Strings and Mirrors

Burrowing into the crumpled-up heart of string theory

By IVARS PETERSON

In the 1920s, two powerful ideas took hold in physics. Quantum theory held that electrons in atoms could have only certain energies. The special theory of relativity insisted that no particles could travel faster than the speed of light.

But there was no equation that combined the two theories to describe the behavior of rapidly moving electrons in atoms.

Paul A.M. Dirac finally found the link in 1929, when he formulated an equation that encompassed both special relativity and quantum mechanics – and created relativistic quantum mechanics. Solutions to the equation not only provided a description of the motion of atomic electrons, but also unexpectedly gave an explanation of their spin and magnetic properties.

Moreover, some technical difficulties in handling the equation led Dirac to postulate the existence of antimatter: For each type of ordinary particle, such as an electron or proton, there exists an antiparticle of opposite charge. The discovery of positively charged electrons (positrons) a few years later vindicated Dirac's daring, controversial prediction.

Dirac's work was strongly influenced by his conviction that a physical theory had to be "beautiful." More than merely a description of a physical phenomenon, a theory had to be formulated as a compact, but richly suggestive mathematical expression of a simple, underlying principle. Dirac's approach to relativistic quantum mechanics met that criterion, and it has survived as an essential part of what is now known as quantum field theory.

Today's theoretical physicists face the challenge of uniting the general theory of relativity – which describes gravity in terms of geometry – with the equations of the standard model of particle physics – which describe the forces between subatomic particles, including quarks and electrons, in terms of quantum field theory.

The leading candidate for achieving this unification is string theory. This theory replaces the point particles of relativity and quantum mechanics with extended objects called strings, which can be visualized as either closed loops or segments with two free ends. These strings are so tiny, however, that they look like point particles when probed at even the highest energies accessible to particle accelerators.

The equations of string theory have proved extremely difficult to solve and interpret. Indeed, no detailed quantitative prediction that would make possible a decisive experimental test of string theory has yet emerged.

Nonetheless, physicists and mathematicians continue working on the theory – in part because, as John H. Schwarz of the California Institute of Technology once stated, "string theory [is] too beautiful a mathematical structure to be completely irrelevant to nature." And there have been modest successes along the way, which have provided intriguing glimpses of the form a definitive, complete string theory might take.

Last month saw the electronic dissemination of two papers demonstrating that string theory allows changes that smoothly alter the topology – the basic shape – of space-time. This result means that, from a physical point of view, nothing disastrous happens during such changes in geometry.

In contrast, the same kind of discontinuity could occur in general relativity only with extreme physical consequences. To remain physically reasonable, "general relativity teaches us that the fabric of space-time can be stretched or shrunk, but it cannot be torn," says mathematician David R. Morrison of Duke University in Durham, N.C.

The new papers suggest that string theory allows radical changes in geometry, while quantum effects shield the physical universe from the potentially catastrophic consequences of such drastic rearrangements of space and time. By such reasoning, it's possible to imagine the universe evolving along exotic paths that seem out of place in general relativity. These results and the innovative methods used to achieve them open up a new avenue of exploration for both string theorists and mathematicians.

Strings arose out of the need in physical theory to evade singularities – places where the mathematics describing the action of a force gives infinity as the answer. Quantum mechanics tamed such infinities in the case of atomic systems by making everything fuzzy, as quantified in the Heisenberg uncertainty principle.

But there was no equivalent method of handling and hiding singularities that arise in general relativity when theorists try to incorporate quantum effects. Until the advent of string theory, this crucial deficiency isolated general relativity (and gravity) from the rest of physics.

By idealizing particles as one-dimensional strings rather than zero-dimensional points, string theory removes those singularities and provides a plausible framework for a quantum theory of gravitation. The trick is to link the behavior of these mathematical objects with the physical world – in other words, to find a way of interpreting complex, exotic geometries as observable physical effects.

