

## A new formula for picking off pieces of pi

Members of the Ancient and Honorable Society of Pi Watchers have something new to gaze upon and ponder.

Mathematicians have discovered a surprisingly simple formula for computing digits of the number pi ( $\pi$ ). Unlike previously known expressions, this one allows them to calculate isolated digits—say, the billionth digit of pi—without computing and keeping track of all the preceding numbers.

"It was something that people just didn't think you could do," says Peter B. Borwein of Simon Fraser University in Burnaby, British Columbia.

The only catch is that the formula works for binary, but not for decimal, digits. Thus, it's possible to determine that the forty billionth binary digit of pi is 1, followed by 00100100001110. . . . But there's no way to convert these numbers into decimal form without knowing all the binary digits that come before the string.

Borwein, Simon Fraser colleague Simon Plouffe, and their coworkers announced the discovery earlier this month by posting it on the Internet.

"It's a curious finding," says Helaman Ferguson of the Supercomputing Research Center in Bowie, Md., who has checked the result. "It's quite surprising that this [formula] exists."

Borwein has long been interested in finding efficient ways of computing pi, the ratio of a circle's circumference to its diameter. In particular, he has focused on methods of performing the computation using only a small amount of computer memory.

$$\pi = \sum_{i=0}^{\infty} \frac{1}{16^i} \left( \frac{4}{8i+1} - \frac{2}{8i+4} - \frac{1}{8i+5} - \frac{1}{8i+6} \right)$$

*New formula that serves as the basis for computing isolated binary (or hexadecimal) digits of pi.*

## Estrogen linked to adult asthma risk

Estrogen replacement therapy may increase a woman's chance of developing asthma, say researchers from Harvard Medical School in Boston. They found that women taking postmenopausal hormones were 50 percent more likely to suffer adult-onset asthma than women who had never taken the hormones.

The finding could also help explain why severe adult-onset asthma afflicts more women than men.

"It is not clear how estrogen affects airways," says study collaborator Frank E. Speizer. "But estrogen usage is one more risk factor to be considered in women with asthma."

About 5 percent of the U.S. population suffers from asthma—a reversible but potentially life-threatening breathing disease characterized by coughing and wheezing. The disease displays a distinctive pattern of gender bias. Severe childhood asthma strikes more boys than girls, but in adulthood, asthma's severe forms strike more women than men.

Scientists have some evidence that the reproductive hormone estrogen may play a pivotal role in these gender differences. In women, the number of asthma cases soars at the onset of puberty, when the ovaries begin to produce large amounts of estrogen. The incidence remains relatively high throughout the reproductive years. Moreover, animal studies indicate that estrogen can inhibit airflow through the lungs.

In light of estrogen's tenuous link to asthma, the Harvard team compared asthma incidence to postmenopausal hormone use in 23,035 women partici-

pating in the Nurses' Health Study. As the researchers report in the October *AMERICAN JOURNAL OF RESPIRATORY AND CRITICAL CARE MEDICINE*, women who reported using hormone replacement therapy at any time were 50 percent more likely to suffer from asthma than women who had never used postmenopausal hormones. And asthma has twice the normal incidence among women who had used the hormones for 10 or more years.

"This is a significant increase in risk of asthma," says Speizer. "Physicians should consider it as one more risk factor, like a strong family history [of the disease]."

Susan Redline of Case Western Reserve University School of Medicine in Cleveland notes that the Harvard study is the first to look at the association between estrogen and asthma by monitoring a group of initially healthy women and comparing differences between those who subsequently suffer asthma and those who do not.

"I think the implication that hormone replacement therapy may modify [susceptibility to] asthma is a very intriguing observation," says Redline. "And it may offer a clue as to why women suffer more asthma than men."

Both Speizer and Redline maintain that more research into estrogen's effect on lung tissue is needed to firmly establish the link between estrogen and asthma. Until then, Speizer notes, estrogen's proven benefits in maintaining bone and cardiovascular health will outweigh any risk of asthma for most women. —L. Seachrist

Working with Plouffe, Borwein identified certain types of mathematical expressions, or series, that would provide the necessary shortcut. An extensive search turned up suitable formulas for pi and several other numerical constants, including  $\log(2)$ .

However, the answer that comes out of the expression for pi gives only hexadecimal (base 16) digits, which can be readily converted to binary. "The frustrating thing is that it doesn't work in base 10 [for decimal digits]," Borwein remarks.

Borwein and his coworkers are still hoping to uncover an expression that gives the decimal digits of pi, but other mathematicians are pessimistic that such a formula will ever be found. Meanwhile, the researchers have been looking for related series to compute other mathematical constants, such as  $e$  and the square root of 2, but with limited success so far.

The existence of such an intriguing formula for computing isolated digits of pi may reveal something mathematical about the nature of the number itself. For example, mathematicians would like to prove that all the decimal digits occur in pi equally often.

"That would be the mathematical prize in all this," Borwein notes. "But at the moment, I can't see [our discovery] leading to a proof."

On the decimal front, Yasumasa Kanada and his coworkers at the University of Tokyo have now computed pi to 4,294,960,000 digits, beating the old record (SN: 8/26/95, p.143). According to their calculations, the four billionth decimal digit of pi is 9, followed by 4375343. . . .

The researchers also show that in the first 4 billion digits, the number 6 appears most often (400,033,035 times) and 2 least often (399,965,405 times).

"We do the pi calculations from scratch because [this] is one of the best benchmark programs for testing the reliability of [our computers] and checking the correctness of calculations, programs, and algorithms," Kanada says. "To be a world record holder is a by-product."

If researchers ever find a decimal equivalent of the new formula for binary digits of pi, Kanada and others would be able to push their calculations much higher. Indeed, because such a formula would enable them to compute isolated clumps of digits, the task could be readily divided up among as many computers as necessary to get the desired result.

Members of Pi Watchers and others interested in the new formula for the rapid computation of pi can obtain additional information from Borwein's World Wide Web site at the following address: <http://www.cecm.sfu.ca/~pborwein/>. Anyone curious about 4 billion decimal digits of pi can check Kanada's site at: <http://www.cc.u-tokyo.ac.jp/>.

—I. Peterson