# Cavities of Chaos

## Sorting out quantum chaos in the microwave lab

By IVARS PETERSON

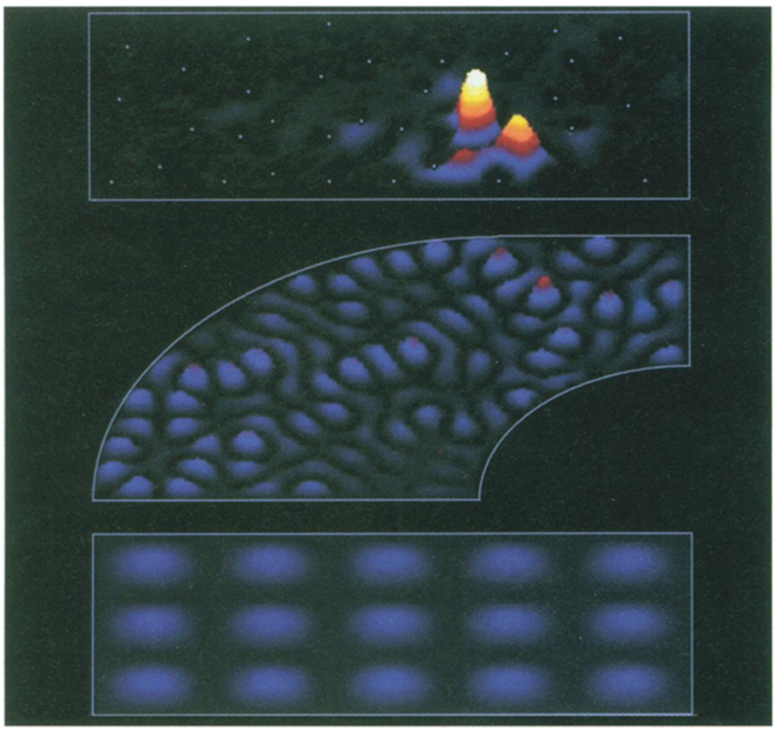
A smooth stroke ending in a sharp tap with a billiard cue sends a ball scooting across the table's green felt surface. The ball bounces off one cushion, then another before smacking squarely into a second ball.

On a rectangular table, an expert player can return the first ball to its original position, repeat the shot, and obtain the same result. However, such consistency proves practically impossible to achieve on a rounded, stadium-shaped table.

In stadium billiards, even the slightest change in starting point can alter the ball's trajectory radically. The ball's path becomes unpredictable — especially after several bounces. If there were no friction, the ball's continuing motion as it rattled around the table would appear random.

Thus, depending on the table's shape, the motion of a billiard ball can be either predictable or chaotic. The same physical laws determine the outcome in either case, but in the chaotic situation the result is sensitively dependent on initial conditions.

The rectangle and the stadium have long served as geometric playing fields for theorists studying differences between regular and chaotic motion. They can readily calculate sample trajectories to investigate the details of these different types of behavior.



*Geometry has a strong effect on the type of wave pattern observed in a microwave cavity. The pattern appears very regular in a rectangular cavity (bottom), disordered in a cavity having a geometry in which ordinary particles would move chaotically (middle), and localized in a rectangular cavity strewn with obstacles (top).*

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Researchers can also use these geometries as models for investigating the link between the classical mechanics of billiard balls and tables and the quantum mechanics of the microscopic world of atoms and electrons.

For example, electrons confined to a tiny volume act in some respects like billiard balls, bouncing around within its walls. But because electrons are quantum particles, they exhibit wavelike behavior too. Where electron waves cross, they can reinforce or cancel each other, creating a quantum interference pattern.

What happens when an electron wave bounces around inside a stadium-shaped cavity, in which a classical particle's motion would be chaotic? More generally, how do quantum systems behave when the corresponding classical systems are chaotic?

Extensive theoretical work, often augmented by computer simulations, has uncovered a wealth of new physical phenomena at the blurry interface between the microscopic and macroscopic worlds — where classical theory may be chaotic and quantum theory is very complicated.

"Although not yet available in your local video store, this new game of Quantum Pinball has captured the imagination of theorists in the field of quantum chaos," physicist Roderick V. Jensen of Wesleyan University in Middletown, Conn., commented in the Jan. 5 Nature.

