The Stability of Matter

Why matter neither collapses nor explodes

By RICHARD LIPKIN

Contemplate, for a moment, a glass of water.

Try to imagine the frenetic atomic activity within this seemingly tranquil liquid. Its trillions of molecules, each containing two hydrogen atoms and one oxygen atom, are continuously crashing into one another.

Each oxygen atom sports, on average, eight electrons whirring around eight tightly packed protons. In each hydrogen atom, a single electron hurtles around a central proton, which pulls it perilously inward.

Despite a strong electrical attraction, the negatively charged electrons do not fall into the positively charged protons; if they did, the resulting unstable mixture might explode. Confined to a relatively small area, pulled by gravity, and pressed by the atmosphere, the liquid teeters, in theory, on the brink of an atomic-bomb-sized explosion.

Nonetheless, the glass of water remains just that—a glass of water.

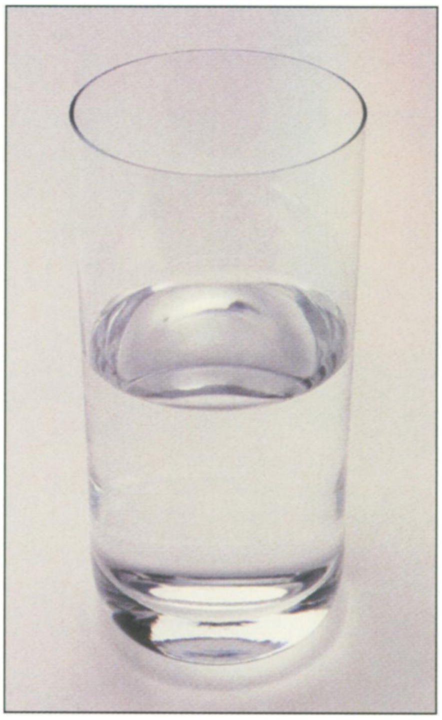
"In some poetic sense, it's a miracle," says Elliott H. Lieb, a mathematical physicist at Princeton University. "When you consider all of the physical forces at work in matter, even in something as simple as a glass of water, it's sort of miraculous that everything doesn't just collapse and then, releasing huge amounts of energy, blow up."

Most people regard this fact as too commonplace to think about, Lieb muses. "It seems so trivial. It's just the way things are. Except, when you think about the problem in detail, it really is sort of amazing."

Putting aside the seeming absurdity of an inquiry into why atoms don't collapse, the question raises serious mathematical issues in quantum mechanics and field theory. "Classical physics can't account for the fact that matter is stable and occupies space," says Mary Beth Ruskai, a mathematical physicist at the University of Massachusetts, Lowell. There's nothing in the classical laws of dynamics and electromagnetism to prevent orbiting electrons from spiraling inward to a nuclear doom.

"People interested in this question have had to rely on some very gnarly mathematics to get answers."

Among such people are Lieb and his collegues Jan Philip Solovej, also at Princeton, and Michael Loss of the Georgia Institute of Technology in Atlanta. In the Aug. 7 PHYSICAL REVIEW LETTERS, the three mathematical physicists tackle the question of why matter remains stable in an intense magnetic field—even one so powerful that it may exist only on a neutron star. To Lieb, whose involvement with this problem spans 28 years, the issue of matter's stability constitutes a "foundational problem" of modern physics.



Speculations about matter's stability hark back to the early days of quantum mechanics. In 1931, Austrian physicist Paul Ehrenfest observed that someone holding a piece of metal or stone should be "astonished that this quantity of matter should occupy so large a volume."

"Admittedly," Ehrenfest remarked, "the molecules are packed tightly together and likewise the atoms within each molecule. But why are the atoms themselves so big?" For an answer, he summoned a principle first stated by Viennese physicist Wolfgang Pauli: No two particles of the same kind can occupy the same quantum state at the same time. Pauli's principle means that electrons can't all fall into the lowest energy, smallest orbital around an atomic nucleus but have to fill successively larger orbitals.

"That is why atoms are so unnecessarily big, and why metal and stone are so bulky," Ehrenfest concluded.

Unfortunately, that intuitively reasonable explanation did not fully satisfy theoretical physicists, who wanted a more mathematically complete answer. So individually and together they picked and pawed at the problem, tinkered with the equations of quantum mechanics and field theories, and eventually, frustrated by a lack of clarity, put the issue to rest for about 30 years.

In the mid-1960s, physicist Andrew Lenard, now at Indiana University in Bloomington, posed a question: Is it possible to prove that matter is inherently stable? Freeman J. Dyson, a mathematical physicist at the Institute for Advanced Study in Princeton, N.J., became enchanted with the question.

Dyson and Lenard mulled the problem over for some time—egged on by the offer of a bottle of wine to anyone who could prove matter's inherent stability—then served up a paper in the August 1967 JOURNAL OF MATHEMATICAL PHYSICS.

Their report proved that with Pauli's exclusion principle, matter is stable; without it, matter collapses into a dense state, creating a situation in which "the assembly of any two macroscopic objects would release energy comparable to that of an atomic bomb."

In effect, Dyson and Lenard had mathematically demonstrated the truth of Ehrenfest's assertion by showing that Pauli's principle of exclusion remained essential to keep matter stable. To build their argument, however, the two researchers had to simplify their model of matter. They had to put aside temporarily some real-world phenomena, such as magnetic and gravitational fields, and consider only the subtle interactions of attractive and repulsive forces between charged particles within an atom.

