A new formula for picking off pieces of pi

Members of the Ancient and Honorable Society of Pi Watchers have something new to gaze upon and ponder.

Mathematicians have discovered a surprisingly simple formula for computing digits of the number pi (π). Unlike previously known expressions, this one allows them to calculate isolated digits—say, the billionth digit of pi—without computing and keeping track of all the preceding numbers.

“It was something that people just didn’t think you could do,” says Peter B. Borwein of Simon Fraser University in Burnaby, British Columbia.

The only catch is that the formula works for binary, but not for decimal, digits. Thus, it’s possible to determine that the forty billionth binary digit of pi is 1, followed by 001001000001110. . . . But there’s no way to convert these numbers into decimal form without knowing all the binary digits that come before the string.

Borwein, Simon Fraser colleague Simon Plouffe, and their coworkers announced the discovery earlier this month by posting it on the Internet.

“It’s a curious finding,” says Helaman Ferguson of the Supercomputing Research Center in Bowie, Md., who has checked the result. “It’s quite surprising that this [formula] exists.”

Borwein has long been interested in finding efficient ways of computing pi, the ratio of a circle’s circumference to its diameter. In particular, he has focused on methods of performing the computation using only a small amount of computer memory.

\[
\pi = \sum_{n=1}^{\infty} \frac{1}{16^n} \left( \frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right)
\]

New formula that serves as the basis for computing isolated binary (or hexadecimal) digits of pi.

Working with Plouffe, Borwein identified certain types of mathematical expressions, or series, that would provide the necessary shortcut. An extensive search turned up suitable formulas for pi and other numerical constants, including log(2).

However, the answer that comes out of the expression for pi gives only hexadecimal (base 16) digits, which can be readily converted to binary. “The frustrating thing is that it doesn’t work in base 10 [for decimal digits],” Borwein remarks.

Borwein and his coworkers are still hoping to uncover an expression that gives the decimal digits of pi, but other mathematicians are pessimistic that such a formula will ever be found. Meanwhile, the researchers have been looking for related series to compute other mathematical constants, such as e and the square root of 2, but with limited success so far.

The existence of such an intriguing formula for computing isolated digits of pi may reveal something mathematical about the nature of the number itself. For example, mathematicians would like to prove that all the decimal digits occur in pi equally often.

“That would be the mathematical prize in all this,” Borwein notes. “But at the moment, I can’t see [our discovery] leading to a proof.”

Estrogen linked to adult asthma risk

Estrogen replacement therapy may increase a woman’s chance of developing asthma, say researchers from Harvard Medical School in Boston. They found that women taking post-menopausal hormones were 50 percent more likely to suffer adult-onset asthma than women who had never taken the hormones.

The finding could also help explain why severe adult-onset asthmatics afflicts more women than men.

“It is not clear how estrogen affects airways,” says study collaborator Frank E. Speizer. “But estrogen usage is one more risk factor to be considered in women with asthma.”

About 5 percent of the U.S. population suffers from asthma—a reversible but potentially life-threatening breathing disease characterized by coughing and wheezing. The disease displays a distinctive pattern of gender bias. Severe childhood asthma strikes more boys than girls, but in adulthood, asthma’s severe forms strike more women than men.

Scientists have some evidence that the reproductive hormone estrogen may play a pivotal role in these gender differences. In women, the number of asthma cases soars at the onset of puberty, when the ovaries begin to produce large amounts of estrogen. The incidence remains relatively high throughout the reproductive years. Moreover, animal studies indicate that estrogen can inhibit airflow through the lungs.

In light of estrogen’s tenuous link to asthma, the Harvard team compared asthma incidence to postmenopausal hormone use in 23,035 women partici-