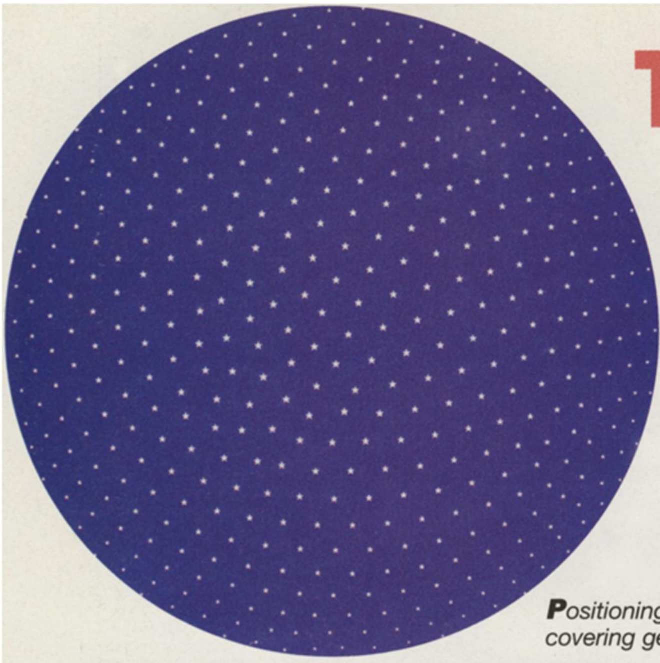


The Codemart Catalog

Arranging points on a sphere for fun and profit

By IVARS PETERSON

***P**ositioning 1,592 points on a sphere to produce the best known covering generates this geometric pattern.*

About 1,500 photodetectors on the surface of a giant, liquid-filled ball peer inward to catch the flashes of light generated by neutrinos interacting with atomic nuclei. What's the best geometric arrangement of detectors to ensure complete coverage?

High-energy laser beams intersect to zap a tumor. How should the lasers be positioned to keep the beams well separated until the point of intersection?

Static interferes with the transmission of digital information. What's an efficient way of encoding the information to minimize the chances of a transmission error?

Production engineers can set the values of six parameters to control the fabrication of silicon wafers. How can they determine the optimal set of values required to maximize production?

All these questions have in common a single mathematical problem: finding a particular optimal arrangement of points on the surface of a sphere. And anyone looking for an answer to any one of them would do well to consult the self-named Codemart team.

Consisting of Neil J.A. Sloane and Ronald H. Hardin of AT&T Bell Laboratories in Murray Hill, N.J., and Warren D. Smith of the NEC Research Laboratory in Princeton, N.J., the team has spent years building up extensive tables of "nice" arrangements of points on spheres. Their work has contributed to solving a wide variety of problems, ranging from digital communication and error correction (SN: 3/12/94, p.170) to experimental design and numerical analysis.

Now, the Codemart trio has established an electronic catalog of their arrangements, open to anyone interested in using the information. "It's quite a large archive," Sloane says. "Arranging points on a sphere is a problem that a lot of people are interested in."

At first glance, the problem seems trivial. But its intricacies become evident upon closer inspection.

For instance, suppose 50 fiercely competitive owners of pizza parlors on a newly colonized, waterless planet want to situate their establishments as far apart as possible. This is an example of the packing problem: placing a given number of points on the surface of a three-dimensional sphere in such a way that the points are separated by the largest possible distance.

Eager customers, however, look at the situation differently. They want the parlors positioned to minimize the distance to the nearest establishment, no matter where someone lives. This is an example of the so-called covering problem.

These are just two of at least 10 different criteria for choosing the positions of points on a sphere's surface. "Except in very special cases, the answers to most of these problems are different," Sloane says. In other words, for a given number of points, the resulting geometric arrangements generally differ from one criterion to another.

Moreover, the problem of distributing points on a surface can be extended to the analogs of spheres in four or more dimensions, using four or more numbers to represent the coordinates of the points. "The problem is a lot more complicated and interesting than you might think," Sloane insists.

Recently, Sloane and Hardin have focused on adapting their point-locating techniques to the design of experiments. "It turns out there are a number of interesting, unsolved problems in statistics that seem amenable to our techniques," Sloane says.