Enter Lieb. He and physicist Walter Thirring of the University of Vienna found a simpler proof of matter's stability than the one put forth by Dyson and Lenard, using mathematical techniques that relied more heavily on physical intuition.

Working in concert with many collaborators, Lieb then turned his attention to examining more carefully those situations in which matter does, in fact, collapse. For example, under the influence of gravitational forces, which only attract and never repel, matter will collapse despite the effects of the Pauli exclusion principle—a situation that presents a problem only for an object as massive as a star.

In 1931, the late Nobel laureate Subrahmanyan Chandrasekhar predicted that stars with more than about 1.4 times the sun's mass would, on running out of nuclear fuel, collapse under their own gravity. More than half a century later, Lieb and H.-T. Yau, a mathematical physicist now at New York University's Courant Institute, proved that, starting from the basic principles of quantum mechanics, Chandrasekhar had indeed been correct.

On earth, however, matter—not subject to star-sized gravitational forces—behaves tamely.

“It's crazy,” says Loss. “Look around and everywhere we see matter extended, proportional to the number of particles. And yet atoms are incredibly tiny, each one being essentially a void.”

To get a deeper feeling for the problem, says Loss, bear in mind that matter is mostly space. Wispy electron clouds hover on the outskirts, relatively speaking, of an atom's tiny core, with vast distances extending between the nuclei of atoms in every molecule—even in a dense solid. And yet, if someone takes 2 quarts of water and pours them into one pitcher, that person will have a half gallon of drinkable beverage rather than, say, one quart of “compact” water that is twice as dense. The water molecules flow together, interact with one another, and respond to each other's electrical charges. But the volume and energy remain proportional to the amount of stuff, or number of atoms, present, says Loss.

“The amazing thing, from a scientific point of view, is that this phenomenon can be explained mathematically,” he continues. Starting with the equations of quantum mechanics, physicists can now show that those mathematical expressions must have stable solutions—implying that matter itself must remain stable.

The fact that someone can't simply push two stones together into one superdense rock falls in with commonsense notions of how solid objects behave on Earth. The situation changes remarkably when matter comes in contact with a very intense magnetic field, however. Such fields, on the order of 1 trillion gauss—or more than 1 million times stronger than the most extreme magnetic fields produced artificially in laboratories—could compress atoms into a state so dense that one would expect them to merge into a single explosive, subatomic stew.

In their Aug. 7 report, Lieb, Loss, and Solovej look at the stability of matter under the influence of an intense magnetic field. Such fields are not the “garden variety found on Earth,” says Loss, but ones trillions of times greater—at an intensity that might occur on a neutron star.

A neutron star, the result of a massive star's collapse, generates crushing magnetic and gravitational fields. Such stars have the power, for example, to squeeze Earth into an object the size of a marble. But such large fields simply do not exist on Earth. Moreover, to create fields of such intensity requires a huge amount of energy.

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>
> **— Elliot H. Lieb**

On Earth, when a person “bangs two stones together, for instance, the most one expects to see is a spark,” says Lieb. “Nothing dramatic happens.” But what if the energy contained in those two rocks were not packaged conveniently in trillions of separate atoms? Under extreme conditions, if those packages of mass and energy were to combine, matter could compress, collapse, and explode. Indeed, there are instances in nature—such as stellar explosions—where matter undergoes just such a sequence of events. Those circumstances, however, involve enormous gravitational forces, for greater than those present on Earth.

“There are two issues here, one weak, one strong,” Lieb says.

“The weaker issue is that matter is stable because there's a limit to the amount of energy that each atom can have.

“The stronger issue is that the energy of matter remains strictly proportional to the number of particles.” In other words, the amount of energy and the number of atomic particles increase at the same rate. This phenomenon of nature is rooted in certain immutable physical constants.

In essence, what Lieb and his colleagues have shown mathematically is that even under extreme conditions matter remains stable. As elucidated by the equations of quantum mechanics, which govern atomic behavior, matter tolerates the onslaught of an extremely large magnetic field without colllapsing.

Earlier research had shown that if, for some reason, a few of nature's critical physical constants ever grew too large, matter would subsequently become unstable. Now the three scientists show that because those constants are indeed sufficiently small, the energies of both the fields and the atomic particles balance out in such a way that, together, they remain stable.

“We have proved that under these conditions, the energy is not only finite but also proportional to the number of particles,” Lieb says, “as it should be in order to have the strong kind of stability.”

Moreover, by clarifying mathematically the conditions under which matter does remain stable, Lieb and his colleagues may help to shed light on those in which it doesn't, such as the conditions believed to be present in stars headed for collapse and explosion.

“This is not a trivial result,” says Ruskai. “Ironically, we need quantum mechanics to explain simple facts about ordinary life.

“If matter weren't stable, we wouldn't exist,” she continues. “And even if we could somehow exist under unstable conditions, it would be virtually impossible to function normally in everyday life. We'd be at risk of setting off nuclear-sized explosions every time we put a glass of water down on a table.”

“Elliott Lieb has become the central figure in this area of research,” says Dyson. “He's done most of the important work during the last 20 years. I'm delighted with the results on magnetic fields. In general, this issue gets right to the heart of the constitution of the universe.”

The subject is not closed by any means, says Dyson. As in every area of science, each solution poses new questions. “There are still open questions regarding the interactions of large numbers of atoms,” he notes.

“But in this field of research, Elliott is like the Moses who has shown us the way to the promised land,” Dyson adds. “Though he's not quite there yet.” ☐