"String theory gives you a complicated set of differential equations that you have to solve," says physicist Brian R. Greene of Cornell University. However, because these differential equations turn out to have many different solutions, it isn't clear which of the solutions – also expressed as equations – to use for deriving (and predicting) the behavior of a given physical system.

"What we have done for many years is to study a whole host of solutions to the [differential] equations to get a sense of what sort of physics can emerge," Greene says.

To complicate matters further, string theorists don't actually know the full differential equations describing the underlying quantum theory. They have to approximate those equations by starting with simplified, bare-bones versions, then introducing finer and finer corrections.

Nonetheless, solutions to the bare-bones equations suggest that strings exist in a 10-dimensional environment. Four dimensions of this peculiar setting – height, width, depth, and time – correspond to the space-time of relativity theory. The remaining six dimensions are somehow crumpled up into tiny, compact balls, which correspond in physical terms to minuscule spaces only 10^{-33} centimeter wide – far smaller than a proton.

Three years ago, Greene and M. Ronen Plesser, now at Yale University, found that pairs of vastly different solutions to the simplified equations – solutions that apparently have nothing to do with each other – sometimes actually lead to identical physical consequences when all possible corrections are added in. For technical reasons, the mathematical spaces represented by such pairs of solutions are called mirror manifolds.

The discovery of this pairing came as a surprise to mathematicians. "We didn't see any reason to believe such things were true," Morrison says. "But it worked."

The results meant that the same physics can be expressed in terms of vastly different geometries. Such a connection hints that these disparate geometries themselves may have as yet unknown mathematical ties.

"From a physical point of view, this [connection] has important practical implications," Greene says. It turns out that when certain calculations prove impossible to carry out by starting with one solution, translating the question into an equivalent problem applied to the original solution's mirror manifold sometimes makes the calculation very easy.

"That gives you a new kind of tool for getting at the core of the physical phenomena [represented by] these solutions," Greene says.

Greene, Morrison, and Paul S. Aspinwall of the Institute for Advanced Study in Princeton, N.J., have applied this new tool to the investigation of the types of radical changes allowed by string theory in the basic geometry of space-time.

Topology emphasizes the characteristics of geometrical shapes that remain unchanged, no matter how much the shapes are bent, stretched, or twisted. For example, the surfaces of a doughnut and a coffee mug have the same topology. It's possible to imagine smoothly deforming a doughnut-shaped piece of clay into a single-handled mug – all the while preserving the hole that is characteristic of both objects.

On the other hand, it would take cutting and gluing to turn a spherical balloon into an inner tube. Thus, the surfaces of a balloon and an inner tube have different topologies.

In general, two shapes have the same topology only if one can be transformed into the other without cutting or tearing.

The equations of general relativity govern the structure of space-time, and any solutions that include tearing would have singularities leading to disastrous physical consequences. Since these disastrous events apparently don't occur, this renders those particular solutions suspect, if not completely imaginary.

"We wanted to know what would happen in string theory," Greene says.

The bare-bones version of the differential equations of string theory looks very similar to the equations connected with the physical interpretation of general relativity. But that's not the full string theory story. One also has to include all the correction terms to see if those additions make a difference.

"When you add in all the . . . corrections demanded by string theory, you get a different theory than general relativity," Greene notes.

In one approach, Edward Witten of the Institute for Advanced Study carefully analyzed what would happen near a singularity – a tear caused by a topology change – and argues that corrections to the basic equations precisely cancel out the singularity that general relativity would encounter.

Greene and his colleagues tried a different strategy. They worked with a solution to the simplified string-theory equations that happened to have a known mirror manifold. One of its two solutions appears to lead to a singularity in the physics, but the other clearly doesn't. "It turns out the mirror picture has no singularity in the physics," Greene says.

Both results demonstrate that in the context of strings, physical theory can accommodate a change in the topology of space-time. Unlike the situation in general relativity, no singularity mars the transition from one topology to another. "The singularities of this kind of topology change can be hidden away by quantum effects," Greene remarks.