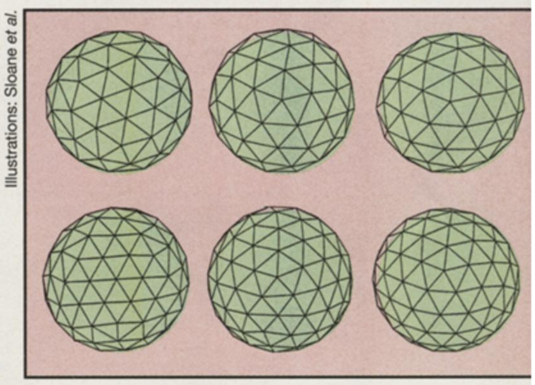
For example, a hypothetical oil refinery's output depends on four variables: temperature, pressure, processing time, and nitrogen concentration. How many trials should one perform to determine the optimal settings for maximizing output?

"The stupid way to do this is to change the variables one at a time. That takes forever," says Sloane. The more appropriate approach is to make a series of tests at different sets of settings.

"Each time you run a test, you pick four values," he says. Each set of values — each experiment — can be thought of as a point in four-dimensional space. By looking at distributions of points in such a space, it's possible to determine how many experiments must be done at which parameter values to find the right setting.

"Hardin and I have a fantastically powerful program that uses our optimizer for designing experiments," Sloane says.

Illustrations: Sloane *et al.*



***F**rom left to right, the best packing, covering, and maximal volume arrangement yet found of 72 points (top row) and 100 points (bottom row) arrayed on a sphere.*

Over the years, the Codemart team has developed computer programs to search for various types of arrangements of points on a sphere. Running in the background, this software takes over whenever computers at Bell

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Labs aren't being used for anything else.

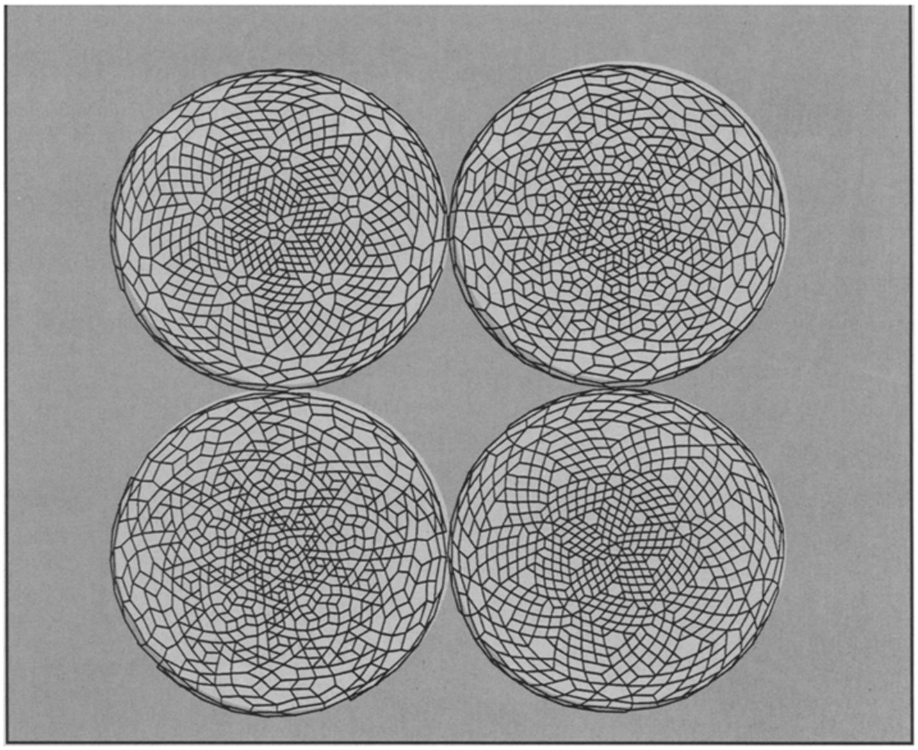
As they explore possible arrangements, the programs automatically notify the team whenever they identify a superior arrangement of points. They also enter the coordinates of these points in the appropriate table.

