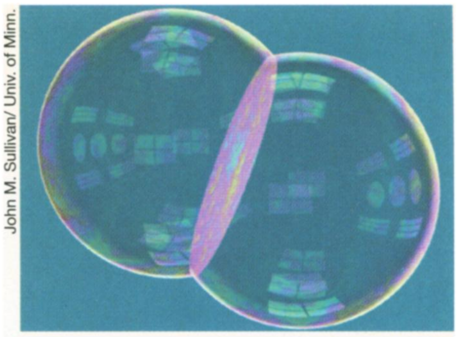
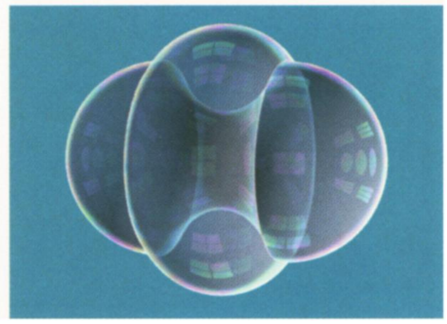
Toil and trouble over double bubbles



John M. Sullivan/ Univ. of Minn.



Standard double bubble (top) and torus bubble (bottom).

What soap bubbles do naturally can create a lot of bother for mathematicians.

Consider the case when two bubbles join to form a double bubble—a sight familiar to any soap-bubble aficionado. The bubbles share a disk-shaped wall, and this wall meets the individual bubble walls at 120°. Mathematicians call this configuration the standard double bubble (see illustration).

But this structure isn't the only candidate for the most economical way of packaging a pair of identical volumes. For instance, one bubble may ring the other—like an inner tube fitting snugly around a peanut's waist—to form a two-chambered torus bubble (see illustration).

Now, Joel Hass of the University of California, Davis, and Roger Schlafly of Real Software in Soquel, Calif., have proved mathematically that the standard double bubble triumphs over the torus bubble as the two-chambered geometric structure of least surface area. Nature's design also turns out to be the mathematician's answer to the most economical way of enclosing and separating two given volumes of space.

Hass described the long-sought proof this week at the Burlington (Vt.) Mathfest, a joint meeting of the American Mathematical Society and the Mathematical Association of America.

Mathematicians have long known that the circle is the shortest way to enclose a region of a given area and that the sphere is the most efficient way to enclose a given volume. They suspected that the standard double bubble was the answer for two given volumes, but they couldn't say so with mathematical certainty.

The campaign to settle the question began when Frank Morgan of Williams College in Williamstown, Mass., suggested to a group of undergraduate students that they tackle the two-dimensional case. "It was one of many unsolved problems in the mathematics of soap bubbles," Morgan says.

In 1991, Joel S. Foisy, now a graduate student at Duke University in Durham, N.C., and his coworkers proved that the standard double bubble in two dimensions—that is, two circles squeezed up against each other—has the least possible perimeter. There are no bizarrely curved configurations that do any better.

Joseph D. Masters, a graduate student at the University of Texas at Austin, then established that the same structure works as the smallest-perimeter arrangement on the surface of a sphere.

Going to three dimensions, Michael L. Hutchings, a graduate student at Harvard University, proved a conjecture that narrowed the candidates to the standard double bubble and the torus bubble.

But a torus bubble has innumerable possible shapes, any one of which could possibly beat the standard double bubble. How could all these possibilities be ruled out?

Hass and Schlafly decided to try to solve the problem by using a computer to check all the torus bubble configurations. They established criteria for comparing the surface areas of the various enclosures and wrote a program to conduct the search.

Normally, mathematicians don't use computers to obtain mathematical proofs, partly because computers generally make slight rounding-off errors when they do calculations. But Hass and Schlafly found a way of circumventing this deficiency.

In the end, they showed that no torus bubble does better than the standard double bubble. "Computer calculations [were] essential to our proof that equal-volume double bubbles minimize area," Hass says.

This result still leaves open the question of what happens when the two volumes are unequal. Preliminary findings suggest that the standard double bubble also wins in this case, but not all possibilities have been checked yet.

The situation for triple bubbles—in both two and three dimensions—is much further from being settled. And mathematicians haven't yet proved that the hexagonal honeycomb is the most efficient way of partitioning a region into equal-area units (SN: 3/5/94, p.149).

Nature probably has the answers already, but mathematicians need to establish them with certainty, and such efforts often entail a lot of toil and trouble.

—I. Peterson

Pine forest thrives on high-CO_2 diet

A forest of the future grows in North Carolina. By pumping carbon dioxide (CO_2) into a stand of loblolly pines, a team of ecologists is testing how mature trees will react to the increase in atmospheric greenhouse gases expected during the next century.

Results collected last summer show that the gassed-up pines boosted their rate of photosynthesis by 65 percent, compared to a nearby control plot, says David S. Ellsworth of Brookhaven National Laboratory in Upton, N.Y. Ellsworth reported the findings last week at a meeting of the Ecological Society of America in Snowbird, Utah.

The logistically complex study—which requires large towers for pumping the CO_2 into the foliage—is the first to examine how mature trees in a forest will respond to air enriched with the gas. Previous experiments, although carried out on a much smaller scale, have shown similar increases in photosynthetic rate among small trees and crop plants growing in CO_2-enhanced air.

In the study, scientists from Brookhaven and from Duke University in Durham, N.C., boosted CO_2 concentrations to 550 parts per million throughout a 0.4 hectare forest plot. Air today typically contains 355 parts per million, whereas CO_2 values prior to the Industrial Revolution stood at 280 parts per million.

The scientists ran the experiment for just one season and thus did not test whether rates of photosynthesis for the forest would remain elevated under prolonged exposure to CO_2. Previous studies have shown that certain plants acclimatize to the higher concentrations of the gas: Within 3 years, their photosynthetic rates fall back to those of control plants. In 1996, Ellsworth and his colleagues will start a long-term experiment to determine how the forest responds over 3 to 5 years.

The new results seem to suggest that pine forests may thrive in the atmosphere of the future. But Fakhri Bazzaz of Harvard University notes that the buildup of CO_2 will produce both winners and losers in natural ecosystems.

Although most plants grow better with extra CO_2, the ones most sensitive to the gas will tend to drive out the less responsive types, thereby winnowing the number of species present in forests. "The production of biomass, of plant material, will probably increase about 10 percent, but the biological diversity will be diminished," says Bazzaz. ***—R. Monastersky***