Now, experiment is beginning to catch up with theory. Several research groups are studying microwaves in thin, metallic boxes and electrons in tiny structures called quantum dots to verify theoretical results and to explore the possibility of unveiling new physics not yet evident to the theorists.

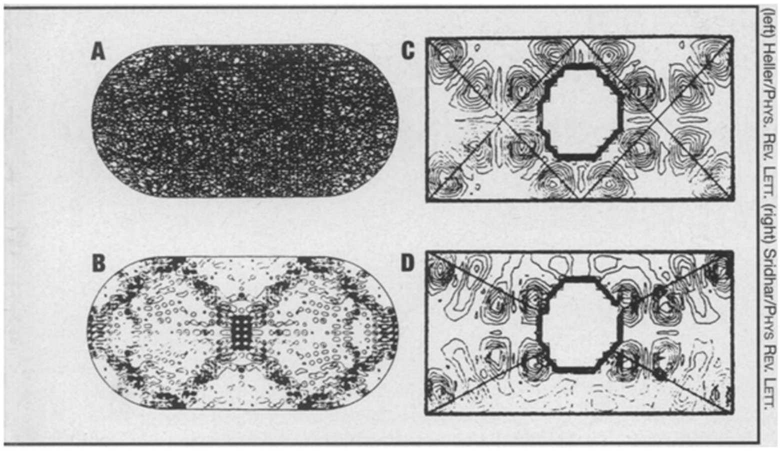
In the realm of electrons, microdots, and nanostructures, "there's a huge frontier of potential applications of quantum chaos," Jensen says. Understanding such systems could lead to novel electronic circuitry and other uses in nanoelectronics.

To explore the manifestations of chaos in quantum mechanics, Srinivas Sridhar, Arshad Kudrolli, and their coworkers at Northeastern University in Boston observe the behavior of microwaves in closed, shallow copper containers, or cavities. They exploit an exact equivalence between the equations of quantum mechanics and those governing the behavior of electromagnetic waves in two dimensions.

"You do it [produce the equivalence] by making the cavity very thin, like a pizza," Sridhar says.

Electromagnetic waves within this microwave cavity resemble oscillations of the head of a drum, which vibrates at characteristic frequencies when struck (SN: 9/17/94, p.184). By distributing fine sand across a vibrating drumhead, researchers can map the drum's normal modes — the patterns of vibration corresponding to the drum's characteristic frequencies. The loose sand tends to settle in more placid regions of the drumhead, away from the areas undergoing strong oscillations.

In the quantum world, these normal modes are known as eigenfunctions, and Sridhar has developed a way to map the eigenfunctions of microwaves bouncing around inside cavities of different shapes.



(left) Heller/Phys. Rev. Lett. (right) Sridhar/Phys Rev. Lett.

***A** billiard ball moving without friction across a stadium-shaped table would follow a highly irregular track that never repeats itself (A). In contrast, a quantum particle traveling in the same geometry has a high probability of following tracks corresponding to a repeated motion, leaving a "scar" in the pattern (B). Sridhar's experiments involving microwaves in rectangular cavities containing a large, central obstacle provided evidence of the existence of such scars in a geometry analogous to the stadium case (C). Different microwave frequencies produce different patterns (D).*

A rectangular cavity produces a set of distinctive eigenfunctions, each one corresponding to a particular energy. "You get exactly what you would expect from quantum mechanics," Sridhar says. There's no chaos in this case.

However, the situation is somewhat different for a stadium-shaped cavity — or a rectangular cavity containing a large disk, which also gives rise to chaotic motion. The classical description of an ordinary particle's motion in this geometry is chaotic, meaning that there are no normal modes. How does quantum mechanics change the picture?

In 1984, Eric J. Heller, now at Harvard University, argued on theoretical grounds that quantum particles in such a geometry would have eigenfunctions that retain some of the wave patterns associated with regular, periodic motion. He described these eigenfunctions as having "scars" — fuzzy tracks following periodic trajectories that are unstable in the classical case (SN: 11/2/91, p.282).

In other words, although the classical trajectories are random, the corresponding quantum motion has eigenfunctions with a distinctive distribution. Heller could explain the existence of these structures in terms of quantum interference effects. Computer simulations furnished a picture of what they look like (see illustrations).

