Chaos in SPACETIME

Looking for answers in the "black box" of general relativity

By IVARS PETERSON

When astronomers survey the distant reaches of space, they see stars, galaxies, and great clusters of galaxies. Yet in whatever direction they look, the universe also appears roughly the same. On a sufficiently large scale, its lumpiness has a remarkably uniform distribution. Moreover, little apparently distinguishes our local sample of the universe from more remote regions.

Faced with these observations and with philosophical and other considerations, cosmologists have generally come to accept that, on the largest scales, the universe is approximately homogeneous. And it looks roughly isotropic — the same viewed from any point in space. In other words, the universe viewed in the large appears simple and regular.

At the same time, several pieces of evidence — especially the characteristic reddening of light that travels from distant sources — suggest that the universe is expanding. These observations fit nicely with theoretical predictions based on Einstein's general theory of relativity. Indeed, the family of solutions to Einstein's equations, discovered independently in the 1920s by Russian meteorologist and mathematician Alexander Friedmann and Belgian cleric Abbé Georges Lemaître, requires an expanding universe.

But this leaves unsolved the puzzle of how the lumpiness now evident in the universe came about. How did such structure arise out of apparent uniformity? What facilitated the agglomeration of matter into gas clouds, stars, galaxies, and clusters of galaxies?

Researchers have proposed a number of solutions, postulating, for example, the existence of dark matter — bizarre, invisible, barely detectable material that could have sped the gravitational collapse of matter into primordial lumps. But the answer may actually lie in the equations of general relativity themselves.

"Our cosmological models may be too simple," says astrophysicist David Hobill of the University of Calgary in Alberta. More comprehensive solutions to the Einstein equations than the simple models already derived may have something to say about large-scale structure formation, he notes.

Einstein's general theory of relativity posits that gravity, time, and three-dimensional space are woven into a single, universal entity. What we perceive as the force of gravity is really a matter of geometry — a consequence of the curvature of four-dimensional spacetime.

This means that time intervals and distances between points are no longer the same as they would be if there were no curvature and no mass present. Moreover, the degree of curvature depends on the amount of mass. Great concentrations of mass, such as galaxies or matter- and light-swallowing black holes, warp space and distort time more strongly than mere planets or moons do.

Einstein expressed these ideas in terms of a set of differential equations that state the relationship between the curvature of four-dimensional spacetime and the extent to which matter and energy distort this geometry. These equations are in some sense roughly analogous to those used in hydrodynamics to describe fluid motion, which can range from smooth to wildly turbulent.

In both hydrodynamics and relativity, the equations involved are nonlinear. In other words, changing the value of a variable on one side of an equation doesn't cause a proportional change in a variable on the other side of the equation. For example, tripling the value of the variable on one side may actually cause a ninefold rather than a threefold increase in the value of the variable on the other side.

Researchers have already established that even simple systems governed by nonlinear equations — for example, three gravitationally interacting, mutually orbiting bodies (SN: 2/22/92, p.120) or liquids flowing through a pipe — can display extremely erratic, seemingly unpredictable behavior. Under certain conditions, these systems display chaos — a sensitive dependence on initial conditions that makes a system inherently unpredictable.

In the case of fluid flow, stable, orderly patterns can exist in the midst of turbulence and chaos (SN: 11/13/93, p.308). Jupiter's Great Red Spot — a remarkably stable whirlpool caught in the fury of the planet's winds — may offer one example of such a phenomenon.

Because Einstein's equations are also nonlinear, "it is important to understand what such . . . knowledge means for general relativity," Hobill says. "Can the nonlinear behavior of the Einstein equations produce structures — much like Jupiter's Great Red Spot — so that some of the structure formation in the universe was not a result of bizarre forms of dark matter but is actually already encoded in the Einstein equations?"

To encourage exploration of this idea, Hobill organized a workshop on the topic of "deterministic chaos in general relativity." Held last July at the Kananaskis conference center in the spectacularly scenic mountains of Alberta, the workshop attracted more than two dozen physicists and mathematicians curious about this new effort.

"This workshop represents the first time that a number of researchers in the field of relativity, together with some mathematicians interested in dynamical systems, have met . . . to discuss the problems that need to be solved to understand the behavior of the Einstein equations in the strongly nonlinear regime," Hobill says.

The trouble is that the Einstein equations are much more difficult to solve than those of hydrodynamics, which themselves are still not well understood. "[Einstein's equations] are nonlinear in a very complicated way," says Beverly K. Berger of Oakland University in Rochester, Mich.

This complexity results from a curious kind of feedback. The presence of a mass changes the shape of spacetime, and the shape of spacetime, in turn, influences what the mass does. In Newtonian terms, gravity generates gravity.

"In other words, the unknown is on both sides of the equation, and that's very bad," Berger says.

Furthermore, every other physical theory "lives" in a spacetime background, which provides a framework for solving problems and making predictions. "In general relativity, the answer *is* the framework. That's the thing you don't know," Berger says. "Even if you get an answer, there's no guarantee that it's a [physically meaningful] answer as opposed to just an effect of having chosen" inappropriate scales for time and space.

With all these complexities present in general relativity, traditional methods of solving equations – analogous to the pencil-and-paper techniques used for solving college calculus problems – work only for the most basic cases. In these special situations, researchers must introduce sweeping assumptions that greatly simplify the equations. To solve Einstein's equations, Friedmann and Lemaître, for example, had to assume that the universe they wanted to describe was homogeneous, isotropic, and expanding. As it happens, the Big Bang model, accepted by most astronomers, has these attributes.

