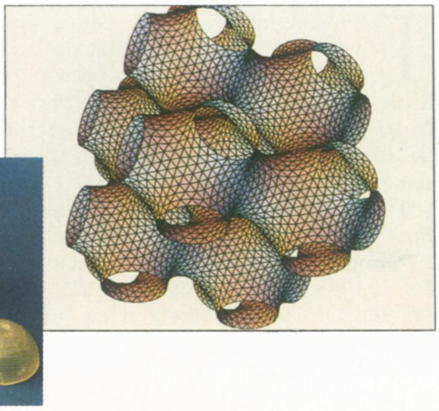


Photo of graphic © Stewart Dickson