Such a restructuring of space and time proves no more unusual than the transition of water into steam, Morrison adds.

By permitting topology changes, this result means that the universe may evolve in far more exotic ways than expected in general relativity. "But they're not really exotic at all from the point of view of string theory," Greene notes. "They're as physically reasonable as what would occur in general relativity."

Although the latest results suggest some intriguing possibilities, string theory itself remains mired in seemingly intractable mathematical difficulties.

"The main barrier is the fact that we don't have the full equations in hand," Greene says. It is often hard to tell whether a given result depends on the approximate solution chosen for study or corresponds to real physics.

The discovery of a large number of different solutions to the equations of string theory presents theorists with another disturbing problem. Within the major assumptions of string theory, there appears to be a tremendous number of paths from the mathematics to the real world.

"One encouraging feature of [the new] result is that it's helping to diminish the numbers of choices," Morrison remarks.

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"Now that we've seen that completely different solutions to the differential equations are connected even if you go through a topology change, an interesting question is whether you can connect all the solutions . . . by such manipulations," Greene says.

"It would be quite beautiful if you could show . . . that although the differential equations look like they have this vast space of different solutions not connected in any obvious manner, through processes that are physically reasonable in string theory but not in general relativity, you can connect them up," he adds.

But the rules of the string theory game are so hard and each match so demanding that only a few can play.

In *Dreams of a Final Theory: The Search for the Fundamental Laws of Nature* (Pantheon, 1992), physicist Steven Weinberg of the University of Texas at Austin remarks: "String theory is very demanding; few of the theorists who work on other problems have the background to understand technical articles on string theory,

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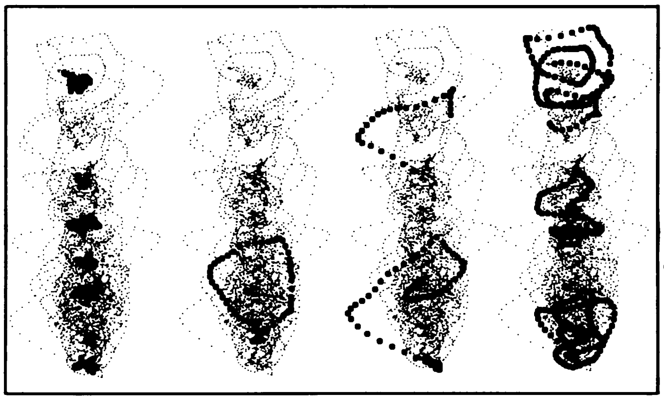
the overall complexity of the woman's heart rate. For this, the researchers had to plot the information in a mathematical universe called "phase space," which plots the heart rate in three dimensions to highlight the amount of variability.



Redington and Reidbord

Darkened points indicate four different patterns, or trajectories, in the same phase-space plot of changes in a patient's heart rate during psychotherapy. Type IV trajectory (far right) exhibits the most complex, spontaneous, random-like motion in the phase space and may reflect specific kinds of mental states patients experience during therapy, say researchers.

In these three-dimensional diagrams, second-by-second changes in heart rate form a continuous series of points. A truly random heart rate, Redington explains, would fill the phase space with an amorphous cloud of data points in no particular order. Conversely, a completely stable heart rate — that of a person under deep anesthesia, for example—would fill only a small, point-like region of the phase space.

"Our data look like something in between these extremes," Redington says.

In a computer-assisted game of connect-the-dots, the researchers identified recurrent patterns, or trajectories, in the behavior of the patient's heart rate in phase space. For example, trajectories the researchers classified as type IV show the most complex, spontaneous behavior. In these trajectories, the heart rate wanders through widely separated regions of phase space. This behavior resembles the "random walk" of a truly unpredictable process.