In 1991, Sridhar provided the first direct observations of these scars in microwave experiments with a rectangular cavity containing a large disk. His subsequent work has revealed similar patterns in cavities having other geometries (see color illustrations).

"The Sridhar experiments are very valuable," Heller says. "At first, the experiments were verifying things which we knew to be true from numerical experiments. Most recently, Sridhar has broken new ground with challenges to theory."

This recent work involves placing a random array of small obstacles inside a rectangular cavity in order to study the connection between chaos and disorder in quantum systems. It's like observing the effect of impurities on the movement of electrons in tiny, two-dimensional structures.

The experiments reveal that these obstacles force the eigenfunctions to concentrate themselves in certain areas of the cavity. They no longer spread themselves throughout the entire container.

"You see enormous localization in certain regions," Sridhar says. "The waves are happy just to sit there." By changing the number of obstacles and the microwave frequency, the researchers can increase or decrease the degree of localization at will, thus making it possible to explore the link between chaos and disorder.

Sridhar and Kudrolli described their work in March at an American Physical Society meeting held in San Jose, Calif. Such findings may help explain the results of experiments involving electron and photon activity in disordered solids (SN: 4/20/91, p.248).

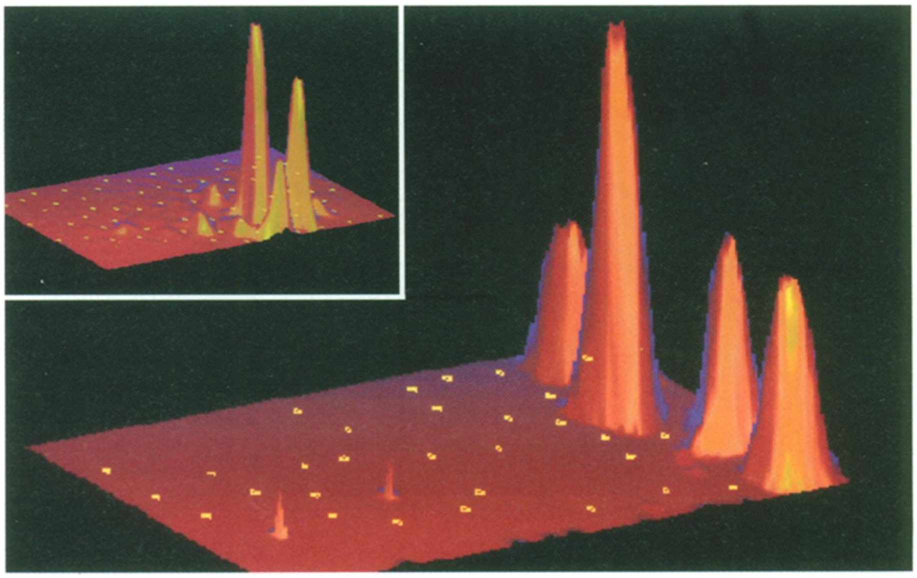
It's in the world of electrons and nanostructures at very low temperatures that quantum chaos may prove of greatest value.

Advances in semiconductor fabrication now enable researchers to investigate the electric properties of solid-state devices less than 1 micrometer across and cooled to temperatures below 0.05 kelvin. Under these conditions, electrons travel freely within the cavity; electric resistance arises from the bouncing of electrons off the microstructure's sides. How well such a device conducts electricity depends strongly on its shape.

"Sridhar's experiments provide a tremendous amount of inspiration and intuition for experiments and theory describing what electron waves look like in small cavities," Jensen says. "You can also gain an enormous amount of insight into these systems and their quantum properties from the classical mechanics, which happens to be chaotic."

For example, researchers have observed clear differences in the electric resistance arising from microstructures in the shape of a stadium, where the classical motion is chaotic, and a circle, where the motion is regular.

"These fruitful combinations of innovative experiments and theoretical games do not just promise to improve our understanding of the classical-quantum correspondence for classically chaotic systems," Jensen contends. "They may also be of practical use in characterizing the unusual quantum mechanical properties of tiny solid-state devices that may be incorporated into the integrated circuits of the future." ☐



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***To** study the role of disorder in determining wave patterns inside a microwave cavity, researchers randomly scattered small metal tiles inside a rectangular container. Such "impurities" can localize the wave pattern to small regions of the cavity. Changing the number of tiles alters the pattern (inset).*