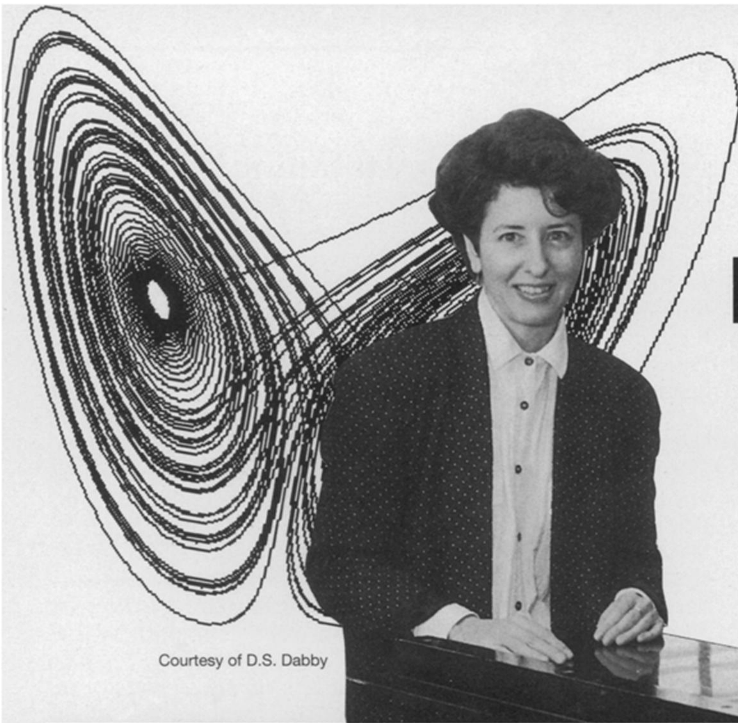
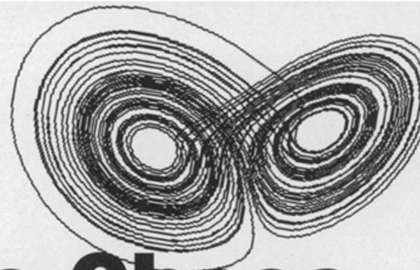


Courtesy of D.S. Dabby



Bach to Chaos

Chaotic variations on a classical theme

By IVARS PETERSON

Diana S. Dabby and "variations" of the Lorenz attractor.

Point by glowing point, the image swirls into view. As it builds up on the computer screen, it begins to resemble a delicate, stylized butterfly with translucent wings held lazily askew.

It's called the Lorenz attractor, named for meteorologist Edward N. Lorenz of the Massachusetts Institute of Technology, who in 1963 discovered this curious form encoded in a set of equations describing air flows in the atmosphere. The computer image arises out of a chaotic — in the mathematical sense — system.

For a given starting point, the computer calculates the coordinates of each successive point as the dynamical system described by the equations evolves. It displays these points as luminous dots on the screen. They appear to sprinkle themselves randomly across the display, but gradually a distinctive butterfly pattern emerges.

Different starting coordinates typically lead to radically different sequences of calculated points. But the overall pattern can always be identified as the Lorenz butterfly. It's an example of both the sensitive dependence on initial conditions and the distinctive patterns that are characteristic of chaotic systems.

When Diana S. Dabby, a graduate student in electrical engineering at MIT, first saw the Lorenz attractor a number of years ago, she was struck by its delicate beauty and elegance. "It appealed to my artistic side," Dabby says.

As a professional concert pianist and composer, she could envision "riding the back of the attractor" to create musical variations that stray in unexpected ways yet do not wander so far as to lose all ties with the original music. She could imagine using the mathematics of chaos to construct a musical space within which to work, create, and play.

As a first step toward realizing such an environment, Dabby has devised a scheme for using the Lorenz attractor to generate variations on the sequences of notes in a piece of music. She described her initial experiments — done on Bach's Prelude in C from the first book of *The Well-Tempered Clavier* — at an Acoustical Society of America meeting, held last June in Cambridge, Mass.

"My vision for this work is to expand it in every way to make a truly dynamic music for the future — one that is always changing, but changing in musical, not random, ways," Dabby says.

A decade ago, there was no chaos (mathematically speaking) in Dabby's life. Her musical career was in full swing. She performed in concerts in the New York City area and abroad, she composed music, and she practiced.

One day, while at the Lincoln Center library in New York City, Dabby came across an issue of a journal devoted to computer music. She noted that nearly all of the contributions to this particular issue came from mathematicians, computer scientists, or electrical engineers. "I wondered what would happen if a professional musician acquired their tools," Dabby recalls.

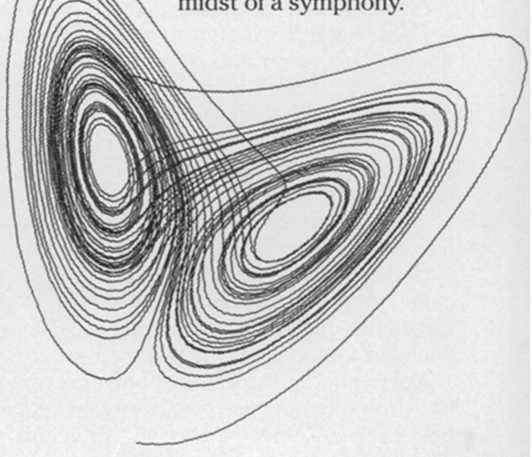
While still playing concerts, she taught herself algebra and returned to college to study calculus and take other courses required for entry into engineering. "I started to get ideas for new music," Dabby says. She also did well enough to get into the electrical engineering department at MIT, where she became a graduate student in 1987.