Theorists have found that one good place to start the search for nonlinear dynamics and chaos in general relativity is a special set of solutions to Einstein's equations that leads to what is known as the "mixmaster universe." Although quite unrealistic as a model of the real universe, this version has features that may carry over to more realistic situations.

Formulated more than two decades ago by Charles W. Misner of the University of Maryland at College Park, the mixmaster model describes a universe that undergoes alternating cycles of simultaneous expansion in two directions and contraction in a perpendicular direction. Pictured as a blob of fluid, this type of universe evolves through a series of randomly oriented pancake and cigar shapes.

"It's a generalization of our standard cosmology," Hobill says. Instead of the entire space expanding all at once in every direction to create more space, every point in space is either expanding or contracting in particular directions. Thus, the mixmaster universe is still homogeneous but no longer isotropic as it goes through its expansion and contraction cycles.

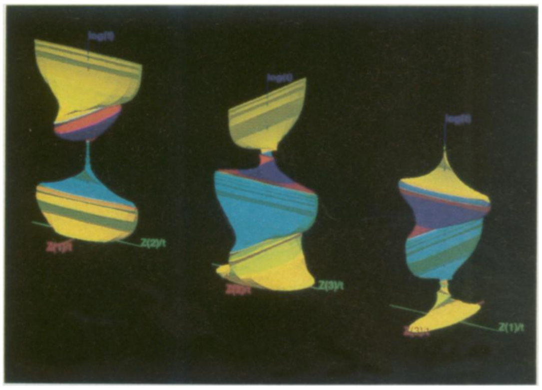
Misner originally proposed that these oscillations eventually mix everything up – hence the name mixmaster. Any matter in this universe would get homogenized into the kind of universe we observe today.

Recent work has focused on the possibility that mixmaster-type oscillations may also occur on a smaller scale in restricted regions of spacetime. Of particular interest are the dynamics of a local gravitational collapse of spacetime – a great crunch to a singularity, a single point in space, near which the gravitational field gets extremely strong.

Mixmaster gravitational collapse is a very simple model, admits Svend Erik Rugh of the Niels Bohr Institute in Copenhagen, Denmark. Nonetheless, the mixmaster model behaves in a dynamically complicated, unpredictable way in approaching the big crunch. "In this way, the [mixmaster model] captures an interesting nonlinear aspect of the classical Einstein equations," Rugh says.

Berger, Vincent Moncrief of Yale University, and their collaborators are starting to use computers to solve the Einstein equations numerically to see if a lumpy model of the universe – more complicated than the homogeneous models usually favored by cosmologists – incorporates mixmaster behavior.

"This may be important as far as providing the seeds for galaxies, but that's completely unknown at this time," Berger says. "I'm not sure we'll be able to answer the question. It depends on how well we can model the physics numerically."



Hobill

To visualize chaotic spacetime oscillations in the mixmaster universe, researchers can plot time along the vertical axis and two spatial directions along the horizontal axes. The colors show oscillations in a third direction.

In general relativity, all sorts of energy contribute to the gravitational field. For example, although light has no mass, it has energy. Thus, because energy is equivalent to mass, a sufficiently strong pulse of light can collapse to form a black hole.

By studying the Einstein equations for this particular situation, Matthew Choptuik of the University of Texas at Austin has discovered that the collapse of radiation to form black holes shows a number of interesting nonlinear effects. It turns out that whether a black hole forms depends on the intensity but not the shape of the initial light pulse. If its intensity is higher than a certain threshold value, the pulse collapses to a black hole. Otherwise, the radiation implodes, then escapes.

"This is a case where there is no chaos," Choptuik says. "But the system's nonlinear behavior has some of the features of chaos."

Although this situation doesn't have any direct cosmological significance, it provides some insight into the kinds of things that can occur in strong gravitational fields, where nonlinear effects and chaos are most likely to occur. Even in Newtonian mechanics, a nonlinear system displays chaotic behavior only under certain conditions. Most of the time, such a system behaves quite predictably.

Researchers face an additional serious problem in trying to identify chaos in general relativity. Conventional measures of chaos require calculating how much difference a tiny change in a starting point has on a system's subsequent behavior. But such comparisons require an unambiguous definition of time, which isn't possible in relativity.

"We're now trying to figure out a definition of chaos that doesn't include time," says mathematician Richard C. Churchill of Hunter College in New York City. "At this point, I can't come up with anything that satisfies me."

What researchers would like most is a definition that distinguishes between chaotic and nonchaotic parcels of spacetime.

Hobill's interest in the nonlinear aspects of general relativity had started when he and his co-workers occasionally obtained unexpected results when trying to solve simplified forms of Einstein's equations numerically using a supercomputer (SN: 9/3/88, p.152; 6/26/93, p.409). "We started asking ourselves whether some of these strange things were artifacts of the numerical methods we used or were really coming from the Einstein equations," Hobill recalls.

This uncertainty continues to plague computational work involving general relativity. Moreover, it isn't always clear whether more general solutions of Einstein's equations – derived on the basis of less restrictive assumptions than those conventionally applied – make sense in the physical world.

"One always hopes that the field one is working in is physically relevant," Berger sighs. "But in this case, there is no such guarantee."

A small group of researchers scattered throughout the world remains committed to this effort. "The Einstein equations themselves still hold a lot of secrets, which have to be unraveled," Hobill declares. "If general relativity has anything to say about our physical universe, then we need to understand the implications of the theory and the predictions that it makes concerning the behavior of the world around us."

Who knows what secrets may lurk in the mysterious black box of general relativity? More comprehensive solutions than those obtained by Friedmann and Lemaître could very well hold crucial clues bearing on the formation of complex structures in space and time. ☐