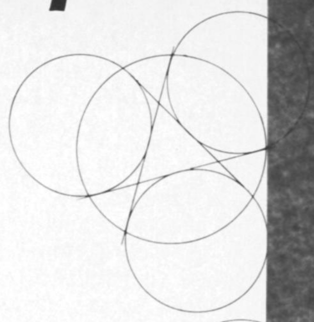
Euclid's Crop Circles

By IVARS PETERSON

It's no wonder that farmers with fields in the plains surrounding Stonehenge, in southern England, face late-summer mornings with dread. On any given day at the height of the growing season, as many as a dozen are likely to find a field marred by a circle of flattened grain.

Plagued by some enigmatic, nocturnal pest, the harassed farmers must contend not only with damage to their crops but also with the intrusions of excitable journalists, gullible tourists, befuddled scientists and indefatigable investigators of the phenomenon.

Indeed, the study of these mysterious crop circles has itself grown into a thriving cottage industry of sightings, measurements, speculations and publications. Serious enthusiasts call themselves cereologists, after Ceres, the Roman goddess of agriculture.



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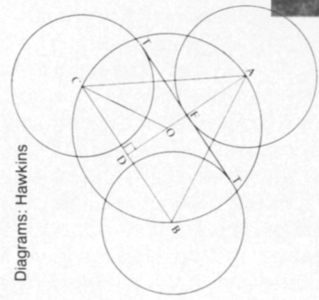


Diagrams: Hawkins

Hawkins' first theorem was suggested by a triplet of crop circles discovered on June 4, 1988, at Cheesefoot Head (above). Hawkins noticed that he could draw three straight lines, or tangents, that each touched all three circles (top left). By drawing in the equilateral triangle formed by the circles' centers and adding a large circle centered on this triangle (bottom left), he found and proved Theorem I: The ratio of the diameter of the triangle's circumscribed circle to the diameter of the circles at each corner is 4:3.

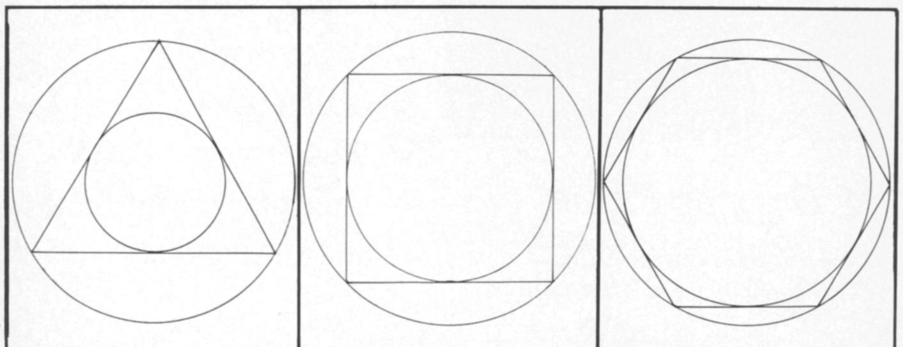
Most crop deformations appear as simple, nearly perfect circles of grain flattened in a spiral pattern. But a significant number consist of circles in groups, circles inside rings, or circles with spurs and other appendages. Within these geometric forms, the grain itself may be laid down in various patterns.

Explanations of the phenomenon range from the bizarre to the unnatural. To some people, the circles – which began appearing about a decade ago – represent the handiwork of extraterrestrial invaders, or crafty tradesmen bent on mischief after an evening at the pub, or even hordes of graduate students driven by a mad professor. To others, the circles suggest the action of microwave-generated ball lightning, numerate whirlwinds, or some other peculiar atmospheric phenomenon.

These scenarios apparently suffered a severe blow late last summer, when two elderly landscape painters, David Chorley and Douglas Bower, admitted to creating many of the giant, circular wheat-field patterns that cropped up over the last decade in southern England. The chuckling hoaxers proudly displayed the wooden planks, ball of string and primitive sighting device they claimed they had used to construct the circles.

But this newspaper-orchestrated, widely publicized admission didn't settle the whole mystery. Gerald S. Hawkins, a retired astronomer who now divides his time between an apartment in Washington, D.C., and a farm in Woodville, Va., felt compelled to write to Bower and Chorley last September, asking how they managed to discover and incorporate a number of ingenious, previously unknown geometric theorems – of the type that appear in antique textbooks – into what he called their "artwork in the crops."

He concluded his letter as follows: "The media did not give you credit for the unusual cleverness behind the design of the patterns."



Diagrams: Hawkins

Theorem II (left): For an equilateral triangle, the ratio of the areas of the circumscribed (outer) and inscribed (inner) circles is 4:1. The area of the ring between the circles is 3 times the area of the inscribed circle.

