Photos: I. Peterson

Chaos in the Clockwork

Uncovering traces of chaos in planetary orbits

By IVARS PETERSON

There's something both reassuring and dreary about the notion of the solar system's planets whirling endlessly around the sun, keeping forever to their approximately elliptical orbits. But the solar system isn't quite as placid or predictable as this classic image suggests.

Strikingly different methods of computing and tracking the evolution of planetary orbits now strongly suggest that chaos lurks in the planetary clockwork. "Computers have been able to simulate the evolution of the orbits of the solar system over quite long times," says Peter Goldreich, a planetary scientist at the California Institute of Technology in Pasadena. "These simulations have uncovered evidence of chaos."

The term "chaos" refers to situations in which the behavior of a dynamical system – whether a pendulum or a collection of gravitationally interacting bodies – depends sensitively on initial conditions. In other words, in a chaotic system, a slight difference in a body's initial position and speed can drastically alter its predicted path.

The presence of chaos would mean that although the solar system has apparently survived for more than 4.5 billion years in some semblance of its present form, nothing guarantees that its future holds no surprises.

The new solar-system computations provide insights into one of the most perplexing, unsolved issues in celestial mechanics: Is the solar system stable? Will the planets continue tracing roughly the same paths they now follow billions of years into the future, or will a time come when Mars catastrophically smashes into Earth, or Pluto escapes the solar system? Could Earth itself drift close enough to the sun to become a twin of veiled, noxious Venus?

In one form or another, such questions have both fascinated and tormented astronomers and mathematicians for more than 200 years. Each step toward resolving them has exposed additional uncertainties and even deeper mysteries.

Nonetheless, spectacular advances in modern computing power have brought about a dramatic increase in the understanding of basic dynamics. Researchers adept at computation can now pinpoint the specific parts of the mathematical machinery that foreshadow irregular behavior, and they can relate these tendencies toward chaos to observed motions in the solar system.

"What's interesting here is not that the solar system is becoming erratic. It isn't," says Myron Lecar of the Harvard-Smithsonian Center for Astrophysics in Cambridge, Mass. "The problem is turning out to be much more fundamental. Even taking everything into account, there's an element you can't calculate. It's hard to make predictions far into the future."

Researchers described and discussed the implications of the new results at an International Astronomical Union symposium on "Chaos, Resonance and Collective Dynamical Phenomena in the Solar System," held last summer at Angra dos Reis in Brazil. Papers presented at subsequent meetings elsewhere furnished additional details.

For many, the need for high levels of computing power to unravel planetary motions appears puzzling at first glance. Ignoring the existence of asteroids, planetary satellites and other itinerant but minor bodies, the solar system consists essentially of just nine planets and the sun. The tug of gravity serves as the only significant force affecting their motion, and the mathematical expression relating this force to the masses and separations of the 10 bodies has been known for more than 300 years.

Given such a precise mathematical formulation, it would seem possible to calculate the positions of the planets at any time in the future and to explore what the laws of physics have in store for the solar system. But it isn't.

If the solar system consisted only of the sun and Earth, the answer would appear in the form of a precise mathematical formula specifying the two bodies' exact movements for all time. Any orbit in such an idealized, two-body system would be stable.

But add another planet to the sun-Earth system, and the situation changes – now there are three bodies, each tugging on the other two. Earth can no longer keep to its precisely elliptical path. It continues to wind around the sun, but depending on its distance from the other planet, Earth experiences different gravitational pulls at different times. Those perturbations distort its trajectory in space, just as Earth's influence perturbs its companion's orbit.

In this case, solving the differential equations representing three gravitationally interacting bodies spawns no simple mathematical formula that describes the paths of all three bodies with unlimited accuracy for all time. The problem grows worse with each additional planet. Traditionally, the best anyone could do was to calculate first the major effects – such as the sun's preponderant influence on each of the planets – then step by step take into account other, less significant perturbations. Such strings of approximations allowed mathematicians and astronomers to close in on the answers they sought.

The prodigious capacity, power and speed of modern electronic digital computers now enable scientists to tackle such questions more directly. Where no formula exists to specify the location of a planet at a given moment, a computer can by brute force calculate its course.

To focus on the solar system's long-term behavior, Jacques Laskar of the Bureau des Longitudes in Paris worked with equations that, in effect, smooth out the recurring wiggles and wobbles in planetary orbits. His "averaged" differential equations take into account long-term trends in the orbits of the eight main planets (only Pluto is missing) and include corrections for the effects of general relativity on planetary motions.

By using such a strategy, Laskar can isolate the parts of a planet's motion that correspond to lasting changes in key characteristics of its orbit, including its eccentricity. Numerically solving, or integrating, the resulting set of equations – which include about 150,000 algebraic terms – shows that the ability to predict the orbits of the inner planets, including Earth, declines sharply within a few tens of millions of years.

Computer simulations demonstrate that under these conditions, the orbits of two nearby bodies will diverge at an ever-increasing, or exponential, rate. In other words, they reveal that a body identical to

Earth but starting at a minutely different position in its orbit could easily end up on a very different trajectory.

Laskar also identified two specific, previously unknown interactions, or resonances, between the motions of certain planets in the inner solar system as the main source of this surprising chaotic behavior. One resonance involves Mars and Earth, and the other concerns Mercury, Venus and Jupiter.

This result doesn't necessarily mean that Earth is likely to wander from its usual path in the next 10 million years or so, perhaps ending up on a collision course with Mars or Venus. It does suggest, however, that the traditional mathematical tools of celestial mechanics would fail to predict such an event far in the future. In a chaotic system, there is no way to prove that something can't ever happen.