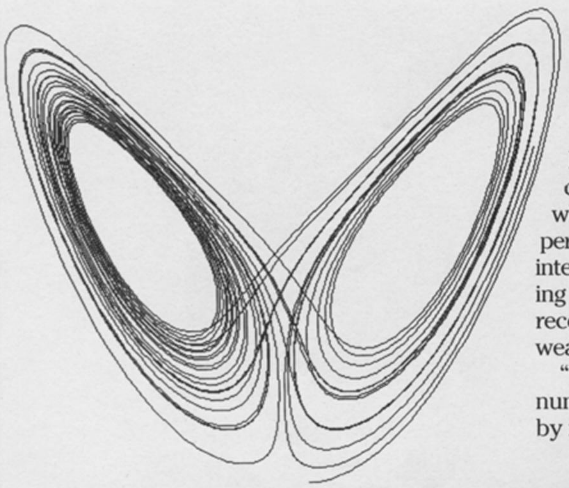
That fall, she encountered chaos in a course on dynamics. It caught her attention. "Chaos has a rich structure that is continually varying," Dabby says. "For a musician, this is much of the essence of music. There are themes, slight variations, and great variations."

"I started to learn everything I possibly could about chaos," she adds. "I went into the hardware [electronic circuitry]. I went into the theory. Anything that I could possibly do at MIT that had to do with chaos, I did."

In the spring of 1993, Dabby composed and performed a piece of music representing a sonic tour of Manhattan. It featured a piano and roughly 200 percussion, orchestral, and electronic sounds.

In her composition, Dabby sought to give the illusion that the piece was always changing with time. She created multiple paths through the music so that on successive hearings, a listener could always choose to follow different musical threads. It is not unlike, but more complex than, following the flute's melodic line in the midst of a symphony.





"In this way, the piece appeared to vary from one hearing to the next," Dabby says. "But in fact, it did not do so. All of the electronic parts were fixed on tape, and the piano part was written out, not improvised."

The process of composing this music led Dabby to think about ways of creating variations — changes in the sequences of notes — such that a piece of music could *actually* differ from one hearing to the next. She ended up inventing a procedure based on the characteristics of a chaotic system to generate these different versions.

The x coordinates of the points that make up the Lorenz attractor for a given starting point fall within a certain range of numbers. Dabby's idea was to list the pitches of all the notes or chords of a musical piece and assign them one by one, in order, to the x coordinates of points belonging to the attractor. In this way, she paired up each of the pitches in the original music with a particular range of x values.

Then she could choose a second starting point only slightly different from the first to produce a new "trajectory" — a new sequence of points — making up the Lorenz attractor. Because this new trajectory generally does not track the original one perfectly, the x coordinates of the two trajectories differ and the musical notes corresponding to these new x coordinates may occasionally change from those in the original piece.

One can imagine that the initial "mapping" step lays down the musical landscape, and the second trajectory, representing the variation, takes a slightly different path through this terrain. Because the landscape incorporates features of the original musical piece, any variations created in this way usually sound consistent with the source piece. Indeed, no variation can ever include a pitch not present in the original.

"What's neat is that [Dabby] can tune the variation to be either very close — by picking [a starting point] very near that of the original reference trajectory — or as different as she likes by choosing one sufficiently far away," says mathematician Steven H. Strogatz, who had worked with her at MIT but is now at Cornell University.

In fact, there are many ways by which one can adjust Dabby's scheme to achieve whatever musical results a composer or performer may wish. A musician can also interact with the computer program producing these variations to select, edit, and record particularly pleasing passages, even weave them into new compositions.

"You can quickly generate an enormous number of variations of a sequence," Dabby notes.

Dabby chose Bach's Prelude in C — a relatively short piano piece familiar to musicians, piano students, and aficionados of classical music — for her first experiments because "it's beautiful, simple, and elegant," she says. "I felt that if I worked with it and produced something nice, it would be a good illustration."

Dabby generated a wide range of variations on the prelude. She chose three for detailed study and recording, with each successive variation departing further from the original, taking unexpected turns, dramatically shifting tones, and presenting musical surprises along the way.

These and other variations continue to echo through her mind. "When I hear the original Bach now, or even when I'm playing the original, I hear an amalgamation of the variations superimposed on the original Bach," Dabby says. "I didn't expect that."

"These variations really sound very musical and creative," Strogatz comments. Most musicians who have heard the music tend to agree. A few, however, are uncomfortable with the role the computer seems to play in these creations.

To Dabby, this method of generating musical variations is both a harbinger of a new music of infinite variety and, in the near term, a technique for jarring a composer's mind into new musical directions. "The variations the computer produces serve as idea generators for composers," Dabby says. "Music can always be heard in a fresh light."

"I think [Dabby] has found something that a lot of people could be interested in," Strogatz suggests.

A favorite piece of music played over and over again inevitably loses some of its freshness. Creating variations may help rekindle a bit of the original passion, and this process can bring one to delve more deeply into the nuances of the music itself.

The essence of chaos is the ease with which one can wander off the beaten path yet remain connected with the familiar and the inceptive. It leads to a kind of music for tickling, sometimes even jangling, the mind. □

The opening measures of Bach's Prelude in C can be compared with the opening measures of three variations generated by Dabby's scheme. Each successive variation strays further from the original music. To emphasize the changes in pitch, Dabby has deliberately written the music without showing the duration of each note.