In most instances, the Codemart team can't claim that it has found the very best geometrical arrangement of points on a sphere to meet a certain criterion. Mathematical proofs that such patterns are truly optimal are few and far between. But in nearly all cases, the team holds the record for the best *known* arrangement, and the programs keep improving on these results.

The Codemart effort has already generated tables far more extensive than any available elsewhere. The new electronic archive includes packings of up to 130 points on spheres in three, four, and five dimensions, coverings of points on spheres in three dimensions, and many special cases.

The team expects eventually to publish their tables in a book. "But in view of the considerable recent interest in these problems, we are making these tables available before the book is completed," the researchers say.

The Codemart catalog is accessible via electronic mail at *netlib@research.att.com.* One can find out what is available by sending to this address a message containing a line such as "send index for att/math/sloane/packings."



The best icosahedral packings now known of 782, 792, 800 and, 810 points.

At present, the tables include only the coordinates of the points in any particular arrangement. Sloane and his colleagues are now considering the possibility of adding pictures of some of the examples.

At the same time, "we're still searching, and we're continually widening our scope," Sloane remarks. "The programs are running day and night." □

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branching, diffusing, tip splitting, and dendrite forming emerge. Moreover, Ben-Jacob's group reports in the Feb. 16 NATURE, the modeled patterns correlate with forms grown in petri dishes.

"This work is very provocative," says Albert Libchaber, a physicist at Rockefeller University. "Fractal analysis is a tool that can give you some intuitions about the cause of a phenomenon. Of course, identical patterns can have different physical causes. So the pattern itself does not determine a phenomenon's origin. But an *evolving* pattern can tell you that something interesting may be going on."

Ben-Jacob argues that patterns visible to the naked eye often convey meaningful clues about molecular processes the unaided eye cannot see. "There is an interplay between information at the macroscopic level and the microscopic level in bacterial colonies similar to what we see in nonliving systems that are changing and evolving," he says.

Using the metaphor of information processors to describe bacterial communication within colonies, Ben-Jacob advocates viewing each bacterium's genome as an "adaptive cybernetic unit." Those units, he asserts, may be influenced by the chemotactic signals generated through the colony's structure. This, he maintains, helps account for the survival of mutations that benefit the colony as a whole.

"There's strong evidence now that in many bacterial systems, genetic changes result from many kinds of chemical regulation and control," Shapiro says. "Mutation is a biochemical process which does not just occur because of random insults to DNA. Biochemical complexes come into play. It would be surprising, in fact, if we found that as conditions change, genomes didn't change."

"We know that bacterial colonies can alter themselves dramatically in response to changing conditions," Shapiro adds. "An area of investigation now is to understand how specific, meaningful, and useful those changes are. This lies at the heart of studies of adaptive mutations."

Shapiro says other findings support Ben-Jacob's models. "His work fits into a trend away from a view of bacteria as isolated, autonomous, and relatively unaware of things around them. Instead, we're seeing that bacteria constantly pick up chemical signals."

As for the usefulness of applying ideas from materials science to biology, Herbert Levine, a physicist at the University of California, San Diego, says that such interdisciplinary techniques often spark clever insights.

"One reason to try to understand nonliving chemical systems undergoing changes is that many scientists think it will lead to useful explanations of how biological systems form structure," Levine says. "If we understand nonliving systems well enough, then maybe we can use the same ideas to make predictions about biological systems."

H. Eugene Stanley, a materials scientist at Boston University, agrees. "I think Ben-Jacob is correct to take seriously what the eye tells us about bacteria," he says. "Given that the bacterial colonies look so much like patterns we already know about from materials science, either the resemblance is fortuitous or it's telling us something valuable. I suspect the resemblance is a powerful clue," he observes.

Shapiro, for one, welcomes the influx of physicists into the biological arena.

"Biologists can learn from people in other fields with different analytical techniques," he says. "The synergism will benefit everyone. The physicists will be forced to give up their dogmas of understanding a whole organism by understanding a small piece of it, and the biologists will have to give up some dogmas about genetics." □