By itself, the Laskar result would have appeared interesting but probably inconclusive. However, Scott Tremaine of the Canadian Institute for Theoretical Astrophysics (CITA) at the University of Toronto and his collaborators, working independently of Laskar, used computers to solve directly the equations of motion for the solar system's planets, tracking the evolution of their orbits over a 6-million-year period. Although this computation couldn't directly confirm the chaotic behavior Laskar had noted, it picked up the resonance between Mars and Earth.

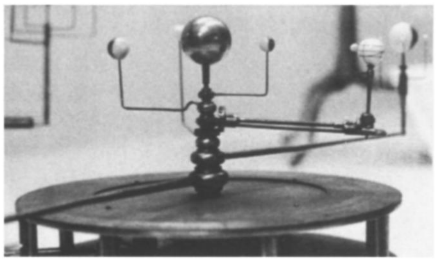
Such agreement provides strong, indirect support that chaos is actually present, Tremaine says.

"When you put together the contributions of Tremaine and Laskar, this is a very interesting, significant result. It's quite an achievement," says André Deprit of the National Institute of Standards and Technology in Gaithersburg, Md., who has made a number of important contributions to celestial mechanics and the computation of satellite orbits (SN: 2/24/90, p.116).

"It's not simply an observation of chaos coming from numerical integration," he adds. "There's also an explanation of it [in Laskar's work]." Laskar, Tremaine and Thomas Quinn of Oxford University in England describe this link in the January ICARUS.

Jack Wisdom and Gerald J. Sussman at the Massachusetts Institute of Technology performed a similar computational feat – using a new, custom-built machine known as the "supercomputing toolbox" to track planetary orbits over a 100-million-year period. These computations also confirm Laskar's results.

Several years earlier, Wisdom and Sussman had found a chaotic motion for Pluto when they used a different custom-built computer to solve the differential equations modeling the motion of the outer planets from Jupiter to Pluto. That solution, or integration, spanned 845 million years.



Mechanical model of orbiting planets and satellites.

The newest solar-system calculations reveal that the outer solar system, although not completely free of chaos, is actually much more regular than the inner solar system, Laskar says.

"If you had asked someone before any of this happened what planets are most likely to be chaotic, most people would have said Mercury or Pluto," Tremaine notes. "Yet the chaos that Laskar discovered involves planets like the Earth, Mars and Venus."

Because Laskar, Tremaine and Wisdom employed very different methods, the close agreement of the three results also furnishes an important, independent check on the techniques they used. Moreover, the calculations demonstrate for the first time that researchers may now have a reasonably reliable method of estimating the long-term variations in Earth's orbit over geological time scales. This information could be used to test theories that attribute major climate changes to minor changes in Earth's orbit.

The power to compute reasonably realistic planetary orbits over long time periods has prompted a new kind of study in which researchers can, in effect, set up fictitious solar systems to gain a sense of the variability allowed within a planetary system.

Gerald D. Quinlan of CITA, working with Tremaine, has traced the orbital evolution of ersatz solar systems consisting of four planets chosen as virtually identical in mass, position and orbit to Jupiter, Saturn, Uranus and Neptune. His simulations show, for example, that slightly shifting Saturn's position to widen its orbit can drive the entire system into chaos.

The resulting orbits look somewhat more erratic than they do in the real solar system, Quinlan says. At the same time, nothing terrible happens to the orbits. They don't cross, and none of the planets gets ejected from the system over a period of a few million years.

Simulations of more than 50 such fictitious solar systems reveal that a majority show at least mild symptoms of chaotic behavior. "Because so many of these [randomly] modified solar systems ... were chaotic, it shouldn't be surprising if our real solar system is chaotic," Quinlan says. "It would be surprising if ours wasn't chaotic."

Such computer experiments may have something to say about the extent to which stability requirements force the solar system to look the way it does. Is the particular distribution of planets in our solar system just one of many possible stable arrangements, or is it the one arrangement that survives because it happens to result in a stable solar system? Such questions remain largely unresolved.

Opinions also differ on whether the solar system has room for an extra planet or two without disturbing its apparent equanimity. "I'm of the opinion that there isn't a whole lot of extra room if you try to put in extra stuff," Goldreich says. "It would get thrown out. That's how we ended up with what we've got."

A deeper question concerns the meaning of the ever-increasing divergence of nearby orbits – as measured by a number known as the Lyapunov exponent – in a chaotic system. Although widely used as an indicator of chaos, the value of this exponent doesn't necessarily reflect the time scale over which a drastic change could occur.

"The fact that you have chaos is sort of a technical fact – related to the rate at which nearby trajectories diverge," Goldreich says. "Whether it has qualitative implications over the age of the solar system is a more interesting but unanswered question."

Indeed, the solar system's survival in roughly its present form for billions of years – many times longer than its computed Lyapunov exponent would suggest – clearly indicates that the underlying dynamical theory requires more work. Chaos seems somehow compatible with the planets going around the sun for a long time without doing anything crazy.

"It's hard to deliver a punch line that tidies it all up," Lecar says. "The last word has not been said yet."

At the same time, efforts to settle the issue of the solar system's stability face a serious, perhaps insurmountable obstacle. "In some sense, you end up having to deal with probabilities," Tremaine says. "You can never rule anything out completely. Even if a system is well-behaved, there's always a small chance of its wandering by some narrow path to just about any configuration."

Long held up as a model of perfection and the symbol of a predictable, mechanical universe, the solar system no longer fits its image as a precision machine. Chaos and uncertainty have overtaken the ultimate clockwork. □