But most intriguing to the researchers, type IV trajectories tend to emerge when the patient seems most focused on the emotions and thoughts at issue during the session — a desirable mental state psychologists call "therapeutic engagement." In contrast, less complex, more point-like trajectories seem to correspond to times when the patient seems more defensive or anxious.

These complex heart-rate patterns may also reveal important information about the mental state of the therapist, Redington notes. In a study slated for publication in an upcoming Journal of Nervous and Mental Disease, the researchers show that type IV trajectories in the therapist often coincide with moments when the therapist experiences strong feelings of empathy for the patient.

"There appear to be interesting patterns in the physiology that may index, or reflect, what's going on inside the head," Redington notes.

The therapist can also affect the behavior of the patient's heart rate. By making what psychiatrists call an "intervention" — offering an insightful interpretation of something the patient has recounted, for instance — the therapist sometimes seems to trigger sudden changes in the complexity of the patient's heart rate.

At these moments in therapy, "there's a dramatic shift in the patient's physiology," says Redington. "It's as if the therapist has metaphorically slapped the patient on the back. And now all of a sudden, the patient is responding in a new way, learning a new way to behave."

What do these dynamic changes say about patients' shifting mental states during psychotherapy? At this early phase of their research, Redington and Reidbord hesitate to tag the trajectories they've observed with specific meanings. In subsequent work, Reidbord says, they will switch to analyzing the phase-space plots with computer-based, numerical methods to avoid the somewhat subjective typing of trajectories by visual inspection.

However, using nonlinear dynamics to study psychological phenomena may prove "intrinsically more difficult" than earlier applications, warns cardiologist Ary L. Goldberger at Harvard Medical School in Boston, who is noted for his research on the chaotic rhythms of the heart (SN: 9/5/92, p.156). Moreover, in all disciplines, not just psychology, the use of chaos theory remains "enormously complicated and controversial," Goldberger says.

"I think the interpretation of the data must be done with great caution," he notes.

Redington and Reidbord say they have indeed proceeded cautiously in their research, largely out of respect for the intrinsic difficulties of training the telescope of nonlinear dynamics on psychological phenomena. In past and ongoing research, they have taken an "ultraconservative approach," which includes collecting heart-rate data carefully and applying the mathematics of nonlinear dynamics rigorously, Redington says.

The researchers, however, are certain that nonlinear dynamics is the appropriate means for exploring the complex interconnections of physiology and mental states and for pursuing their goal of describing mathematically the "pushes and pulls" that shape human thought and action.

"When you look through a telescope, you start to see in much finer detail, you're better able to describe things," says Redington. "And that's exactly what I think nonlinear dynamics is all about." □

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and few of the string theorists have time to keep up with anything else in physics, least of all with high-energy experiments."

Nonetheless, he continues, "string theory provides our only present source of candidates for a final theory.... It is a pity that it has not yet been more successful, but string theorists like everyone else are trying to make the best of a very difficult moment in the history of physics. We simply have to hope either that string theory will become more successful or that new experiments will open up progress in other directions."

The attempt to construct string theory has also led to fruitful collaborations between physicists and mathematicians. Mathematicians can take advantage of the intuitive insights that physicists bring to the solution of mathematical problems, and physicists benefit from the rigor that mathematicians bring to what initially may be little more than speculation.

"This kind of jumping ahead when you don't really know what you're doing is really useful," Greene says. "If you wait for the mathematical rigor to be there, it might take a long time, and by that time the question you started with may not be important anymore."

"I think that over the last decade it has become apparent that we mathematicians can actually learn a lot in interacting with physicists if we suspend disbelief for a while," Morrison says. He describes some of the mathematical surprises emerging from recent developments in string theory in the January Journal of the American Mathematical Society.

In 1989, Schwarz remarked, "It is very satisfying to witness the growth of interaction between mathematicians and physicists after a long period of separation. I think it is fair to say that the study of string theory holds great promise for the unification of particles and forces, but it has already done a great deal to unify disciplines."

Those remarks still ring true. □