Theorem III (middle): For a square, the ratio of the areas of the circumscribed and inscribed circles is 2:1. If a second square is inscribed within the inscribed circle of the first, and so on to the m*th square, then the ratio of the areas of the original circumscribed circle and the innermost circle is* 2^m*:1.*

Theorem IV (right): For a regular hexagon, the ratio of the areas of the outer circle and the inscribed circle is 4:3.

Hawkins' first encounter with crop circles occurred early in 1990. Famous for his investigations of Stonehenge as an early astronomical observatory, he responded to suggestions by colleagues that he look into crop circles, which were defacing fields suspiciously close to Stonehenge.

Of course, there was no connection between crop circles and the stone circles of Stonehenge, but Hawkins found the crop formations sufficiently intriguing to begin a systematic study of their geometry. Using data from published ground surveys and aerial photographs, he painstakingly measured the dimensions and calculated the ratios of the diameters and other key features in 18 patterns that included more than one circle or ring.

In 11 of these structures, Hawkins found ratios of small whole numbers that precisely matched the ratios defining the diatonic scale. These ratios produce the eight tones of an octave in the musical scale corresponding to the white keys on a piano.

"That was surprise number one," Hawkins says.

The existence of these ratios prompted Hawkins to begin looking for geometrical relationships among the circles, rings and lines of several particularly distinctive patterns that had been recorded in the fields. Their creation had to involve more than blind luck, he insists.

His first candidate, which had appeared in a field in 1988, consisted of a pattern of three separate circles arranged so that their centers rested at the corners of an equilateral triangle (see illustration). Within each circle, the hoaxers had flattened the grain to create 48 spokes.

Hawkins approached the problem experimentally by sketching diagrams and looking for hints of geometric relationships. He found that he could draw three straight lines, or tangents, that each touched all three circles. Measurements revealed that the ratio of the diameter of a large circle – drawn so that it passes through the centers of the three original circles – to the diameter of one of the original circles is close to 4:3.

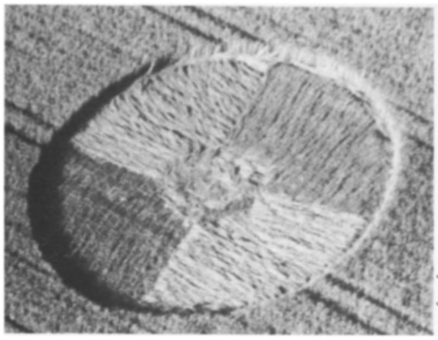
Was there an underlying geometric theorem proving that a 4:3 ratio had to arise in such a configuration of circles? Armed with his measurements and statistical analyses, Hawkins began pondering the arrangement.

"I ground on week after week – in the shower, while driving. Then, eureka, it came," he recalls. "And it's a very simple proof."

That was just the beginning. Over the next few months, Hawkins discovered three more geometric theorems, all involving diatonic ratios arising from the ratios of areas of circles, among various crop-circle patterns (see diagrams). For Hawkins, it was a matter of first recognizing a significant geometric relationship, and then proving in a mathematically rigorous fashion precisely what that relationship was.

"That was the approach I had taken at Stonehenge," Hawkins says. "It wasn't just one alignment here and nothing there. That would have had no significance. It was the whole pattern of alignments with the sun and moon over a long period that made it ring true to me. Once you get a pattern, you know it probably won't go away."

There was more. Hawkins came to realize that his four original theorems, derived from crop-circle



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This pattern, discovered Aug. 12, 1989, suggested Theorem III.

patterns, were really special cases of a single, more general theorem. "I found underlying principles – a common thread – that applied to everything, which led me to the fifth theorem," he says.

Remarkably, he could find none of these theorems in the works of Euclid, the ancient Greek geometer who established the basic techniques and rules of what is known as Euclidean geometry. He was also surprised at his failure to find the crop-circle theorems in any of the mathematics textbooks and references, ancient and modern, that he consulted.

"They really are not there," Hawkins says. "I found nothing close. I don't know where else to go."

This suggests that the hoaxer or hoaxers "had to know a tremendous lot of old-fashioned geometry," he argues.

Hawkins himself had the kind of British grammar-school education that years ago instilled a healthy respect for Euclidean geometry. "We started at the age of 12 with this sort of stuff, so it became part of one's life and thinking," he says. That doesn't happen nowadays.

The hoaxers apparently had the requisite knowledge not only to prove a Euclidean theorem but also to conceive of an original theorem in the first place – a far more challenging task. To show how difficult such a task can be, Hawkins often playfully refuses to divulge his fifth theorem, inviting anyone interested to come up with the theorem itself before trying to prove it.

"It's a good test," he says. "It's easy to prove the theorem but so difficult to conceive it."

What Hawkins now has is a kind of intellectual fingerprint of the hoaxers involved. "One has to admire this sort of mind, let alone how it's done or why it's done," he says.

Did Chorley and Bower have the mathematical sophistication to depict novel Euclidean theorems in the wheat?

Perhaps Euclid's ghost is stalking the English countryside by night, leaving his distinctive mark wherever he happens to alight. □