1,000 times and calculated an acceptable range of variation. Henneberg and Thackeray then repeated this statistical exercise for sample sizes of 4 to 19 skulls.

They found, for example, that the extent of cranial variation in modern human samples met or exceeded the corresponding variation in 10 *Australopithecus africanus* skulls, 12 skulls from both *A. africanus* and *A. afarensis*, 13 skulls from early *Homo* and the robust australopithecines (considered a separate hominid genus by some scientists), 19 australopithecines from several commonly accepted species, 18 *H. erectus* skulls from Africa and Asia, and 10 *Homo* specimens from various regions dated at 1 million to 500,000 years old.

Any of these samples could therefore have come from a single species composed of separate populations bearing distinctive features, as occurs in modern humans, Henneberg maintains.

The range of body heights and weights in different sets of hominids that lived at about the same time also falls within the spread for modern humans, the South African researchers hold. Even fossils grouped by their ages rather than their species designations—such as all those from 4.5 million to 3.1 million years old or from 1.75 million to 1.25 million years old—differ no more in the extent of their body sizes than do modern humans.

The findings indicate that species have emerged over long periods in a single hominid lineage, according to Henneberg. "Rather than trying to arbitrarily decide where one species ends and another begins, we need to concentrate on studies of the processes of continuous evolution," he asserts.

Henneberg's opinion draws sharp criticism. For starters, counters Bernard Wood of the University of Liverpool in England, modern humans inhabit diverse environments around the world and thus display much more anatomical variation than any other species, past or present. For this reason, Wood doubts that *H. sapiens* provides a reliable anatomical yardstick for earlier hominid species that covered smaller portions of the globe.

Henneberg and Thackeray also fail to consider that specific clusters of anatomical features act as unique tags for hominid species, in Wood's opinion. Consider *A. robustus*, he says, which had large, peg-shaped teeth and a bony crest atop its skull that together distinguished it from *Homo* species of its time.

"We haven't recognized enough early hominid species," Wood argues.

Ian Tattersall of the American Museum of Natural History in New York City agrees. Subtle body differences that do not show up on bones may often have distinguished hominid species from one another, he says.

Even Milford H. Wolpoff of the University of Michigan in Ann Arbor, who wants to merge *H. erectus* into a single species of *H. sapiens* that evolved over the past 2 million years, suspects that earlier hominids belonged to numerous species. Any group of animals that migrates to a new locale typically evolves survival-enhancing traits that gradually result in a new species, Wolpoff holds. In the unique case of *H. sapiens*, however, survival has increasingly hinged on culturally acquired knowledge and tools; thus people have settled in all corners of the world without evolving into separate species.

Alan G. Thorne of Australian National University in Canberra supports Wolpoff's view of *H. sapiens*, but he also welcomes Henneberg and Thackeray's findings. Only the robust australopithecines—which died out about 1 million years ago—show clear anatomical signs of having branched off the main stem of hominid evolution, Thorne holds.

Conclusions such as that of the South African scientists undermine arguments that some hominids, such as Asian *H. erectus* and Neandertals, did not contribute to human evolution, according to Thorne. Instead, anatomically distinct races capable of interbreeding have evolved over at least the past 2 million years, in his view.

"This new study provides a powerful argument in favor of a single, hardly branching hominid lineage," he remarks. "We'll have to start looking at teeth and other skeletal features to test this theory further." □

Mathematics

Ivars Peterson reports from Alexandria, Va., at a meeting of the International Society for the Interdisciplinary Study of Symmetry

A new twist on outside in

It's a regal but ghostly transformation: A golden ball turns itself inside out to reveal its purple inner surface without suffering any rips or creases along the way. Mathematicians call this process sphere eversion. The sphere acts as if it were made of a stretchy though delicate material that readily passes through itself but self-destructs if punctured or sharply pinched.

Until 1957, mathematicians were unsure whether it was possible to turn a sphere inside out without making a hole. Then, Stephen Smale of the University of California, Berkeley, proved that such an operation is feasible, although his proof furnished no clear picture of how to do it.

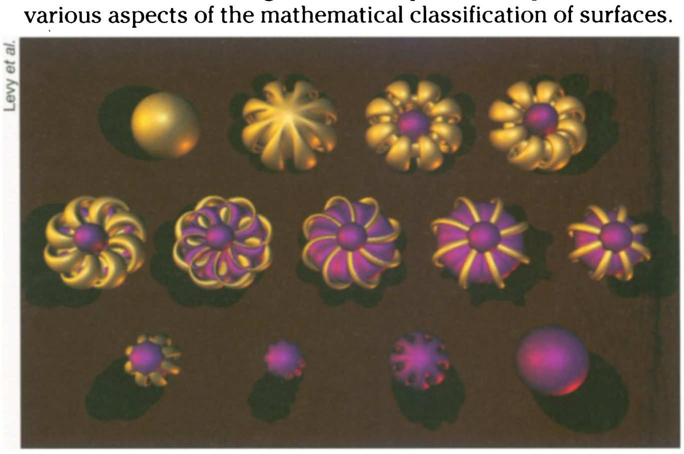
In subsequent years, mathematicians developed a number of different ways to visualize sphere eversion, gradually simplifying the steps to make it easier to follow the process (SN: 5/13/89, p.299; 6/20/92, p.404). The latest version comes from Silvio Levy, Delle Maxwell, and Tamara Munzner of the Geometry Center at the University of Minnesota in Minneapolis, who have created a dramatic computer animation that shows a sphere eversion in its full glory.

Levy and his coworkers based their visualization on a geometric technique developed by William P. Thurston of the Mathematical Sciences Research Institute in Berkeley, Calif., to help understand certain smooth curves and surfaces known as immersions. Thurston found it useful to imagine such curves and surfaces as springy, meaning they could be moved and bent at will. This strategy allowed him to introduce corrugations—wavy bends—to make these shapes extremely pliable, gaining insights into how immersions maintain their smoothness during transformations such as eversions.

In an eversion, a sphere's initially unwrinkled surface develops a symmetric set of bulges, or corrugations (see illustration). The poles push part way through each other, creating loops at the equator and revealing patches of the sphere's purple inside. The two polar caps then twist in opposite directions to undo the loops, and the equatorial region collapses and pushes through itself. Finally, the corrugations disappear, and the eversion is complete.

Why turn a sphere inside out? "The short answer is that it is a mathematical puzzle that is interesting and counterintuitive, and therefore, challenging to solve," Maxwell explains. This exercise and the corrugation technique also help to elucidate various aspects of the mathematical classification of surfaces.

Levy et al.



A golden sphere turns itself inside out to reveal its purple interior.